SCIENTIFIC MEMOIRS,

SELECTED FROM

THE TRANSACTIONS OF

FOREIGN ACADEMIES OF SCIENCE AND LEARNED SOCIETIES,

AND FROM

FOREIGN JOURNALS.

EDITED BY

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VOL. V.

LONDON:

PRINTED BY TAYLOR AND FRANCIS,
RED LION COURT, FLEET STREET.

SOLD BY LONGMAN, BROWN, GREEN, AND LONGMANS; J. RIDGWAY; FISHER, BROTHERS AND CO.; SIMPKIN, MARSHALL AND CO.; H. FELLOWES; S. HIGHLEY AND SON; WHITTAKER AND CO.; J. WEALE; AND J. B. BAILLIERE, LONDON:—AND BY A. AND C. BLACK, AND THOMAS CLARK, EDINBURGH; SMITH AND SON, GLASGOW; HODGES AND SMITH, DUBLIN:—DOBSON, PHILADELPHIA; AND WILEY AND PUTNAM, NEW YORK.

1852.

'Every translator ought to regard himself as a broker in the great intellectua traffic of the world and to consider it his business to promote the barter of the pro duce of mind For whatever people may say of the inadequacy of translation, it i and must ever be one of the most important and meritorious occupations in the grea commerce of the human race —Goethe, *Kunst und Alterthum*

PREFACE TO THE FIFTH VOLUME

THE present Part completes the Fifth Volume, and with it terminates the present Series of these Memoirs. This Volume has extended to an unusual length, owing to the impossibility of including in the four Numbers Professor Plateau's interesting Researches on the Figures of Equilibrium, which, as the first part had appeared in our Fourth Volume, it was desirable to complete in the present Series. Besides the interesting researches just alluded to, the present Volume contains, among other valuable papers, Fresnel's celebrated treatise on Double Refraction, Plucker's various papers on Diamagnetism, Weber's important memoir on the Measurement of Electro-dynamic Forces, and Knoblauch's Investigations on Radiant Heat, which have been pronounced by one of the most eminent philosophers of this country as "not to be surpassed for extent and accuracy of detail."

The management will now pass into other hands, and the work will receive that attention which of late I have been prevented from devoting to it by increasing years and other circumstances. It is hoped, that by a more regular publication, and the

separation of the Physical from the Natural History Sciences, the New Series may acquire a wider circulation, and accomplish more fully the original purpose of the work,—that of making the friends of Science in this country acquainted with the investigations and speculations of their fellow-labourers on the Continent.

In retiring from the field as Editor, it affords me much pleasure once more to acknowledge the valuable assistance which I have received from many kind friends since the commencement of this work, and especially from Colonel Sabine, the Rev. F R. Robinson, D.D., Professors Faraday, Wheatstone, Lloyd, Forbes, Miller, Baden Powell, Challis, Owen, Sir J. Lubbock, and the Rev. A. W. Hobson It now only remains for me to commend the New Series to the attention and support of the English scientific public.

August 28, 1852.

RICHARD TAYLOR.

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SCIENTIFIC MEMOIRS.

VOL. V.—PART XVII.

ARTICLE I.

Contributions to the Comparative Physiology of the Invertebrate Animals, being a Physiologico-Chemical Investigation. By Dr. CARL SCHMIDT.*

[Published as a separate work at Brunswick, 1845.]

INTRODUCTION.

If we survey the animal creation—if we discern in the infinite variety of external aspect the necessary result only of an internal structure—if, from numerous observations on the development of these forms, from original unity in the cell to their utmost complexity, we distinguish certain common morphological periods which we unite into *typical laws of formation*—lastly, if ascending from the simplest to the most compound, we endeavour to comprise forms corresponding to similar stages of development *as natural orders or families*; the question urges itself upon us, Does an analogous combination of the *chemical* go hand in hand with the homonymous development of the *morphological* elements or not? in short, what connexion is there here between form and composition, between the elementary constitution of matter and its external, mathematically definable and appropriate limitation in space? Although physiological chemistry has made such extraordinary progress within the last few years, nevertheless in this direction barely anything has been done; and deductions by analogy from existing observations on the Vertebrata are, as we shall see, inapplicable to the more simple structures of the Invertebrata from the Cephalopod down to the Monad. Lastly, although sound logic and natural philosophy

* Translated from the German by J. W. Griffith, M.D.

forbid us to base lofty edifices of theories and laws upon but few observations, on the other hand it compels us not to be satisfied, like the mason, with the mere collection of the building-stones, and to lose sight, over accumulating details, of a higher object, but at certain stages to look around, to arrange the results obtained, to compare them with known phænomena, and thus to extend our intellectual horizon.

The present treatise forms an attempt at this, viz. to test experimentally Reil's celebrated position, "that the phænomena of individual life are the necessary result of form and composition," to introduce a new element, *comparative chemistry*, together with *comparative anatomy*, into the physics of organized beings, and thus to obtain new points of support for a rational philosophy of nature. Of course the real value of rough and minute comparative anatomy, especially the latter, ought not to be depreciated. Where it appertains to the subject, I have considered it necessary specially to detail the researches of others as well as myself on this point.

I have, however, avoided unnecessary anatomical detail. My desire was merely to state my own observations, and especially to show how *comparative anatomy* and *chemistry* mutually support each other, and must go hand in hand in order to form a *physiology* of the animal kingdom, which, for its part again, can then alone suffice to satisfy the higher mental claims when in combination with *psychology* and *speculative philosophy* generally. Unfortunately but little has been effected in regard to the latter; in fact, contrary to the broad path of empirical investigation, it constantly recedes from us; of course abstracted from the unfounded phantasies of the incompetent followers of the *youthful* Schelling, which being now out of date, merely deserve mention as forming historical records for our future warning

I have first given a general sketch, then proceed to the details, and finally recur to all that has been previously stated, where I shall attempt to develope some interesting positions in general physiology based upon them.

I. *General View.*

We so often find in the animal and vegetable kingdom a remarkable connexion between matter and form, *i. e.* a peculiar form and arrangement of the *morphological* elements so frequently corresponds to a *definite* combination of the *chemical* ones, that

we are compelled to regard this connexion as *essential*; and we might even now, in a modern garb, yield the first place in our experimental sciences to the ingenious ideas with which Reil formerly commenced his Archives.

The greater the importance of an organ, the more does the variety in the combination of its chemical elements disappear. The nervous system, i. e. the primitive fibres and the ganglion cells, does not appear to present any chemical differences; however, nothing certain can be based upon mere microscopic reactions. The muscular system, i. e. the primitive bundle (both the *smooth* and the *transversely striated*), exhibit the same composition. In the vascular system, i. e. the walls of the tubes, we also find no difference—both belonging to the proteine compounds, or being nearly related to them. The intestinal canal with its appendages forms the transition to the cutaneous system; the epithelia follow next, horny plates, and certain membranes which are situated between the epithelia and the muscular laminæ, or rather which themselves perform the functions of epithelia, exhibit the same composition; whilst the appended glands (the pancreas, liver and salivary glands), excluding their separate secretions, consist of proteine compounds. The same applies to the respiratory system. The external tunics of the laminæ of the gills, as also the tracheæ, correspond to the cutaneous system. Lastly, the latter, i. e. the teguments destined as a protection from external influences, exhibit the utmost variety in form and combination. In *the highest grade of the animal kingdom* this system consists of proteine compounds; *it is purely animal*; in the *intermediate* ones it is *combined* with the cutaneous system of *plants*; finally, in the *lowest* it is *identical with the latter*. Hence the *Mollusca* stand *higher* than the *Articulata*, the *latter* occupy the *intermediate station*, the *Zoophytes*, in the true sense of the word, are *plant animals*.

The transition stages are all extremely interesting. Thus, in the *Cirripedia*, from the *cirri* alone they should, in a histologico-chemical point of view, be arranged among the *Articulata* (*Crustacea*), whilst from the shells they should be placed with the *Bivalves*. Again, the *Ascidia*, which form the transition from the Mollusca to the Zoophytes, are arranged, from the delicate structure and the chemical properties of their tunics, among those *animals* which have *a vegetable mantle*. Lastly, also the most simple forms of the animal world (*Bacillaria*) form trans-

itions to the primary vegetable-cell (mother of vinegar, yeast-cell), among which, with our scholastic definition of the notion of animal and plant, we fall into a most remarkable difficulty, for *there are organic beings which combine the organic re- and decomposing forces* (Stoffwechsel) *and the chemical constituents of the plant with the locomotion of the animal!*

II. *Special Observations and Deductions*

A. *Nervous System.*

As is well known, we find a great uniformity in the minute structure of the nervous elements of the Vertebrata, and, judging from microscopic reactions, also in their chemical composition. In all we find ganglionic bodies and primitive tubes; these when fresh are filled with homogeneous, highly refractive contents, which after death coagulate and become granular. Alkalies make the external outline of the ganglion-cells, also that of the primitive fibre (cell-wall), swell, become pale, transparent, then disappear (solution); the finely-granular contents become converted into large highly refractive drops, which are unchanged by acids and alkalies, and are dissolved by æther; acetic acid acts in a similar manner, but does not cause true solution, which points out that the wall of the primitive tube, as also that of the ganglion-cell, is entirely composed of the cellular tissue of the adjacent substance, whilst fat, in a peculiar state of combination with albumen, forms the fluid contents.

If we regard the difference between the ganglion-cell and the primitive fibre as an *essential* morphological fundamental condition of the mechanism of the nervous system generally, as the originator and conductor of an active system of forces (nervous agency, nervous principle, &c.), we should naturally find it wherever the effects of this system are perceptible; and, in fact, we do find it in the animal series generally, so far as we are able to trace these effects*. It is *à priori* extremely probable that this peculiar system of forces requires a peculiar material substratum, in addition to a structurally distinct one, in order to be apparent in its actions, consequently to be perceptible to us.

* Valentin, Course and Terminations of the Nerves, tab. 8, and Wagner, *Handwörterbuch*, p. 700 (Cray-fish). The latter author and Henle, Müller's *Archiv*, 1840, p. 318 (*Distoma* and *Echinorhynchus*). Henle, *Allgemeine Anatomie*, p. 773. Ehrenberg, Description of a remarkable and hitherto unknown Structure of the Brain, tab. 7.

Chemical analysis * has proved the existence of the former, and the microscope that of the latter in vertebrate animals, the extraordinary quantity of peculiar fat and the large amount of phosphoric acid are not found elsewhere in the animal body. By the above reactions I satisfied myself in the œsophageal ring of Anodonta, *Helix* (*pomatia*) and *Limnæus* (*stagnalis*) as representatives of the Mollusca, in the Craw-fish, Cockchafer and geometrical spider (*Epeira diadema*) of the Articulata, of the identity of the chemical composition of the nervous elements in these different families, so that I consider the conclusion of the chemical identity of the nervous system in the animal series at least, as not too hazardous; that nerves, which can only with great difficulty be isolated sufficiently for microscopic examination, cannot be subjected to elementary analysis, is self-evident.

B. *Muscular System*

As we know, two morphological muscular elements are distinguished in the vertebrate animals,—transversely striated primitive bundles and smooth fibres, which moreover exhibit numerous intermediate stages, as in the heart. The question of the existence of a chemical difference corresponding to this morphological one, has as yet neither been suggested nor experimentally determined; the latter would also be effected with very great difficulty, especially with the active assimilative changes in the higher vertebrate animals, the intermediate products of which adhere very intimately to the morphological elements. We find fewer difficulties in the more simple organization of the Invertebrata. The Articulata have transversely striated, and the Mollusca smooth muscular elements †; nevertheless the development of the two ‡ exhibits great uniformity; in fact, in the young stages of the Crustacea we find plane primitive fibres, which subsequently acquire the transversely striated aspect. The next question was, whether the same uniformity occurred as regards their composition. I therefore separated the large thoracic muscles of the Cockchafer, the muscles of the posterior abdominal segments of the Craw-fish, and the adductor muscles of Anodonta,

* Fremy, *l'Institut*, No. 311, p. 1[illegible].
† R. Wagner, Muller's *Archiv*, 183[illegible], p. 315.
‡ For the Vertebrata see Valentin, History of Development, p. [illegible]07, and Muller's *Archiv*, 1840, p. 138; Schwann, *Mikroskopische Untersuchungen*, p. 166; Henle, *Allgemeine Anatomie*, p. 600. In the Cephalopoda, A. Kölliker, *Entwickelungsgeschichte der Cephalopoden*, Zurich, 1844, p. 70.

carefully from the abdomen, sternum, large branches of nerves, &c., exhausted them of the nutritive fluid by macera in water, and of the fat in the minute nervous twigs by alc and æther, the residue necessarily constituted the pure prim fibre. When dried at 266° F., and burnt in oxygen gas small platinum vessel, it yielded as follows[*] :—

a. *Craw-fish.*

Determination of the Ash.

0·360 of the substance gave 0·0115 ash (pure phospha lime) = 3 194 per cent.

Nitrogen.

I. 0·349 of substance gave 0 819 ammonio-chloride of tinum = 15 22 per cent nitrogen.

II 0 3845 of substance gave 0·915 ammonio-chloride of tinum = 15·34 per cent. nitrogen

Combustion.

a 0·7525 of substance gave 1·391 carbonic acid and 0·

water $= \left\{ \begin{array}{ll} \text{carbon . .} & 52\ 14 \\ \text{hydrogen. .} & 7{\cdot}10 \end{array} \right\}$ per cent.

b. 0 7165 of substance gave 1 331 carbonic acid and 0·

water $= \left\{ \begin{array}{ll} \text{carbon .} & 52\ 39 \\ \text{hydrogen. .} & 7\ 18 \end{array} \right\}$ per cent.

b *Cockchafer*

Determination of the Ash.

0 2435 substance gave 0 008 ash (phosphate of lime w little phosphate of magnesia and a trace of oxide of iron) = 3 per cent.

Nitrogen.

I. 0·378 substance gave 0·885 ammonio-chloride of plat = 15 20 per cent. of nitrogen.

II. 0·367 substance gave 0·867 ammonio-chloride of plat = 15·34 per cent. of nitrogen.

Combustion.

a. 0·720 substance gave 1·335 carbonic acid and 0·4515

$= \left\{ \begin{array}{ll} \text{carbon . . .} & 52\ 35 \\ \text{hydrogen .} & 7\ 20 \end{array} \right\}$ per cent

* The nitrogen was estimated by Varrentrapp and Will's method Th culations, in this as in all the following analyses, are made after the ded of the ash

b. 0.6123 substance gave 1.1695 carbonic acid and 0.3805 water $= \left\{ \begin{array}{lr} \text{carbon} & 52.08 \\ \text{hydrogen} & 7.14 \end{array} \right\}$ per cent.

c. *Anodonta*.

Determination of the Ash.

0.402 substance gave 0.0075 ash (pure phosphate of lime) = 1.866 per cent.

Nitrogen.

0.3555 substance gave 0.852 ammonio-chloride of platinum = 15.33 per cent. nitrogen.

Combustion.

a. 0.6178 substance gave 1.220 carbonic acid and 0.420 water $= \left\{ \begin{array}{lr} \text{carbon} & 52.10 \\ \text{hydrogen} & 7.31 \end{array} \right\}$ per cent.

b. 0.593 substance gave 1.119 carbonic acid and 0.380 water $= \left\{ \begin{array}{lr} \text{carbon} & 52.50 \\ \text{hydrogen} & 7.26 \end{array} \right\}$ per cent.

We thus have,—

	Primitive muscular bundle.					
	Transversely striated.				Smooth.	
	A. *Astacus fluviatilis.*		B. *Melolontha vulgaris.*		C. *Anodonta cygnea.*	
	1. *a.*	*b.*	1. *a.*		1. *a.*	2. *b.*
Carbon*	52.11	52.39	52.35	52.08	52.10	52.50
Hydrogen	7.10	7.18	7.20	7.14	7.31	7.26
Nitrogen	15.22	15.11	15.20	15.31	15.33	

We thus see that, in these representatives of the Articulata and Mollusca, there exists a uniform composition in those organic elements, through the medium of which spontaneous motion is effected. Among the Zoophytes the lowest form of the animal world at my disposal was *Frustulia salina*, Ehrbg.†, to which I shall hereafter minutely refer in the consideration of the cutaneous system. I found in it 15 per cent. of a substance resembling proteine and abounding in nitrogen, and which in its reactions

* The equivalent of carbon being = 75.1, that of hydrogen = 12.5, and nitrogen = 17 (from Erdmann and Marchand's determination), according to which the logarithm for calculating the nitrogen from the ammonio-chloride of platinum formed (which must be added to the logarithm of the latter) is = 79780 1.

† Ehrenberg, *Die Infusionsthierchen als vollkommene Organismen*. Berlin, 1838, p. 232. Ehrenberg saw a broad and thick foot, which served for locomotion, project from the carapace in the closely allied *Navicula fulva*.—l. c. ii. 17, 178.

(solubility after swelling, and becoming transparent in alkalies, the same phænomena without subsequent solution in acetic acid, and the production of a lemon-yellow colour on being heated with nitric acid) agreed with the muscular elements. I shall subsequently state how elementary analysis and the estimation of the nitrogen were rendered impracticable.

At all events, I think I have rendered the chemical identity of those organic elements which effect spontaneous motion, hence the purely vital functions of the animal, at least extremely *probable*, although, as in every case, many more examinations are requisite to *establish* it. If with these results we compare the composition of fibrine, albumen and caseine, as found in the numerous experiments made under the direction of Liebig in Giessen*, and by Mulder†, we find a remarkable difference. All these secondary elementary substances of the animal organism contain 55 per cent. of carbon and somewhat more nitrogen. My own analyses throughout have been performed on such considerable quantities of anatomically-pure material, and the application of the platinum vessel with the current of oxygen ensured both an accurate determination of the hydrogen, and so sure a control over the perfect combustion of the carbon, and finally I have made them with such care, that I place full confidence in them, nevertheless I obtained only 52·2 to 52·5 per cent. of carbon, and 15·2 to 15·4 per cent. of nitrogen. As we know, Scherer‡ has rendered it probable that the chemico-physical difference in the modifications of the fibrine in chyle from that in arterial and venous blood depend upon a definite compound of the albumen with oxygen in some form, so that the arterial fibrine which is relatively most consolidated yielded the largest amount of oxygen with the same relative proportion of the carbon to the nitrogen. Playfair and Boekmann's§ analyses, the only ones which have been instituted on muscular fibre, had quite a different object in view, in which histological purity of the substance was not requisite, their purpose was the comparison of the *entire* muscle wit

* The analytical results are in Wohler and Liebig's *Annalen*, vol xl recommend Liebig's exposition of these relations, especially of the *true* import elementary analysis and the value of their expressions in equivalent formul (see Animal Chemistry), to the consideration of those calculators of the atom in tubercle of the liver, brain, lungs and abdomen, and other such absurdities

† *Natuur en Scheikundig Archief*, for several years after 1836

‡ Wohler and Liebig's *Annalen*, vol xl part 1.

§ Liebig's Animal Chemistry, Analytical Proofs

the *entire* blood. That fresh fibrine absorbs oxygen with extraordinary ease has been experimentally shown by Scherer: the analyses I have adduced lead to the assumption of the occurrence of a similar metamorphosis in the organism, whence the pure primitive muscular fibre would appear as the medium of transition of albumen through all the modifications of fibrine into chondrine from constant absorption of oxygen (perhaps partly with hydrogen, in the proportion for forming water). Thus we have—

	Protein	Muscular fibre	Chondrine
Carbon	55	52 3	50 5
Hydrogen	7	7 1	6 8
Nitrogen	16	15 3	14 5

I shall return to this point in the subsequent consideration of the cutaneous system.

C. *Reproductive Organs*

In the ovum we have differentials in magnitude of the future organism; hence we ought also to find in it the sum of the fundamental constituents of the latter, and with the exception of phosphate of lime these do not present any essential differences; but even the latter, however, is never perfectly absent. As is known we are indebted to the investigations of R. Wagner * for the knowledge of the uniform structure of the primitive ovum in the animal series; an identical, or at least very similar grouping of the chemical elements appears to correspond to this. The occurrence of true crystals of stearine, as observed by Vogt † in *Alytes*, appears to stand isolated. The unimpregnated ova of *Astacus* (*fluviatilis*), *Melolontha* (*vulgaris*), *Musca* (*vomitoria*), *Epeira* (*diadema*), and *Tegenaria* (*domestica*), as representatives of the Articulata; *Unio* (*pictorum*), *Anodonta* (*cygnea*), *Helix* (*pomatia* and *nemoralis*), *Limax* (*ater*), and *Limnæus* (*stagnalis*), from the series of Mollusca, exhibited similar reactions, which were as follows:—Acetic acid caused the chorion and the vitelline membrane to swell without effecting their true solution; potash acted in the same manner; the contents at the same time swelled to such an extent as to burst the softened membranes, and numerous drops of fat came into view, the former being dissolved, the drops of

* [illegible] Lips 1836. *Beit[illegible] G[illegible]* [illegible] *Abhandl d[illegible]* [illegible] vol ii 1837

† [illegible] Solothurn 181[illegible]

oil were readily taken up by æther. I was fortunate enough to isolate the germinal vesicle in *Anodonta*, it completely disappeared when treated with potash, excepting some drops of fat in the situation of the germinal spot, the contents of the germinal vesicle were coagulated by alcohol or nitric acid. Hence the chorion and vitelline membrane would consist of proteine compounds, the contents of the yolk abounding in fluid fat; the germinal vesicle, with its transparent contents, consists of albuminates, the germinal spot would consist of one or more vesicles of fat*. On incineration, they all left comparatively large quantities of ash, consisting principally of phosphate of lime.

If we add these experiments as a slight contribution to Ascherson's† important observations on the formation of membrane around globules of fat in albuminous fluids, and above all to Wagner's profound researches in this most difficult branch of the history of reproduction, the view of the latter upon the formation and import of the individual parts of the ovum becomes more deeply impressed upon our conviction

Cannot then the earliest formation of the ovum-cell, in accordance with the observations which have been made, be explained by known mechanico-chemical laws? Wherever heterogeneous bodies come into contact, condensation occurs at the surface of contact; the fact has been proved in the case of coercible gases and fluids. If now a fluid, in consequence of its chemical constitution, possesses the property of becoming comparatively solid even by slight condensation, every drop of a heterogeneous fluid which gets into it becomes surrounded on all sides by a condensed mass, *i. e.* forms the contents of a cell. That the required property is probably possessed by a combination or mixture of albumen with phosphate of lime, I hope subsequently to prove, but that fat and albumen are extremely heterogeneous bodies is evident. In the glandular tubules of the ovary this fluid (albumen + phosphate of lime) exists; each globule of fat which reaches it condenses a portion to form the membrane of a cell By the separation of solid constituents, the remaining albuminous solution must become more dilute, an effort at the restoration of equilibrium, endosmose, must occur, and a portion of fluid must get between the oil-globule and the

* Most of these reactions have already been given by Wagner (*Lehrb d Physiologie*, 1843, S 40)

† Muller's *Archiv*, 1840, S 44 *et seq*

membrane which has just been condensed and which closely surrounds it, if we denominate the oil globule the *germinal spot* the vesicle thus formed corresponds to the *germinal vesicle*. If we place a solid body in a fluid filled with suspended molecules the latter are rapidly deposited upon it. this phenomenon can be readily observed in any liquid in which we suspend a little powdered chalk or wood and immerse a piece of chalk or wood. We find similar molecules in the tubules of the ovary, but they are innumerable and consist of oil globules surrounded with condensed albuminous coatings. When these are deposited around the newly formed *germinal vesicle*, we have the *yolk*, which, after the deposition of the fatty molecules present, is surrounded by new albuminous layers the vitelline membrane and chorion, just as a crystal in a saline solution.

I consider that the formation of the *ovum cell as such* may be arranged among known physico chemical processes, but it does not *then* possess vitality. the totality of the motor phenomena, which we call life, results primarily from that peculiar combination of the above mentioned with new masses and forces which are in motion, by the addition of a new system of the same kind,—the spermatic fluid in impregnation.

Lastly, if we study the yellowish masses on both sides of the siliceous carapace in the gelatinous envelope of *Frustulia salina*, which were pointed out by Ehrenberg as ovaries, we find the interesting circumstance that *elementary analysis* aids us when our present optical resources (magnifying 1200 diameters[1]) carry us no further, i e that with the assistance of the former we can ascertain the physiological nature of organs, the isolation and further anatomical tracing of which would be impossible even to an Ehrenberg, with his wonderful skill in the vivisection of microscopic objects. Thus these yellowish masses are in reality only fat, they disappear on treatment with ether, and the latter contains considerable masses of a brownish fat in solution. The whole process of solution can be directly traced under the microscope in such specimens as have been previously placed in alcohol to remove the water. If we observe in the same manner the action of potash, we see that the remaining mass (proteine substance, probably the food observed by Ehrenberg), which fills the siliceous carapace, dissolves, whilst the yellow masses continue to run together assuming a spherical aspect, and finally issuing from the apertures of the siliceous carapace in the form

of large oily globules. This fat is fluid, of the consistence of human fat, saponifiable by alkalies, and when heated is decomposed, evolving the characteristic odour of acroleine and a fatty acid (thus containing oxide of glyceryle). The fatty acid, when separated from the potash-soap, formed a brownish oil, which when in solution reddened litmus, and yielded on analysis as follows:—

0·413 of the substance gave 1·150 carbonic acid and 0·4315 water; hence,—

Carbon . . .	76·03 per cent.
Hydrogen . . .	11·61 per cent.

i. e. very nearly the composition of oleic acid, so that there is no further doubt about its nature. In the simplest animal forms in which we are able to recognise the ovary anatomically with certainty, it is the only organ in which such abundance of fat occurs accumulated in one spot, we have therefore every reason to regard Ehrenberg's *idea* as a *well-founded observation*. Further remarks on the quantity of this fat (15 per cent.) in proportion to the weight of the coats, the muscular substance and the siliceous carapace, which is determinable with accuracy, also on the manner of ascertaining it, will be found in connexion with the cutaneous system.

D. *Vascular System.*

The walls of the tubular conductors, as also the pulsating central organ, appear, as regards chemical and in many respects histological identity (layers of longitudinal and annular fibres), to belong to the muscular system, and the former systems generally. The heart and its auricles, with the largest vascular trunks of *Unio*, *Anodonta*, the Craw-fish, as also the dorsal vessel of *Squilla* (*mantis*) and of *Scolopendra* (*morsitans*), reacted in the same manner as regards solubility in alkalies, mere expansion and acquiring transparency in acetic acid, burning with the disagreeable odour of the albuminates, and becoming yellow by nitric acid; however, I could not make any elementary analysis for want of sufficient substance.

E. *Respiratory System.*

In the animal series, as is known, to allow the exchange of the gaseous products of the alteration of matter with the oxygen of the atmosphere, we have external or internal sacs, in which, on

the principle of the greatest possible extent of surface numerous anastomosing canals containing the formative fluid run. The latter belong to the vascular system, whilst the former belong to the cutaneous system, of the chemical composition of which they partake. This relation is naturally most striking when the contrast of the cutaneous system in general is most distinctly perceptible, as in the Articulata. The tracheal system of insects, as also that of the tracheal spiders, the respiratory sacs of the pulmonary spiders and the gills of the Crustacea, consist of *Chitine* according to investigations made on the cock chafer, the common house fly and *Ateuchus sacer* among insects, the craw fish and crabs among Crustacea, and *Phalangium* (*parietinum*) and *Epeira* (*diadema*) as tracheal and pulmonary spiders. This *Chitine* is a peculiar substance resembling woody fibre but containing nitrogen and forming the cutaneous skeleton of these animals, further details of which will be given when treating of the latter. Its insolubility in potash, even after continued boiling, is highly characteristic of this substance; the organs under consideration may thus be easily isolated and prepared for microscopic analysis. The chitinous tissue does not exhibit the least change, and the elegant ramifications of the trachea especially may thus be exquisitely separated and examined.

1. *Organs of Digestion*

The substance of the alimentary canal, the other tube which is in direct communication with the external world, appears to belong to the cutaneous system. This conclusion is based upon the examination of the stomach of the Craw fish. It consists of an external thin transparent, difficultly separated mucous membrane and an internal transparent membrane which unites the separate parts of the complicated framework of the stomach, and is covered with hairs of various forms. The latter membrane is cast annually; the former produces the new stomach, or rather the new epithelium.

Von Baer*, with his usual acuteness of observation and clearness of description, first examined it, and at the same time refuted numerous fables regarding the change of the stomach of the Craw fish which had been current since the period at which Van Helmont† and Geoffroy‡ (the younger of the two elder)

* Müller's *Archiv* 1834 p. 510 *et seq.*
† *Lithiasis* cap. vii.
‡ *Mém. de l'Acad. roy. des Sciences* 1709 p. 309

lived Oesterlen * subsequently gave a full description and terminology. The last-mentioned transparent, and in other respects structureless membrane, with its manifold appendages (teeth, scales, hairs, &c.), forms the innermost layer of the intestinal tube; upon this lies the reproducing mucous membrane which we have mentioned, and lastly upon the latter, from the pylorus to the anus, layers of transverse and longitudinal smooth muscular fibres. Glands, cylindrical epithelium and such like, cannot be detected upon it; with difficulty we recognise slight indications of hexagonal cells, which enable us to determine its mode of development. The whole of this internal apparatus consists of chitine, that peculiar substance which forms the cutaneous system of the same animal, and consequently, of which are composed all those parts which are annually cast off and must be reproduced. Probably the same holds good with all Crustacea and perhaps with all the Articulata; I made the observation too late in the year to be able to test its applicability to other families and genera.

The intestinal canal of the Mollusca, as also their cutaneous system, resembles muscle. There is nothing remarkable in *Unio*, *Helix*, *Limnæus* or *Limax*; the smooth elements of the layers of longitudinal and transverse fibres are narrower than those of the adductor muscle. The intestine of *Ascidia mammillata* exhibited the same relations.

G. *Cutaneous System*

The external coverings of the invertebrate animals exhibit extraordinary variety in their minute structure, as also in their chemical composition. We here meet with phænomena which no one would *à priori* expect, and which, when combined with others, overthrow any remaining chemico-physiological distinction between animals and plants. We shall consider the chemical relations according to the great natural orders, which, on the other hand, are characterized by those relations.

1. *Articulata.*

We cannot make any use of the older observations in this department; they were adapted to the existing state of knowledge at that time, but are now merely of historical interest. Odier's †

* Muller's *Archiv*, 1840, p. 387 *et seq.*
† *Mémoire de la Société d'Histoire Naturelle*, tom. i. p. 29 *et seq.*

investigation upon the elytrum and horny tegument of the Cockchafer, which in correctness of observation and modesty of style excels many of those of his followers, forms an exception. He first found that the parts we have mentioned, after treatment with water, alcohol and potash, left a colourless transparent substance retaining the original form, which being characterized by the essential reactions of woody fibre, was considered by him, in consequence of a readily explicable mistake, as free from nitrogen, and which as a peculiar modification of it, he designated by the name of Chitine.

In 1843 Lassaigne* renewed the investigation; he asserted that he had detected this substance in the skin of the silk worm and spider, and having repeated merely the same reactions, he drew up such a magniloquent account as to render it almost doubtful whether he or Odier was the discoverer, and called it *Entomoderm*, as the former name did not appear to him sufficiently suitable. He however found nitrogen in it.

It is clear that so long as we are unacquainted with the elementary composition and the true chemical relations of this substance, we know *nothing* about it; and also that we cannot have the slightest idea of its physiological import, nor of the method of its formation from those animal and vegetable substances, &c. with which we are acquainted; much less ought we to *assert* anything of the kind. This defect, which we could not attribute to Odier in 1821, renders Lassaigne's statements at present useless.

Not long since Payen† resumed this subject in a notice; he estimated the amount of nitrogen in this substance in comparison with the cellular membrane of plants; it was 8.935 per cent in the shell of the craw fish, and 9.05 in the silk worm.

Lastly, there is an analysis by Children and Daniell‡, which like Payen's is also incorrect; they obtained,—

Carbon	46.08
Hydrogen	5.96
Nitrogen	10.29

I found Odier's statements almost entirely correct. The elytra consist of the true wing plates and the muscles by which they are moved; the vessels of the latter of course contain blood,

* Comptes Rendus, tom. xvi. p. 1087.
† Comptes Rendus, tom. xvii. p. 7.
‡ Todd, Cyclopædia of Anatomy and Physiology, vol. ii. p. 88.

which held the substances soluble in water. The recently prepared ash does not however effervesce with acids; it contains soda and phosphoric acid, as proved by the yellow precipitate with salts of silver. The alkaline reaction is thus understood, and the effervescence observed by Odier is explained by the easy decomposition of the tribasic phosphates. The substances soluble in potash consist of the proteine of the above-mentioned muscles and a brown resinous matter which unites the fibrous tissue.

So much for the illustration of the historical points. I shall now proceed to my own observations.

At first I made use of the Cockchafer; the histological elements of the tegument and elytra are the same, it is however difficult to decide positively as to this point before having recourse to the potash; we find several superimposed fibrous membranes, which become distinct on disintegration; their upper surface, which is especially impregnated with the resinous brown colouring matter, and covered with a thin epithelium consisting of six-sided cells, exhibits cylindrical depressions placed at regular distances, from which simple elongated cells, "hairs," arise.

A portion of the elytrum was exhausted successively with water, alcohol and æther, and lastly with a tolerably concentrated solution of potash with heat, until it appeared colourless and transparent; during the last operation a little ammonia was evolved, evidently from a small portion of the muscles of the wings remaining. I examined it microscopically; the epithelium, hairs and their cylindrical depressions were unaltered, the brown resinous substance had disappeared; several layers of sharply defined muscular fibres were perceived superimposed in such a manner, that a layer of transverse fibres was placed over each layer of longitudinal ones, and so on, so that the whole, with the hair-cells which remained unaltered in the uppermost layers, presented the appearance of a regular and elegant trellis-work. H. Meyer* has fully described this structure in *Lucanus cervus*; his illustration applies to *Melolontha* and the elytra of most of the beetles, so that I consider further description of their form (which is very uninteresting without the history of development) as superfluous.

The brown colouring matter with which the fibrous layers are impregnated and by which they are united, is precipitable from

* Muller's *Archiv*, 1842, p. 12–16

its alkaline solution by acids is insoluble in water, alcohol and æther, amorphous and of a resinous aspect; it requires separate examination, which would be especially interesting in regard to the possibility of its metamorphosis into the other colouring matters of beetles. As regards the true chitine, *i. e.* the colourless transparent residue of the elytra which is insoluble in water, alcohol, æther and potash, the sharp outline of its histological elements and especially the perfect preservation of the hair cells, which can easily be confirmed by admeasurement, are in favour of this substance being a compound of carbon, hydrogen, nitrogen and oxygen. This chitine is soluble without change of colour in concentrated muriatic or nitric acid, and may be kept boiling for some days in the strongest solution of potash without undergoing any change. When heated with water to 536° F. in hermetically sealed metallic tubes, it becomes brown and brittle; the water, however, does not contain a trace in solution, and the minute structure when magnified appears unchanged. Strong solution of potash with increase of the heat to 110° F. in strong glass tubes yields the same result; water at lower temperatures of course exerts as little action. When immersed in concentrated sulphuric acid it swells and dissolves without any change of colour; the solution gradually becomes coloured, and in twenty-four hours we obtain a fluid which is coloured black by a slight but extremely fine powder in a state of suspension, is of a pungent odour, and ammonia can be detected in it by excess of potash or chloride of platinum, whilst the fluid obtained on distillation, when treated with sulphuric acid and alcohol, evolves acetic æther; peroxide of mercury is dissolved in it without reduction, forming a persalt of mercury, and it has the odour of acetic acid, in fact, it contains a considerable quantity of this acid. It does not, however, evolve any sulphurous acid; neither does it contain any formic acid, as is evident from its action upon peroxide of mercury, nor could the formation of the latter be detected even after exposure to the air for fourteen days. Submitted to destructive distillation, water, acetic acid and acetate of ammonia pass over, and lastly an empyreumatic oil, but in comparatively small quantity; the remaining cinder so accurately preserves the form of the elytra, that we can obtain the entire beetles reduced to a cinder either in the walking, running or flying attitude, and without the least structural alteration, by drying and properly laying out the colourless and transparent chitine skeletons ob-

tained by means of potash. The peculiarity of the products of distillation caused Odier to overlook the nitrogen present, as this was evolved as acetate of ammonia mixed with free acetic acid, no alteration in the colour of the red litmus paper could occur.

However, it is easily seen that this substance is principally shown to be peculiar by negative characters; the cortical substance of the hair, skin, nails and epidermic scales of the Vertebrata are soluble with difficulty in potash, and its peculiarity could only be considered as proved when all the analyses of the substance obtained from different organs and animals agreed. I therefore subjected the entire tegument of the Cockchafer, after removing the intestinal tract, as also the tegument and elytra of *Ateuchus sacer*, to the same treatment; by this means we should moreover ascertain whether the winged inhabitants of Algiers produce the same chemical substance, notwithstanding the difference in food and climate. The following are the analytical results:—

a. Melolontha. Elytra alone.

Determination of the Ash.

0·206 of the substance gave 0·001 ash = 0·5 per cent.

Nitrogen.

I. 0·317 substance gave 0·318 ammonio-chloride of platinum = 6·33 per cent. nitrogen.

II. 0·403 substance gave 0·429 ammonio-chloride of platinum = 6·72 per cent. nitrogen.

Combustion.

0·292 substance gave 0·4975 carbonic acid and 0·175 water, hence per cent. $\begin{cases} \text{Carbon . . 46·69.} \\ \text{Hydrogen 6·69.} \end{cases}$

b. Melolontha. Elytra, wings and cutaneous tegument.

Determination of the Ash.

0·271 substance gave 0·0018 ash = 0·664 per cent.

Nitrogen.

I. 0·366 substance gave 0·3685 ammonio-chloride of platinum = 6·36 per cent. nitrogen.

II. 0·418 substance gave 0·4285 ammonio-chloride of platinum, = 6·48 per cent. nitrogen.

Combustion.

a. 0·716 substance gave 1·220 carbonic acid and 0·425 water,

hence per cent. $\left\{\begin{array}{ll}\text{Carbon} & 46·70\\ \text{Hydrogen} & 6·51\end{array}\right.$

b. 0·583 substance gave 0·9905 carbonic acid and 0·341 water,

hence per cent. $\left\{\begin{array}{ll}\text{Carbon} & 46·80\\ \text{Hydrogen} & 6·63\end{array}\right.$

c. Ateuchus sacer. Tegument and wings.

Determination of the Ash.

0·068 substance gave 0·000 ash.

Nitrogen.

0·237 substance gave 0·248 ammonio-chloride of platinum = 6·57 per cent. nitrogen, or at one view:—

	Melolontha vulgaris.				*Ateuchus sacer.*
	Wings alone.		Entire tegument.		Entire tegument.
	1 *a.*	2.	1 *a.*	2 *b.*	
Carbon	46·69		46·70	46·80	
Hydrogen	6·69	6·72	6·51	6·63	
Nitrogen	6·33		6·36	6·18	6·57

Lastly, the absence of sulphur and phosphorus was proved by heating it to redness with a mixture of burnt marble and nitre in the manner proposed by Wöhler[1].

The agreement is perfect, and we have every reason to regard this substance as distinct. As confirmatory experiments in the examination of other members of this family, the most striking reactions, viz. the insolubility in potash, the action of heat and concentrated acids may now suffice. In this manner I examined the following:—

Order.	Species.
ELEUTHERATA.	*Carabus* (*hortensis*, *auratus*, &c.), *Calosoma* (*Sycophanta*), *Cicindela* (*campestris*), *Meloe* (*proscarabæus*).
ULONATA.	*Forficula* (*auricularia*), *Gryllus* (*campestris*), *Locusta* (*viridissima*), *Gryllotalpa* (*vulgaris*).
SYNISTATA.	*Ephemera* (*vulgata*), *Libellula* (*depressa*), and many species of *Phryganea*.
PIEZATA.	*Vespa* (*Crabro*), *Apis* (*mellifica*), *Formica* (*rufa*).

[1] Liebig's *Annal.* vol. lii. p. 1 7, in his investigation of the Quinone series.

RHYNCHOTA ..	*Aphis* (*rosæ*), *Nepa* (*cinerea*), *Hydrometra* (*paludum*)
ANTLIATA ...	*Simulia* (*reptans*), *Musca* (*domestica* and *vomitoria*), *Sargus* (*cuprarius*).
GLOSSATA ..	*Tinea* (*pallionella*), *Hybernia* (*brumata*), *Bombyx* (*pini*), *Cossus* (*ligniperda*), *Sphinx* (*ligustri*), and some others.

In addition to these, many larvæ and pupæ, partly from those genera and species enumerated above, partly from others, the systematic names of which I did not note at the time and have now forgotten. In all, the minute structure presents great analogy in the elegant grouping of the layers of longitudinal and transverse fibres which has been mentioned. On treating them with potash, we find the most amusing metamorphoses, the most splendid *Vanessa Antiopa*, *Sphinx* or *Papilio* becomes as colourless and transparent as the commonest bee; the Swallow-tailed butterfly (*P. Machaon*) with its most elegant play of colours cannot be distinguished from the common moth. On directing our attention to the Crustacea we obtain the same remarkable result. If we extract the lime-salts from the thoracic shield of the Craw-fish with dilute acid and macerate it for a couple of days in hot solution of potash, we obtain a colourless skeleton of chitine, in which, with the aid of the microscope, numerous interwoven layers of longitudinal and transverse fibres may be distinguished. In this case, the lime-salts appear to occupy the place of the resinous colouring matter of the beetles, as a uniting medium. The number of these fibrous layers increases with the age and thickness of the shield, and hence is very considerable in the Lobster. The shield of the craw-fish, lobster and a *Squilla* (*mantis*) was prepared in considerable quantity in the various ways above mentioned, the basic substance in all, as the following data will show, is perfectly identical :—

a. Astacus fluviatilis. Tegument.

Determination of the Ash.

0·247 substance gave 0·005 ash = 2·0 per cent.

Nitrogen.

I. 0·412 substance gave 0·424 ammonio-chloride of platinum = 6·59 per cent nitrogen.

II. 0·360 substance gave 0·357 ammonio-chloride of platinum = 6·35 per cent. nitrogen.

Combustion

a 0·391 substance gave 0·656 carbonic acid and 0·229 water,

hence per cent $\begin{cases} \text{Carbon} & 46·71 \\ \text{Hydrogen} & 6·64 \end{cases}$

b *Astacus marinus* Claws

Determination of the Ash

0·1705 substance gave 0·003 ash = 1·7 per cent

Nitrogen

0·168 substance gave 0·179 ammonio chloride of platinum = 6·51 per cent nitrogen

Combustion

a 0·812 substance gave 1·409 carbonic acid and 0·479 water,

hence per cent $\begin{cases} \text{Carbon} & 46·18 \\ \text{Hydrogen} & 6·43 \end{cases}$

b 0·592 substance gave 0·994 carbonic acid and 0·342 water,

hence per cent $\begin{cases} \text{Carbon} & 46·61 \\ \text{Hydrogen} & 6·53 \end{cases}$

c *Squilla mantis* Tegument, claws and pairs of feet

Determination of the Ash

0·2007 substance gave 0·0012 ash = 0·6 per cent

Nitrogen

0·320 substance gave 0·344 ammonio chloride of platinum = 6·79 per cent nitrogen

Combustion

0·3795 substance gave 0·643 carbonic acid and 0·230 water,

hence per cent $\begin{cases} \text{Carbon} & 46·51 \\ \text{Hydrogen} & 6·77 \end{cases}$

Or —

	Astacus fluviatilis 1 *a*	2	*Astacus marinus* 1 *a*	*b*	*Squilla mantis* 1 *a*
Carbon	46·71		46·18	46·61	46·51
Hydrogen	6·64		6·43	6·53	6·77
Nitrogen	6·59	6·35	6·51		6·79

The shield of these animals however still contains a certain quantity of lime salts, viz carbonate and phosphate of lime with a little phosphate of magnesia The proportions by weight of the latter to one another, as also to the surrounding chitine

tissue, are of physiological importance, it will not therefore be superfluous to state them.

1·710 of the thoracic shield of the Craw-fish (dried at 248° F.), when heated to redness, yielded, after the deduction of the carbon left undissolved on exhaustion with dilute acid, 0·911 fixed substances, 0·120 of which consisted of phosphate of lime with a little magnesia (precipitated by ammonia); 0·4615 of *Squilla mantis* gave 0·1715 fixed residue, containing 0·090 phosphate of lime.

3·023 of the claws of the Lobster gave 2·3295 fixed residue, containing 0·281 phosphate of lime; thus we have—

	Craw-fish	Squilla	Lobster
Chitine . . .	46·73	62·84	22·94
Lime salts . .	53·27	37·17	77·06

100 parts of ash consisted of—

	Craw-fish	Squilla	Lobster.
Phosphate of lime .	13·17	47·52	12·06
Carbonate of lime	86·83	52·48	87·94

We find here the interesting result, that the amount of earthy phosphates increases in proportion to the quantity of organized chitine-tissue; this is confirmed by former analyses of the shell of the lobster, craw-fish and *Cancer pagurus* made by Mérat-Guillot*, Chevreul† and Gobel‡.

This fibrous chitine-tissue is however the result of an active process of cell-formation during the change of the shell, the quantity of phosphate of lime also increases with the intensity of this process, for which the relative amounts of tissue formed yield the standard. Hence the phosphate of lime must be in intimate relation with the process of cell-formation.

It is evident from the following observations, that the chitine-tissue really owes its origin to such a process.

By carefully removing a portion of the thoracic or mandibular shell in layers down to the uppermost pigment layer of the membrane lying beneath it, I induced a new process of cell-formation. This soon took place; in eight hours, a thick, tenacious, transparent mass (*cytoblastema*) had already exuded; in it I found

* *Annales de Chimie*, xxxiv. p. 71.

† *Ann. Gen. des Sciences Phys.* iv. p. 124, also Schweigger's Journal, xxxii. p. 495.

‡ Schweigger's Journ. xxxix. p. 411. They are all collected in Heusinger's *Histologie*, ii. p. 253.

numerous globules (vesicles of fat) which were insoluble in potash and acetic acid as also other molecules (albuminates) soluble in these media but no other solid particles, when incinerated it left a considerable quantity of phosphate of lime (by approximative determination 8 per cent) with a little alkaline phosphate and carbonate of lime, which did not pre exist as such This phosphate of lime existed in it in a state of solution, for ammonia rendered the mass, placed under the microscope, very turbid In fourteen to sixteen hours the soluble molecules (albuminate perhaps also phosphate of lime) had accumulated around the fatty vesicles forming globular masses, some of these globular masses were already surrounded by a membrane, others not, at the same time it contained numerous rhombohedric crystals (of carbonate of lime) which effervesced with acids When treated with potash, the primary cells, as also their granular contents (albuminates?), swelled considerably, became transparent and dissolved in each the fat globule appeared as a nucleus hence they were not yet composed of chitine, unless perhaps this, in its early and perfected condition, preserves relations similar to those of gum to cellular membrane, *i e* is soluble Lastly, in from twenty four to thirty six hours, several of these primary cells were found lying beneath the same elements they were spindle shaped and elongated still swelled in potash, but did not now dissolve they appeared therefore to consist of chitine I was not able to trace the process any further, as the animals died from want of attention, and it was too late in the year to procure others

We thus found a considerable quantity of phosphate of lime in a state of solution in the cytoblastema, also some lime in organic combination (probably with albumen as albuminate of lime) I shall return to the import of these facts in the consideration of the Mollusca

Finally, two other membranes, which run beneath the tegument, belong to the cutaneous system of the Craw fish, their basis consists of the same substance, viz chitine The external one covers the whole tegument internally of which it forms the matrix as the dura mater does the cranial bones It is covered on both sides with a layer of dark roundish epithelial cells, containing a sharply defined, dark granulated nucleus these consist of a proteine compound (are dissolved by potash) Its texture is made up of numerous intimately interwoven longitudinal

and transverse fibres, of about the thickness of the cellular tissue of the conjunctiva, these are composed of chitine. In the upper epithelial layer, which is towards the tegument, we find the blue and red pigment in the form of small angular granules (crystals?) $\frac{1}{800}$ to $\frac{1}{1200}$ of a line in diameter, the former in the cell-nuclei (Kolliker's primary cells), the latter in peculiar branched cells, resembling those of the *lamina fusca* of the sclerotica. This uppermost epithelial layer appears to have the function of separating the phosphate of lime and the lime-salts (albuminate of lime) generally from the blood; for 0·214 of the mucous membrane, after having been carefully separated and dried at 248° F., left 0·025 ash, in which there was 0·019 phosphate of lime, *i. e* in 100 parts,—

Organic matter	88·32
Phosphate of lime	8·89
Carbonate of lime with a little phosphate of soda	2·79

This separation evidently occurred in consequence of the process of cell-formation which was going on during the regeneration of the shells (it was the middle of September).

I was unable to ascertain anything further regarding the physiological import of the innermost transparent membrane, which is covered with peculiar hairs, and much resembling the innermost intestinal wall which we have mentioned above (Heusinger's respiratory membrane *), the hairs and the membrane consist of chitine The former, as on the inner lining of the intestine, appear to be merely simple secondary cells which have grown perpendicularly between the others, which have extended themselves and disappeared in the direction of the surface. The dark basis is perfectly homogeneous, sharply defined compared with the colourless contents of the hair- (cells?), and it appears to me that it ought to be considered as a primary cell (nucleus); the cylindrical marks on the membrane, from which the hair-cells arise, are depressions, into which the former are inserted like the hairs of plants in their epidermis. The same applies to the so-called hairs of insects and also of

Spiders.—These, forming the last family of the Articulata, remained yet to be examined I could not obtain sufficient substance for elementary analysis, our native representatives are too small and too difficult to render anatomically pure in suffi-

* *System der Histologie,* ii p. 251

cient quantity. The cutaneous system however, of all those species which were examined (*Phalangium parietinum*, *Attus scenicus*, *Epeira diadema* and *Tegenaria domestica*) give the reactions of chitine. In *Epeira* the fibrous layers are very distinctly seen even before they have been treated with potash: the separate fibres here form elegant undulating lines, which are coiled around cylindrical depressions in the upper layer (these are for the reception of the long hairs). The entire aspect, as also the hairs remain unaltered after treatment with potash, the pigment disseminated between them being dissolved.

We have, then in the remarkable agreement of form and composition, another common link in the characteristics of the Articulata. A comparative histo-genesis would also be of great interest: but with reference to this very little has been done: older works, which are still classical in another point of view give us no assistance here.*

Now in what relation does this chitine, a substance which, as we have seen, is widely diffused through the animal kingdom, stand to the other important constituents of the animal or vegetable organism, to albuminates, the so called hydrates of carbon &c.? The solution of this question is of great interest. We find it as we have seen only in the Articulata, those three families of the animal kingdom which, being inclosed in a more or less solid tegument, are compelled to overcome this obstruction to their internal growth by periodically casting off their armour. In many, and these are the largest (Crustacea), the *annual* formation of the tegument is well known: an enormous quantity of formative material must be generated in a short time for the reproduction of these cast-off envelopes. This material, as we have seen, is chitine, a substance which *cannot* be generally proved to exist under a *similar* arrangement of its elements in the animal or vegetable cell, and yet these chitinophores form their mantle from both animal and vegetable food.

If for comparison we admit man to form "the standard and measure of creation," we here *apparently* find for a short period an enormous *distinct* production of matter—I allude to that of the milk during the earlier periods after child-birth. But, as I have stated, this is only apparent; it is in reality a mere alteration in *position* and *form* which strikes us, which the former

* As Rathke, History of the Development of the Cray-fish, Treviranus's treatise on Spiders, &c.

disturbance of equilibrium in the female organism produces, and the result of which we designate "milk-fever" The sugar is taken up in a similar arrangement of its elements, the fat and albuminates of the blood, which a short time before flowed to the uterine vessels, now pass to the mammary glands, thus no comparison can be admitted

But chitine contains exactly the elements of carbon, water and ammonia, or, what is the same, acetic acid, sugar, gum, starch or woody fibre, and ammonia,—in our experiments with the test-tubes of the chemist it is resolved into these elements; we might in fact be induced to attribute to the simple organism of an articulate animal the capability of forming its tegument from woody-fibre and ammonia, did not the above-mentioned observations on the new formative process oppose such a view. We may regard the formula $C_{17} H_{11} N O_{11}$ as the most simple expression of the analysis, which corresponds with sufficient accuracy to the results obtained.

	Calculation As $C_{17} H_{11} N O_{11}$	Experiment. Maximum	Minimum	Mean	Number of experiments.
Carbon .	46·83	46 80	46·48	46·66	7
Hydrogen	6·42	6·77	6·43	6.60	
Nitrogen .	6·12	6·79	6·33	6·53	9

The formula contains the elements of—

$$\begin{array}{lll} \text{Carbon . .} & C_{17} & \\ \text{Water. . .} & & H_{11} \quad O_{11} \\ \text{Ammonia. .} & & H_3 \quad N \\ \hline & C_{17} & H_{11} \; N \; O_{11} \end{array}$$

from which the equations representing the decomposition by a high temperature and concentrated acids are self-evident. If we compare the empirical formula, *i. e.* the simplest expression of the former analyses of muscle in equivalents $= C_8 H_6 N O_3$:

	Calculation As $C_8 H_6 N O_3$	Mean of experiments
Carbon	52·22	52·24
Hydrogen . .	6 52	7·15
Nitrogen . .	15·21	15 30

with the value of chitine as found in the same manner, we have—

$$\text{Craw-fish}\begin{cases} \text{Chitine } C_{17} H_{11} N O_{11} \\ \text{Muscle } C_8 \; H_6 \; N \; O_3 \end{cases}$$
$$\overline{C_9 \quad H_5 \quad\quad O_8}$$

and we thus arrive at the interesting result, that *the substance of the tegument of an articulate animal contains the elements of its primitive muscular bundle, plus one of the so-called hydrates of carbon, i. e. sugar, gum, woody fibre, &c.*; and thus that we can very well explain the formation of this substance in such enormous quantity and so comparatively short a time by the coalition of muscle, i. e. blood or proteine, and woody fibre, into this peculiar combination. Would not the Craw-fish, if its tegument were reproduced merely from the albuminates of its organism, perish from loss of substance on changing its shell? Do we not see here a wise œconomy of nature in causing a large part of the cytoblastema to be formed of calcareous salts, two thirds of the remainder by hydrates of carbon (Algæ, Confervæ, &c.) which are at hand, and the latter third only by the fluid mass of the animal? We do not find the stomach and intestinal canal of these animals, at or soon after the period of the casting of the tegument, filled with stems of Charæ, pieces of Confervæ, &c. without a reason! Those which consume vegetable substances (as the Cockchafers, so many thousands of which we frequently find living on the leaves of one tree, that we cannot resist the idea of the principal constituents of the vegetable cell, gum and woody fibre, being assimilated by them) therefore produce their cutaneous system from woody fibre and vegetable albumen; whilst on the other hand those which feed upon animal matters mostly devour the weaker members of their families, and from them obtain the requisite chitine, already prepared and formed. May we not have here the same relation as in the higher Vertebrata? Does not the total effect appear here also to be diminished by the withdrawal of a certain amount of power for the production of the formative material, so that as regards the faculty of perception and volition we are obliged to yield to the Carnivora a position above the Phyllophaga?

Of course these views will remain hypothetical, although highly probable, until they have been proved to be correct by direct observation. Now this proof may be obtained with sufficient accuracy in two ways:—

1. By tracing the history of development in a chemical point of view, for instance, of the Lobster. This method would not be very difficult, for according to Rathke's observations on the Craw-fish*, its cutaneous skeleton is not formed until the latter

* *Loc. cit.* pp. 11 [illegible] 63.

stages; and in these the embryo of the lobster must be of sufficient size to allow of our tracing the transition stages by elementary analysis.

2. By the accurate study of the relations accompanying the annual formation of the skin, likewise in those species which can be obtained of the largest size, and in quantity, where the embryonic process of formation of the tegument must be repeated, at least in its essential features. I must leave this for future accomplishment; it requires long residence at the sea-side, which I have been unable to obtain during the course of the last summer.

2. *Mollusca*

In the general part I stated that the cutaneous system of these animals was purely animal. This position is based upon the following observations: the shells of *Unio* and *Anodonta* consist of superimposed layers of calcareous salts (carbonate of lime) and albuminates. The latter are brought into view by the action of acid solvents; they then remain behind as white structureless lamellæ. The lime probably exists in the shell in the form of acute rhombohedra, which are arranged in rows*; at least, when treated with acetic acid before undergoing solution, it is resolved into fibres, in which I thought I could distinguish the separate elements composing it. The iridescence of the shells, a phænomenon resulting from interference, is effected by the delicate interstices of these fibres. These calcareous shells are a product of secretion from the mantle; they are externally covered by a membrane resembling horn, which thickens into a ligament at the hinge: this, in minute structure and chemical properties, exhibits the reactions of a duplicature of the mantle. Thus, its external layer consists of an epithelium composed of five or six-sided cells, containing nuclei and filled with a bluish-green or brown pigment, and between which we find one or more layers of fibres resembling those of cellular tissue. It is impossible to free it completely from the finely-divided silicates which adhere to it, and the presence of which does not at all interfere with the determination of the amount of nitrogen.

0·213 of this duplicature of the mantle removed by the forceps (and dried at 248° F.) gave 0·037 ash = 17·4 per cent.

0·369 gave 0·739 ammonio-chloride of platinum = 15·22 per

* For a description of the beauty of these crystals in *Teredo gigantea*, see Home, Philosophical Transactions, 1806, p. 276.

cent. of nitrogen. Of the structureless membrane which remained after treatment with acids—

0.165 dried at 218° F. gave 0.0195 ash = 11.82 per cent. (the above mentioned silicates).

0.261 substance gave 0.554 ammonio-chloride of platinum = 15.11 per cent. of nitrogen.

Thus both essentially belong to the same class of substances (muscle, cellular tissue). The same applies to the naked Mollusca; for 0.311 of the folds of the mantle of *Limax*, purified by exhaustion with water, alcohol and ether, and dried at 218° F., left 0.014 ash, for the most part consisting of phosphate of lime = 4.5 per cent.

0.367 of the same gave 0.837 ammonio-chloride of platinum = 15.00 per cent. of nitrogen.

Nothing can be done with the Mollusca which live in water (*Limnæus, Planorbis* and *Paludina*), as they are covered with a complete fauna and flora of microscopic forms (*Bacillariæ* and *Confervæ*). On the other hand, in *Helix* (*pomatia, nemoralis* and *hortensis*) we find the inner layer of the calcareous shell composed of a transparent structureless membrane upon which in the embryo the earliest calcareous layers are formed; it may be easily isolated by extracting the carbonate of lime with dilute acids.

0.203 of it (in *Helix nemoralis*) dried at 218° gave 0.0032 ash = 1.58 per cent.

0.289 of the same (*Helix nemoralis*) dried at 248° gave 0.692 ammonio-chloride of platinum = 15.27 per cent. nitrogen.

The wide difference between the cutaneous system of these families and those of the Articulata is evident. We shall stop a moment at the calcareous shells, and examine the proportion of the carbonate to the phosphate of lime.

3.186 of the shells of *Anodonta*, dried at 218° F., when heated to redness, left, on deducting the carbon remaining after the solution of the inorganic matter, 3.131 incombustible residue, which contained 0.019 phosphate of lime.

1.831 of the shells of *Helix* (*nemoralis*) left 1.760 incombustible residue containing 0.0165 earthy phosphates (lime with a trace of magnesia). Hence—

	Anodonta.	*Helix.*
Structureless membrane	1.49	3.88
Incombustible residue	98.51	96.12

In 100 parts of the incombustible residue—

	Anodonta	*Helix.*
Carbonate of lime . .	99·45	99·06
Phosphate of lime	0·55	0·94

We have here scarcely any process of cell-formation, mere amorphous, hardened mucous masses (albuminates) separated by calcareous layers, and scarcely any phosphate of lime, the coincidence is so striking that we cannot avoid regarding it as a confirmation of the view of the physiological import of this salt proposed above. I believe, as already stated, that a definite combination of albumen with phosphate of lime, or rather, that an albuminous solution saturated with a certain portion of the latter, possesses the power of condensing into comparatively solid membranes around heterogeneous bodies when brought into contact with them, *i. e.* of forming the wall of primary cells However, I have not yet succeeded experimentally in ascertaining the "why" and "wherefore" with sufficient accuracy Before we leave the Mollusca, I shall say a few words on the physiological import of the folds of the mantle in *Anodonta* and *Unio*, which is very interesting

This mantle consists of an intermediate scanty layer of fibrous tissue, resembling cellular tissue, which is covered internally by ciliated epithelium, but next the shell by the so-called glandular epithelium, *i. e.* epithelial cells containing nuclei, and resembling the cells of the liver. Now whilst the former has constantly to supply the gills with fresh water, the function of the latter is evidently that of decomposing the blood, of secreting a compound of albumen with lime next the shell, which is decomposable even by the carbonic acid of the air or of the water, but of retaining the phosphate of lime and returning it to the organs which require it for the process of cell-formation (testicle and ovary). This view appears to me to be supported by the following facts —

0·7745 of the folds of the mantle of *Unio*, after careful separation, when dried at 248° F. left 0·136 of ash, containing 0·115 phosphate of lime.

0·610 of the same from *Anodonta* gave 0·112 ash, containing 0·091 phosphate of lime.

Hence, in 100 parts of the folds of the mantle,—

	Unio	*Anodonta*
Phosphate of lime	11.85	11.91
Carbonate of lime, phosphate of soda, chloride of sodium and sulphate of lime	5.71	6.45
In all	17.56	18.36

per cent. of incombustible residue.

We thus see that the amount of phosphate of lime is constantly so enormous, that it cannot be considered as accidental.

On the other hand the *amorphous* mucus which is found between the shell and the mantle, and which is mixed with but few epithelial cells, when incinerated left the greater part as a colourless ash, (the characteristic odour of burning albuminates being evolved at the same time,) which was soluble in acids with considerable effervescence, and consisted almost entirely of carbonate of lime. The smallest quantity only of this pre-existed in the mucus, inasmuch as acids caused but slight evolution of gas in the latter, whilst oxalic acid instantly produced a dense white precipitate, consisting of oxalate of lime and albumen. Hence the lime was contained in it, in the form of a readily decomposed compound with albumen, as a soluble, perhaps basic, albuminate of lime.

If we add these two secretions together, we ought again to obtain their sum, and thus the confirmation of our view, in the blood of these animals.

7.560 of the blood from the heart and auricles of about forty *Anodontæ* (obtained by puncture just before the systole) when stirred with a glass rod, formed a small colourless clot, which, when dried, weighed 0.002. After the removal of these, the whole was dried in a water bath finally at 218° F. and weighed 0.061. On incineration, this left 0.0302 of white ash, 0.0025 of which was soluble in water; the residue, which dissolved in acetic acid with considerable effervescence, yielded 0.0026 of phosphate of lime.

I must remark, that the blood when freshly drawn from the heart was perfectly clear and colourless, but did not effervesce with acids, consequently contained no carbonates, although it had a slightly alkaline reaction; the part soluble in water contained sulphate of lime, phosphate of soda and chloride of sodium.

In another portion, which I accidentally set aside over night between watch glasses, I found the next morning the whole surface covered with a thin crystalline film. The crystals under

the microscope exhibited the most beautiful regular forms, although it was difficult to determine to which system they belonged; they dissolved in acids with considerable evolution of carbonic acid, and Prof. Wohler drew my attention to their great resemblance to the crystalline form of Gay-Lussite. In fact, judging from the reaction with perchloric acid, they appeared to contain soda, together with excess of lime, but it was certainly neither the first nor second rhombohedron of calcareous spar.

The first-mentioned coagulum reacted towards alkalies, by which it was dissolved, and towards nitric acid, which coloured it yellow, as an albuminate. The same applies to the organic matter of the dried residue, which, by forming pellicles on evaporation, and becoming but slightly turbid when first heated, appears to be related to caseine.

If we sum up what has been stated, it is evident that this blood contained essentially a compound of albumen with lime—which is decomposed even by the carbonic acid of the air, of the water, or of that produced by the chemico-vital reactions—together with phosphate of lime and soda, which amounted by weight in 1000 parts to—

	Water	991·46	
	Fibrine	0·33	
with	Albumen	5·63	in a peculiar state of combination.
	Lime	1·89	
	Phosphate of soda, sulphate of lime and chloride of sodium	0·33	
	Phosphate of lime	0·34	

This peculiar albuminate of lime, which for greater clearness of consideration we shall call neutral, is thus decomposed by the above-mentioned epithelial cells into free albumen and basic albuminate of lime: the latter is secreted as an amorphous mass next the shell, as such, in an almost unorganized condition, obeying the laws of crystallization, to contribute to its increase in thickness; the former (the free albumen) returns with the phosphate of lime into the circulation, to serve purely animal functions; either the process of cell-formation of the primitive ova in the ovaries, or of the maternal cells of the seminal animalcules in the glandular system of the testicle.

We have yet to investigate the Zoophytes; but we shall first glance at the two transition forms of the Cirripeds and Ascidia,

which are extremely interesting in this point of view: the former as being intermediate between the Mollusca and the Crustacea, the latter as forming the transition of the former to the Zoophytes.

3. *Cirripeds.*

I have examined *Lepas* (*lævis*). The stalk and extremities (cirri), when treated with potash in the manner so frequently mentioned, become colourless and transparent; so do the branched, jointed and simple hair-cells: they prove to be tubes of chitine, serving for the protection and support of the numerous muscles governing their segments, which latter are arranged in a sheath-like form, and in which the former play. The inner surface of this chitine tube is covered with a layer of pigment-cells, resembling those of the choroid coat, and such as also covers the concave surfaces of the articulated calcareous shells lying next the body, which, corresponding to the analogous coverings of the bivalves, nevertheless appear to be joined together by chitine ligaments, *i. e.* Crustacean ligaments. The analogy of these articulated calcareous shells with those of the Conchifera is evident from the following analyses:—

1·766 dried at 218° F. after being heated to redness and deducting the amount of carbon left on solution, gave 1·7115 incombustible residue, containing 0·012 phosphate of lime.

Hence the shells contained per cent.—

Albuminates	3·09
Incombustible residue	96·81

and 100 parts of the latter contained—

Carbonate of lime	99·30
Phosphate of lime	0·70

The above-mentioned albuminate remained in the form of structureless white films after treatment with dilute acids, just as in *Unio*; but the calcareous shells of *Lepas* are not furnished with the horny investment on the outer surface of the *Anodontæ* (hardened duplicature of the mantle), but at this period are unattached to the last formed calcareous layer (or rather the earliest formed calcareous lamella).

Thus, even in a purely chemical point of view, the Cirripeds retain their position in the animal kingdom.

4. *Ascidia.*

These animal forms, which in regard to the history of their development have been too little investigated, present us with

extremely interesting phænomena. I examined *Ascidia (Cynthia) mammillaris** The thick fleshy sac, in which the gill and intestinal tubes, as also the liver and ovary are fixed, consist of a conglomerate of large unnucleated cells, strikingly like the parenchyma of the Cacti or many fruits On its inner surface numerous vascular ramifications are spread, these communicate with the gills. If this entire external sac is treated with water, alcohol, æther, dilute acids and alkalies, in succession, the walls and contents of the vessels are dissolved, and the transparent colourless tissue of the above large globular cells is left, without its minute structure having undergone the least change It is *not* altered by *nitric, muriatic,* or *acetic acids,* nor the *most concentrated solution of potash*; in fact, an excellent method of obtaining it clear and transparent is *ebullition for several hours* with nitric acid However, in concentrated sulphuric or fuming nitric acid it slowly deliquesces into colourless fluids, the nature of which I was unable to examine for want of a sufficient quantity The amount of water contained in this capsule is so great, that 3·3175 of it left only 0·0355 = 1·07 per cent. of solid residue, so that the mantle of one entire animal of the size of half the fist, and 2 lines in thickness, when dried weighed barely 0·5 grm. The substance of this remarkable tissue, which was obtained chemically and anatomically pure in the manner mentioned first, is *free from nitrogen,* as I assured myself in two experiments upon 0·105 and 0·2065 heated with soda-lime, when heated in a glass tube it carbonizes, perfectly retaining its form, and evolving the peculiar odour of carbonizing cellular tissue of plants, and in the air it burns away readily and completely on account of its fine state of division. When heated in glass tubes to 392° F. it remains unaltered; lastly, when burnt in the small platinum vessel in a current of oxygen, as above, it yielded as follows:—

0·2168 of substance gave 0 357 carbonic acid and 0 125 water, leaving 0·002 of ash (sulphate of lime in the platinum vessel).

Hence 100 parts of the tissue, free from ash, contains—

Carbon .	45·38
Hydrogen .	6·47

i. e. *the composition of the cellular membrane of plants* †.

* With his well-known liberality Prof Wagner gave two specimens from his private collection (from Genoa and Marseilles) for this examination

† This remarkable fact has since been fully confirmed by MM Lowig and Kolliker —Ed

We here find, in regard to the minute structure, a remarkable agreement between the form and elementary constitution of the material substratum, and an infinitely more remarkable fact for general comparative physiology, and especially for these animal forms. For these organisms the whole life of which can scarcely be considered as more than a mere vegetation, a constant process of assimilation, and the whole nervous system of which is reduced to its simplest elements, a single ganglion (sympathetic?) with a pair of primitive fibrous bundles running from it, are placed in a vegetable envelope. According to the observations of Milne Edwards*, the Ascidia in their young state swim about unattached, and do not become fixed until a certain period of their existence. We might imagine that in this case a luxurious condition of simple cellular tissue, which we might call an Alga, or something of that kind, surrounded the animal in the form of a pouch, and thus formed with it a zoophyte in the true sense of the word, did we not find in this sac on the one hand, the perfect branched vascular system,—this organic connexion with the purely animal organic systems of the animal,—and did not, on the other hand, the observations of Sars† and Milne Edwards‡, on the development of the compound Ascidia (*Botryllus, Polyclinum*, &c.), oppose this view; for in them the earliest formation of this sac appears during the process of bifurcation in the form of a transparent colourless gelatinous layer, between the envelope of the ovum (*chorion*?) and the yolk.

Chemistry has here done all that it can; further explanation must be obtained from morphology. A new fundamental study of the development of this animal, with especial regard to the histogeny of its envelopes, must solve the enigma, and under the present circumstances would prove of the greatest interest.

We shall conclude this investigation with a consideration of—

5. *The Zoophytes*,

in one of their simplest representatives, which has been before mentioned, *Frustulia salina*, Ehrbg.§ Its discoverer first found it in quantity in the Königsborner saline spring. As is well known, Wöhler|| two years ago made the observation which is of

* *Observations sur les Ascidies composées des côtes de la Manche*. Paris, 1841. Condensed by Von Siebold in the Annual Report of Müller's Archives, 1842, p. clxxx.

† Froriep's *Notizen*, vol. 1837, p. 100.

‡ *Loc. cit.*

§ *Loc. cit.* p. 232.

|| Wöhler and Liebig's *Annalen*, 1843, p. 206.

such importance in general physiology, of *the evolution of oxygen as the final result of an inverted vital reaction* (Stoffwechsel) or respiratory process of these organisms. He had the kindness to draw my attention to the phænomenon itself, as also to the excellent material for minute examination, and pointed out to me the locality and the very spot (in the Rodenberg saline spring); the undertaking and happy issue of this investigation are owing indeed to his friendly advice and assistance. I arrived at the saline spring at three o'clock one afternoon in the end of September. A whitish mucous mass covered the bottom of the brine reservoirs, between the layers of which bubbles of gas 1″ to ½′ long and 2‴ to 2″ broad were inclosed. Stirring with a stick caused an enormous evolution of gas. A glowing chip of wood was set in a flame three times in succession when inserted into an ale-glass which had been filled a few seconds previously. Observation made with a good Oberhauser's microscope upon the spot, showed that no trace of *Conferva* or other forms than the *Frustulia*, could be detected in the fresh mucous masses which were uppermost, especially in those which were filled with this air loaded with oxygen.

The central, round, eye-like masses, which Ehrenberg pointed out to be male seminal glands, as also the narrow ones lying on the lateral walls of the siliceous carapace towards its apex, and which this philosopher considers as ovaries, were yellowish brown. Microscopic reactions, as also the combustion tube, appear to confirm the correctness of this view; these masses as mentioned at p. 11, consist of fat. It was noticed at the same time, that potash appeared to dissolve the other contents of the siliceous carapace. The residue, after treatment with æther and dilute solution of potash, was considerable. It was proved (0.415 being heated to redness with soda lime) to be *free from nitrogen*; the results of elementary analysis were:

0.6275 of the substance dried at 218° gave 0.527 carbonic acid and 0.186 water.

0.6275 left 0.316 ash (silica) in the small platinum vessel. Hence in 100 parts of the substance free from ash there was—

Carbon	46.19
Hydrogen	6.63

This result agrees perfectly with that obtained by Rochleder and Heldt as a mean of seven determinations for the cellular membrane of lichens; they found—

Carbon	16.08
Hydrogen	6.67

However the slight excess of carbon and hydrogen may be ascribed to impurity arising from a portion of the fat, colouring matter, &c. being mixed with it. At all events *this residue is identical with the membrane of the vegetable cell!* The contents of the siliceous carapace which were soluble in potash, behaved like proteine judging from the reactions with potash, ammonia, acetic acid and nitric acid (formation of xantho proteic acid), but their elementary constitution could not be determined with the necessary accuracy, inasmuch as the residue free from nitrogen is only relatively, not absolutely insoluble in potash, a property which belongs also to Payen's pure cellulose; hence on neutralizing the alkaline solution with acetic acid, a quantity of the latter is precipitated with silica and proteine. This mixture yielded from 8 to 12 per cent. of nitrogen, and

Carbon	48	49.7
Hydrogen	6.7	6.9

results which agree perfectly with my supposition. Hence, by determining the amount of ash and nitrogen, the relation of the siliceous carapace to the fat, proteine and cellulose may be ascertained with sufficient accuracy and elegance. The pure mucous masses (i. e. of course freed from the contents of the brine ley by washing with pure water), dried at 218° *before* treatment with æther, gave as follows,—

0.4235 substance yielded 0.191 ash, containing 0.1795 silica, 0.0115 phosphate of lime with a little peroxide of iron = 45.1 per cent.

0.1375 of the substance gave 0.1665 of ammonio-chloride of platinum = 1.35 per cent. of nitrogen (after deducting the ash).

The same mass, when dried at 218°, *after* treatment with æther, hence after the removal of the fat, yielded,—

0.2015 substance, 0.1095 ash = 53.515 per cent.

Proteine, fibrine, albumen and caseine contain on an average 15.8 per cent. of nitrogen. Taking this as a base, we have in 100 parts of the *Irustulæ*,—

Siliceous carapace	45.10
Fat (ovary, testicle?)	15.77
Proteine substance (foot?)	15.12
Vegetable cellular matter (mucous envelope)	24.01

I therefore believe that the position advanced at the end of the

General View, that "these *Frustuliæ* are beings having the substance and the organic re- and decomposing forces of plants with the locomotion of animals," is satisfactorily proved.

Are we, however, in the present state of our knowledge, justified in accurately defining the above line of limitation between animals and plants? Is it not high time to overthrow this Chinese structure, as an obsolete descendant of systematic scholasticism, and to consider that from *man to the primary animal and vegetable cell, there exists no gap in the realization of a general idea upon which nature as a whole is based?*

In what does the spore of *Vaucheria clavata**, that simple cell with its vibrating cilia which moves about for hours together in water, differ from the young Medusa, the not less simple vesicle which cleaves the waters of the North Sea with its ciliated bulbs? In what does the embryonic cell of the swimming Ascidia differ from both of these? Do not all three, with the utmost probability, possess the same elementary form and composition? The mantle of the Ascidia exhibits to us the substance and structure of the plant; it must pre-exist essentially as such in the ovum, for in the earliest stages of development of the latter, in the earliest change of that indefinite chaos towards the future organism, we find it already separated as a protective formation to its contents (the bifurcation globules)†. It is highly probable that the transparent mantle of the Medusæ possesses the same elementary composition, hence the embryo of an Alga, as regards its *material substratum* (form and composition), is identical with that of a Medusa or Ascidia; in the former, we have the highest stage of development of the plant; in the latter, the simplest form of the animal! Cannot we apply the idea, so important in its consequences, by which Steenstrup ‡ not long since combined numerous observations, heretofore isolated and apparently paradoxical, into an harmonious whole, in the same manner to the simplest forms of the animal world? I mean, cannot we regard the *Alga* as the nurse of its *more highly developed embryo?* The *nurse of a Campanularia* § *exhibits no trace* of the phænomena which we necessarily connect with the idea

* Dr. F. Unger, The Plant at the Moment of its Animalization. Vienna, 1843 (in Letters to Endlicher).

† Milne Edwards, *l. c.*

‡ J. J. Sm. Steenstrup on the Alternation of Generation, translated for the Ray Society, 1846. London.

§ Steenstrup in reference to *Campanularia geniculata*, p. 31, fig. 52.

of "animal" we have here no stomach, no internal cavity for the assimilative process, no spontaneous motion; in short, it is a perfect *parent cell of an Alga*. The embryo which on the bursting of this so called parent animal begins to pass through its independent vital cycle, exactly resembles *Vaucheria** like the latter, when the ciliary motion has continued for two hours it becomes fixed, and thus attached becomes developed into perfect polypes: in the first stages of this process it is a true alga, in the latter an animal organism†. We may regard *the Alga as an interrupted formation of the polype, as polypes with a simple alternation of generation, whilst* Campanularia *possesses a double one!* We probably have exactly the same relation in the *Medusæ, Salpæ* and *Ascidiæ*, and also as experimentally proved in numerous parasites (*Ascaris*) ‡, the consideration of which here would lead us too far, and which is at once seen when the views we have detailed are compared with the ingenious ideas and excellent observations of Steenstrup in the work we have quoted.

Lastly, these *Frustuliæ*—with their vegetable mantle and their vegetable alteration of matter—even with regard to their only animality, the feeble spontaneous motion, are 100 times surpassed by the embryo of the Alga! There can be no doubt that they must possess the faculty of converting constituents of the atmosphere into the substance of their organism; the water of the spring hardly contains traces of organic compounds; when the air is excluded and it is removed from the influence of light and heat, it remains clear and colourless in sunshine, without the previous formation of *Conferva*, without a trace of any other previously formed formative matter, the few germs of these beings (*Frustuliæ*) which have accidentally fallen into it become developed into millions of individuals; they reduce the carbonic acid of the atmosphere to fats and hydrates of carbon; they assimilate the ammonia, or even produce it from the nitrogen of the atmosphere, and combine it with the elements of the fats and hydrocarbons, so as to produce proteine and albuminates; they separate the oxygen in excess, and man, investigating and reflecting from the final product on the "essence" of the process, sees

* Steenstrup, *l. c.* fig. [illegible]1 and in Unger, *l. c.*

† Steenstrup, *l. c.* figs. [illegible] and 7.

‡ *Ibid. id. l.* p. [illegible]0 *et seq.* Development of the Trematoda.

the possibility of his own existence being partly mediated by the above most simple beings restoring the equilibrium of the atmosphere.

III. *Conclusions.*

The facts which have been discussed in the preceding pages may be briefly expressed as follows:—

1. That the Articulata are characterized by the presence of a peculiar substance, chitine, which constitutes the whole of their external investments, as also the tracheæ, the gills, and probably the innermost layer of the intestinal tube; this substance, which resembles woody fibre, is *not* found elsewhere in either the animal or vegetable kingdom, and it contains exactly the elements of proteine and starch or of ammonia and sugar.

2. The substance of the cellular membrane of plants (cellulose) is by no means peculiar to plants; in fact it appears to be very widely diffused in the lower classes of animals, and has been experimentally proved to be a constituent of the mantle of the *Ascidiæ* and *Frustuliæ*.

3 The smooth and transversely striated muscular elements (primitive fibres) of the Invertebrata (Cockchafer, Craw-fish and *Unio*) are identical in composition.

4. Phosphate of lime is in intimate relation with the process of cell-formation, and probably a soluble combination of albumen with it in definite proportions alone possesses the physico-chemical qualities necessary for this process.

These facts lead to the following deductions:—

I. *No* chemical or physical difference can be instituted between animals and plants; *psychology* alone must define the boundary limits, if any. All those distinctions which have hitherto been made, and which have long been untenable before the tribunal of sound *natural philosophy* are also without *experimental* foundation, and have arisen from confusion of the relations of causality they are all mere consequences of the psychical constitution of the individual, of the species or genus; merely the means necessary for the attainment of an object which the soul of the individual or of the universe aims at.

Proof. The most important differences in form and composition which have hitherto been instituted relate to—

a Motion.

b An internal cavity for the assimilative process

c The ultimate products of the metamorphosis of matter (products of the respiratory process)

d The substance of the cell wall

Ad a The Oscillatoriæ and the Spores of the Algæ have a spontaneous motion as perfect as, and even considerably more so than that of the *Bacillariæ* and the fixed marine animals (*Ascidiæ*, &c) This motion is a necessary fundamental condition of the *physical* existence of these beings what the atmosphere is to plants the ocean is to the adherent marine animals If the land animals lived in a sea consisting of albumen and hydrates of carbon, they would not require a locomotive apparatus to enable them merely to replenish their formative matter, if the atmosphere contained no carbonic acid, plants would stand in need of locomotion

Spontaneous motion is the *consequence* of the presence of the Will The will without the apparatus requisite for the realization of its ideal activity, would be an extremely useless gift of nature and if we adopt the maxim that "everything in existence is judicious and perfect" it would be inadmissible Cuvier has beautifully treated of the relations of causality in the introduction to his Comparative Anatomy '

Ad b What, then is the principle of this internal cavity in the assimilative process? Evidently the greatest possible increase of surface so as to favour the most perfect assimilation in an endosmotic apparatus Do we not perhaps find it realized in plants? Undoubtedly the whole system of intercellular spaces, with their outlets in the stomata, exhibits exactly this arrangement, except that in their case, preserving the same kind of comparison, we have the lungs and intestinal tube combined Carbonic acid, the formative material of plants, passes freely through the stomata of the elongated canals of the intercellular spaces, so as to be taken up into the surrounding cells by diffusion as formative material just as albumen and the hydrates of carbon pass through the *sphincter oris* into the intestinal cavity that which is designated diffusion in the former corresponds to endosmose in the latter, the unnamed cells of the former constitute the epithelia of the intestinal villi in the latter

The Vibrions are usually denominated animals They exhibit the most active motions, they permanently exist as simple cells without a trace of contraction even when magnified to the great

est extent: but the fact that the intestinal tube and respiratory apparatus (lung, gill and trachea) are mere inflexions of the outer surface to afford increase of surface, ought to be proved in the Articulata to a demonstration, in addition to other facts, by physiology and the history of development, for in them they entirely consist of that remarkable substance which is characteristic of these animals, viz. chitine

Ad c. Wohler has clearly proved that elimination *of oxygen* is the ultimate product of the metamorphosis of matter in the *Frustulæ*; on the other hand, Drs. Schlossberger and Dopping* have proved the *exhalation of carbonic acid* to occur in Sponges and Fungi. Thus we have the exact antithesis of the required separation of carbonic acid in animals, and the excretion of oxygen in plants.

Ad d. I have proved the identity of the substance of the cellular membrane of plants with that of the mantle of the *Ascidiæ* and *Frustulæ*, and rendered it probable with that of the *Medusæ* and *Polypes*.

II. Reil's position, "that the vital phænomena are the result of form and composition," is even now correct when put into the following form:—"The working of the animal machine itself, independent of another sphere of motive phænomena of a distinct immaterial substance, psychical activity, is the necessary result of the structure and composition of its elements."

Proof. This is afforded by a comparison of the minute structure of the mantle in the *Ascidia* with that of plants having the same composition; not less striking is the systematic position of the Cirripeds compared with the relations of their composition.

Moreover, the doctrine of Vital Force has gone out of fashion; a "metabolic power in the cell," &c. is now substituted for it, *i. e.* it has received another appellation, or is designated "as the unknown cause of a series of phænomena which we call life." Every motor phænomenon is however merely the result of the reaction of at least two masses in motion (the first position in mechanics), to explain a motor phænomenon, and to refer to its causes, means to analyse its intensity and direction according to the parallelogram of the forces of its components: this implies at least two forces; this is the province of physiology, as well as of every physical science. It is clear that from *one primum movens*, from one mental phænomenon (force), assumed as a causal mo-

* Wohler and Liebig's *Annalen*, vol lii p 119

mentum for the sake of convenience we cannot explain a single motor phenomenon, much less a sum of them This fundamental idea of Reil's excellent position regarding the so called vital force "that it is the necessary result of form and composition" will remain as the sure basis of a rational physiology (*i e* the physics of the organism) It was the identification of the *soul* (the sum of the *psychical* motor phænomena) with the *vital force* (that of the *physical*) which necessarily led Reil, an inductive philosopher, into numerous conclusions contradictory to experiment —with him physiology and psychology were synonymous

We see that the mechanism of the organism in the simplest vegetable form (*Conferva Protococcus*) proceeds *ad infinitum* with mathematical precision, just as a curve according to its formula, if but a differential of magnitude be given But in the animal world we find a substance added the mechanism of which we call psychology, a sum total of motor phænomena with as many points of commencement, directions and intensities, as there are reacting masses of the corporeal organism, like this, developing itself from a differential in magnitude according to stated formulæ, which formulæ being peculiar to each species according to the magnitude of the substituted value and the duration of the real construction admit in it of an infinite variety

The only rational difference which we can make between an animal and a plant appears to me this that for each species of plant we have from the commencement (which it is the province of geology and palæontology to determine) *one* differential of magnitude and one formula (cell), truly only a single differential, for this by integration produces only *one* definite curve, be the substituted values ever so different, whilst in the animal *two* of them are given (the cell *plus* the soul atom), the *integrals* of which we designate *vegetable life* in the former and *animal life* in the latter

Article II.

*Memoir upon the Colours produced in Homogeneous Fluids by Polarized Light** By Augustin Fresnel †.

[Presented to the Academy March 30th, 1818.]

M. BIOT was the first to remark that several homogeneous fluids possessed the property of colouring polarized light, and of reproducing the extraordinary image in the same manner as crystalline substances. This beautiful discovery proved that the polarizing action of bodies could be exercised independently of the arrangement of their particles, and solely in virtue of their constitution.

Reasoning from analogy, I have long suspected that these phænomena of polarization ought to be accompanied by double refraction, in fluids as in crystals. The colorization of the light is moreover explained in so satisfactory a manner on the undulatory theory by the interference of two systems of waves, that it was natural to suppose their existence, even in homogeneous fluids, on seeing that colours were produced by these fluids. Nevertheless no hypothesis stood more in need of confirmation by direct experiment.

The theory of interference points out several very simple modes of observing the slightest differences in the course of two systems of waves emanating from a common source. For this purpose, for instance, the phænomenon of coloured rings may be employed, or that of the fringes produced by the meeting of two pencils of rays.

At first I followed the former process. Having tightly squeezed two prisms together so as to produce the coloured rings, I caused the light of a lamp to fall upon the surfaces in contact, at the angle of complete polarization. The rays thus reflected traversed a tube $1^{m}\cdot715$ in length, filled with essential oil of turpentine. I was obliged to make use of an opera-glass to distinguish the rings, in consequence of the distance at which the prisms were situated.

* This Memoir was supposed lost. It has been found recently amongst the papers of M. Léonor Fresnel, the brother of the illustrious Academician.

† Translated by E. Ronalds, Ph.D. The Editor is indebted to the Rev. Professor Lloyd, President of the Royal Irish Academy, for his kind assistance in revising this translation for the press.

With the glass alone, I did not perceive more rings through the oil of turpentine than before the interposition of that liquid but on placing a rhomboid of carbonate of lime in the interior of the telescope, so as to produce two separate images I perceived in each of them a considerable increase in the number of rings they were perceptible even when the film of air was of that thickness at which I had previously never been able to discover them.* Now, one can only explain the appearance of these new rings by supposing a diminution in the interval of the two systems of waves which combine to produce them or, what comes to the same thing, by supposing that the one part of the system of waves reflected by the first surface of the film of air, traversed the tube a little more slowly than a part of those reflected from the second surface. Thus it must be admitted that the oil of turpentine retards like crystals, the passage of light in two different degrees. As the rays reflected by the first and second surface of the film of air must equally suffer double refraction in passing through the liquid, the new rings can only be formed by one half, at most, of the light which reaches the eye so that they ought to be much more feeble than the others.

It may be objected to the deductions which I have just made from this experiment, that the circumstances giving rise to the new rings being precisely those which cause the colours in the oil of turpentine, the apparent augmentation of the number of rings may possibly be due to the *simplification* of the light. But in the first place, I reply, that these colours were exceedingly feeble in consequence of the great length of the tube, and that even, in certain positions of the rhomboid of calcareous spar, they became insensible, the two images then appearing to have no other colour than that peculiar to the liquid. It will be seen besides that several other phenomena confirm the hypothesis of double refraction in the oil of turpentine.

Having carried the same tube into a dark room, I directed it

* M. Arago made a long time ago a perfectly similar experiment upon plates of rock crystal cut at right angles to the axis. The same phenomenon can be produced with laminæ of rock crystal or sulphate of lime cut parallel to the axis and of but slight thickness. When they are only 1 or 2 millimetres thick the new rings are perfectly separated from those which surround the point of contact and establish with certainty the double refraction of the crystal. This property of crystalline laminæ may be equally well applied as a measure of their double refractive powers, their thickness, and of the curvatures of the object glasses of the telescope.

towards a luminous point, before which I had placed a series of glass plates to polarize the incident light. At the other extremity of the tube I placed, at the angle of complete polarization, two plates of glass not silvered and very slightly inclined towards each other, so as to produce fringes of sufficient breadth. Then observing with a magnifying glass the light thus reflected, I discovered the existence of three systems of fringes which touched and mixed slightly with each other, in consequence of the tube not being sufficiently long.

The middle system, proceeding from the superposition of the fringes produced by the meeting of the rays which had suffered the same refraction, was much more intense than the two others, which resulted from the coincidence of the rays oppositely refracted. The light was not sufficiently intense to enable me easily to discover in these the position of the dark bands of the first order; but it appeared to me, as far as I could judge, that the distance of the centre of each of the systems on the right and left from the centre of that in the middle was the breadth of seven fringes. Another more precise experiment, detailed at the end of this memoir, shows that the feeble colours produced by this tube belong to the sixth order.

Although the existence of double refraction in the oil of turpentine establishes a great analogy between the phænomenon of its colorization and that presented by crystalline laminæ cut parallel to the axis, yet nevertheless they differ essentially in many respects. In the crystalline laminæ, the rotation of the rhomboid of calcareous spar produces a variation in the intensity of the tint without changing its nature; in the oil of turpentine, on the contrary, the same motion of the rhomboid changes the nature of the tint without diminishing its intensity. Lastly, the tube containing this liquid may be made to turn upon its axis without producing any change either in the nature or in the vividness of the colours; whilst on turning the crystalline laminæ in its plane, the colours are augmented or lessened until they are reduced to a pure white.

The singular modification which double total reflexion at an azimuth of 45° impresses on polarized light, and which imparts to it the appearances of complete depolarization when analysed by a rhomboid of calcareous spar, does not deprive it, as is known, of the properties of colouring crystallized laminæ. These tints have even as much vividness as those produced by ordi-

nary polarized light, and are only different in kind Now here is another characteristic difference between the action of crystalline laminæ and that of oil of turpentine Light thus modified is no longer coloured by this liquid, and appears when subjected to this trial, as completely depolarized as when caused to pass immediately through a rhomboid of calcareous spar

At the extremity of a tube 0m 50 in length, filled with oil of turpentine I placed a glass parallelopiped, in which the incident rays previously polarized, suffered two complete reflexions in a plane inclined at an angle of 45° to that of primitive polarization In looking through the other extremity of this tube with a rhomboid of calcareous spar, I could perceive no trace of colorization, when the rays had been reflected at a proper incidence in the glass parallelopiped, whilst polarized light which had not suffered this modification gave rise in the same tube to the most vivid colours Rock crystal cut perpendicularly to the axis produced under these circumstances the same effect as oil of turpentine

Polarized light modified by double total reflexion being no longer coloured in this fluid, analogy indicates that it should no longer produce more than a single system of fringes with the apparatus described above and this is confirmed by experiment

It is natural to conclude from these two experiments, that light thus modified suffers only a single refraction in the oil of turpentine To verify this conclusion and to assure myself that the light on leaving the tube really did not contain more than a single system of fringes I made it traverse a thin crystalline lamina, and I then saw that it gave rise to the same colours as when it had not traversed the oil of turpentine, or at least the tints differed very little, and this slight difference was due to the peculiar colour of the liquid, as was seen by causing incident light to traverse this fluid before its primitive polarization

But here is another sufficiently remarkable experiment, which shows perhaps still better, that in the present case the oil of turpentine gives up the light just as it received it When the polarized rays have suffered total reflexion in an azimuth of 45° to the primitive plane of polarization, if they are again submitted to two total reflexions in a second glass parallelopiped, they reassume all the appearance and properties of complete polarization, this is a phænomenon easily explained upon the theory put forth in my last memoir But the same phænomenon still

occurs, when a tube, however long, filled with oil of turpentine is placed between the two glass parallelopipeds. Thus the modifications imparted to the incident rays are not altered in this case by the interposition of the fluid.

When, instead of placing the glass parallelopiped at the foremost extremity of the tube, it is placed at the end nearest the eye, the polarized light, which, after traversing the oil, is reflected twice in this parallelopiped, presents the characters of a pencil of light which has traversed a thin lamina parallel to the axis, for, on turning the rhomboid of calcareous spar, the nature of the tints is no longer varied, but only their intensity, which pass into a perfect white in two rectangular positions of its principal section, when it is inclined 45° to the plane of double reflexion The tints arrive, on the contrary, at their greatest intensity when the principal section of the rhomboid is parallel or perpendicular to this plane Their nature depends upon the position of the glass parallelopiped, and is precisely that of the colours obtained directly without its interposition, when the principal section of the rhomboid of calcareous spar is brought to the same azimuth.

In thus modifying by double total reflexion the polarized light which has traversed oil of turpentine, the effects of this liquid may be combined with those of a crystallized lamina cut parallel to the axis, in the same manner as the effects produced by two such laminæ are combined But in order that the addition or subtraction of the tints may be effected in a perfectly similar manner, to obtain, for instance, the total disappearance of one of the images with a lamina of suitable thickness, it is necessary that the plane of double reflexion should be turned in a certain azimuth depending upon the length of the tube, this azimuth, in the particular case of perfect compensation, is that which gives the same tint as the crystallized lamina. When the axis of the lamina is to the left of the plane of double reflexion, the tints are added, when it is to the right, they are subtracted. This order would be inverted with a fluid like oil of citron, in which the polarizing action is in a contrary direction to that of oil of turpentine.

In the last memoir which I had the honour of presenting to the Academy, I described an apparatus, by means of which, with a crystallized lamina cut parallel to the axis, the phænomena of colorization produced by oil of turpentine and plates of rock-

crystal cut parallel to the axis, could be imitated. It consisted of two glass parallelopipeds arranged at right angles, between which the crystalline lamina was placed, so that the polarized pencil of light suffered double total reflexion on leaving the lamina as on entering it, but in a plane perpendicular to the former, both these planes being inclined at an angle of 45° to the axis of the crystal. This system of a crystalline lamina and two glass parallelopipeds thus arranged, possesses the singular property, that it can be turned upon its axis between the two planes of extreme polarization like a plate of rock crystal cut perpendicularly to the axis, without changing either the nature or the intensity of the colours; whilst by varying one of these planes in relation to the other, all the various tints are obtained which under similar circumstances are presented by plates of rock crystal cut perpendicular to the axis and by oil of turpentine. Moreover, when the incident light has suffered double total reflexion in a plane inclined at 45° to that of primitive polarization, it is no longer coloured in traversing this apparatus in whatever azimuth it may be turned; and when it suffers this modification on leaving the apparatus, instead of receiving it at its entrance, it takes, as does also the oil of turpentine in a similar case, the same appearances as if it had been received immediately upon the rhomboid of calcareous spar after leaving the crystallized lamina.

Lastly, when the incident light, after having been completely depolarized by two successive reflexions before entering this apparatus, is again at its exit twice totally reflected in a glass parallelopiped, it is found to be again brought to a state of complete polarization, as if the apparatus had not been used, or been replaced by a tube filled with oil of turpentine. It would appear then from these numerous and varied phænomena that this apparatus possesses all the optical properties of oil of turpentine. This was also what I at first thought, but a more attentive examination convinced me that a notable difference existed between these two kinds of phænomena.

Having placed a glass parallelopiped at the extremity of a tube 0^{m}.50 in length filled with oil of turpentine, so that the rays which traversed it suffered double total reflexion parallel to the primitive plane of polarization, I caused the extraordinary image, which was of a violet red, to disappear by the interposition of a lamina of sulphate of lime, about 0^{mm}.12 in thick-

ness, which gave nearly the same tint in the extraordinary image, that is to say, the extreme red of the second order, or the purple of the third. But on calculating from these data the apparent rotation of the plane of polarization of the red rays in oil of turpentine, on the theory of the apparatus which I have just described, I found an angle more than double that which M. Biot had determined by direct measurement, and which he had the goodness to communicate to me. To discover what could occasion so great a difference, I wished to observe the series of colours produced by different lengths (from 0 to 50 centimetres) of oil of turpentine. Having placed the tube in a vertical position, and fixed the principal section of the rhomboid of calcareous spar in the primitive plane of polarization, I caused the fluid which it contained gradually to flow out, and was very much astonished to see the extraordinary image pass through a white slightly coloured, and finally arrive at black without showing at all the red of the first order.

It is sufficiently different from the red of the second order to be easily distinguished, and by the simple inspection of the tints, it is easy to observe that that which corresponds to 50 centimetres of the oil of turpentine is not of the first order. Besides, what still better determines its rank, is the thickness of the crystallized lamina which causes the extraordinary image to vanish. It may be objected, perhaps, that this disappearance only taking place when the glass parallelopiped is used, it is possible that double reflexion may alter the tint produced by oil of turpentine, and cause it to descend in the order of the rings. But, in the first place, on examining at the same time the direct and the reflected images, one must be convinced that their colour is absolutely the same; secondly, experiment and theory both show that double reflexion, at the incidence which produces complete depolarization, modifies all the rays in the same manner, and that, if it changes, in general, the interval which separates two systems of waves polarized in contrary planes, this change for each kind of rays is proportional to the length of their waves; so that it can neither raise nor lower the tint, the rank of which solely depends upon the relation of the constant part of the interval to the lengths of the different luminous waves. Therefore it remains confirmed that the extraordinary image passes from black to the red of the second order, without passing through the red of the first.

This succession of colours, so odd in appearance and so opposed to that observed in reflected rings, may be explained in a very simple manner, if we admit that the double refraction in the oil of turpentine is not the same for rays of different kinds, and that it is strongest for those whose waves are shortest. It is known that the double refraction of the violet rays in calcareous spar is more marked than that of the red; it is probably the same in other crystals; but these differences are too slight in relation to the difference of velocity between the ordinary and extraordinary ray. It is for this reason that we have supposed until now that the interval which separates two systems of waves was sensibly the same for rays of various colours. But when the double refraction becomes extremely feeble, as in oil of turpentine, where the velocities of the ordinary and extraordinary rays scarcely differ by the one millionth, it is very possible that the dispersion of the double refraction (if I may so express myself) becomes a considerable part of the double refraction itself. It would result from some approximative measurements to be mentioned in the sequel of this memoir, that the double refraction of the extreme violet rays ought to be about one and a half that of the extreme red rays. This hypothesis does not appear to me improbable, or even contrary to analogy, which ought not properly to be stretched to its greatest length; and in adopting it we are enabled to account for that singular anomaly of which I have just spoken, and which without it appears to me inexplicable.

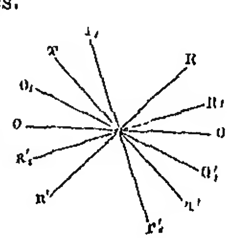
It is easily conceived that the interval between the two systems of waves being no longer the same for all the rays, as in the phænomenon of coloured rings, or in that presented by thin crystalline laminæ, but changing with the length of the luminous waves, the succession of the colours may be quite different, as this interval is so much the greater in proportion as the waves are short, which alters doubly the relation between its length and that of the luminous waves. Thus we arrive at the red of the second order, when the interval between the two systems of red waves has not yet exceeded that which would produce the red of the first order, if it were the same in the rays of different colours.

This hypothesis enables us to apply to the polarization exercised by homogeneous fluids, the theory which I set forth in the

preceding memoir in explanation of the colours produced by crystalline laminæ placed between two glass parallelopipeds at right angles to each other. It is natural to think, from the intimate relation which exists between these two classes of phænomena, that they result from the same general modifications communicated to the luminous rays, and that the difference which they present in the succession of the colours is alone due to the double refraction not being the same for the different rays in the fluid particles, whereas, on the contrary, it is sensibly the same in the crystalline lamina.

It is evident that the cause of the phænomena of colorization to which they give rise must be sought for in the individual constitution of the particles, as they are entirely independent of their arrangement, and yet at the same time so dependent upon their form, that, to use the expression of M. Biot, according to the nature of the fluid the light is turned from left to right, or from right to left. I shall suppose therefore that they are so constituted as to produce in the luminous rays which traverse them the modifications which they undergo in the apparatus that I have just described, that is to say, that the light on entering and on leaving each particle undergoes the same modification as that produced by double total reflexion, and that it suffers, besides, double refraction within it.

I shall at first show, as the result of this hypothesis, that the rays which have been ordinarily or extraordinarily refracted in a particle thus constituted always suffer the same refraction in the particles of the same nature which they successively traverse, whatever may be the azimuths of their axes.



Let OO' be the principal section of the first particle, RR' and TT' the two planes which correspond to those of double reflexion in the apparatus, and which I shall call the *plane of entrance* and the *plane of exit*, these are, by hypothesis, perpendicular to each other, and inclined at an angle of 45° to the principal section. Let $O_1O'_1$ be the principal section of the second particle traversed by the pencil of light, $R_1R'_1$ and $T_1T'_1$ the two planes in which it suffers, at its entrance and exit, the modification just spoken of. It consists, as was seen in the foregoing memoir, in each pencil

of light being divided into two systems of polarized waves, the one parallelly, the other perpendicularly to the plane, the first being a quarter of an undulation behind the second

Let us consider that part of the incident ray which has been ordinarily refracted in the first particle and thus polarized in the direction of OO', and let us represent it by O On leaving the particle it divides itself into two systems of polarized waves, the one parallelly the other perpendicularly to $I\,I'$ the intensities of which, as also the relative positions, are represented by the following expressions

$$\begin{array}{cc} \sqrt{\frac{1}{2}}\,O_{\frac{1}{4}} & \qquad \sqrt{\frac{1}{2}}\,O \\ O\ I & \qquad O\ R \end{array}$$

In fact, as I observed in the preceding memoir when a system of waves is thus decomposed into two others, the velocities of the molecules of æther, in their oscillations, are not proportional to the square of the cosine and sine of the angle OCI but simply to the sine and cosine, so that it is not the sum of the velocities which is constant but the sum of the squares of the velocities This is a consequence of the principle of the conservation of living forces in the vibrations of elastic bodies

By the action of the plane of entrance $R_1\,R_1'$ of the second particle each of these pencils of light divides itself into two other systems of waves making in all four if the angle OCO_1 which the principal section of the second particle makes with that of the first be represented by p, the intensities of their vibrations will be—

$$\begin{array}{cccc} \sqrt{\frac{1}{2}}\sin p\ O_{\frac{1}{4}} & \sqrt{\frac{1}{2}}\cos p\ O_{\frac{1}{4}} & \sqrt{\frac{1}{2}}\cos p\ O_{\frac{1}{4}}, & \sqrt{\frac{1}{2}}\sin p\ O \\ O\ I\ R_1 & O\ I\ I_1 & O\ R\ R_1 & O\ R\ I_1' \end{array}$$

In virtue of the double refraction of this particle each of these pencils divides itself again into two, polarized, parallelly and perpendicularly to the plane $O_1\,O_1'$ The intensities of the systems of waves ordinarily refracted in the second particle will be represented by the following expressions

$$\begin{array}{cccc} +\frac{1}{2}\sin p\ \ O_{\frac{1}{4}} & +\frac{1}{2}\cos p\ \ O_{\frac{1}{4}}, & +\frac{1}{2}\cos p\ \ O_{\frac{1}{4}}, & -\frac{1}{2}\sin p\ \ O \\ O\ I\ R_1\ O_1 & O\ I\ I_1\ O_1 & O\ R\ R_1\ O_1 & O\ R\ I_1'\ O_1' \end{array}$$

Adding the expressions which have the same characteristic,

and recollecting that the $\frac{1}{2}$ in the characteristic is equivalent to the sign minus, we obtain, $-\sin p \,.\, O$ and $\cos p \,.\, O_{\frac{1}{4}}$. But the resultant of these two systems of waves differing by the quarter of an undulation, is $\sqrt{O^2 \sin^2 p + O^2 \cos^2 p}$, or O. Hence the waves arising from the ordinary refraction of the first particle suffer ordinary refraction in the second, because, both in the one and the other, the principal section is turned towards the same side as regards the plane of entrance.

This principle may be further verified by calculating the intensity of the polarized light in the plane $E_1 E_1'$ perpendicular to the principal section $O_1 O_1'$. We then obtain for the four constituent pencils,—

$$-\frac{1}{2}\sin p \,.\, O_{\frac{1}{4}}, \quad \text{or} \quad +\frac{1}{2}\sin p \ O, \qquad\qquad +\frac{1}{2}\cos p \ O_{\frac{1}{4}},$$
$$O.T\ R_1.E_1'. \qquad\qquad O\ T.T_1.E_1'.$$

$$-\frac{1}{2}\cos p \,.\, O_{\frac{1}{4}}, \qquad\qquad -\frac{1}{2}\sin p \,.\, O.$$
$$O\ R\ R_1.E_1' \qquad\qquad O.R.T_1'\ E_1'.$$

The expressions having the same characteristic are equal and of contrary signs, so that these four systems of waves mutually destroy each other. Thus no one of the ordinary rays issuing from the first particle can suffer extraordinary refraction in the second. If the latter be turned in such a manner that the plane of exit becomes the plane of entrance, it is evident that it will still be placed upon the same side in relation to the principal section, and consequently the rays will still be refracted in the same manner.

It should be noticed that the calculations which have just been made, and the results to which they lead, are independent of the relations of intensity of the double refractions exercised by these particles, and that we have only supposed them to be constituted in the same manner; that is to say, that their axes were turned towards the same side in relation to their plane of entrance. Hence, whatever may otherwise be their inclinations, or even whatever the nature of the particles successively traversed by the incident light, the rays which have in the first instance suffered ordinary or extraordinary refraction continue to undergo the same kind of refraction throughout the whole extent of the fluid. The hypothesis which we have adopted will ex-

plain (what at first appears difficult to conceive) how it happens that the double refraction excited by particles so irregularly arranged does not give rise to more than two systems of luminous waves in the fluid

When it is homogeneous, the effects produced by all the particles are added, and the interval between the two systems of waves ought to be increased in proportion to the length of the passage When the fluid is composed of two different kinds of particles, the axes of which however are turned in the same manner with relation to the plane of entrance, their effects are added if in both it is the same refraction that is the most rapid and they are subtracted on the contrary, if the most rapid refractions are of opposite natures The inverse takes place when the particles have their axes turned in contrary directions relatively to their planes of entrance

It is likewise seen that the mixture of any number of fluids of different kinds, the particles of which are thus constituted ought to produce the same effect upon light as that which it would suffer if it traversed successively these different fluids Hence the problem in this general case may always be reduced to the particular case of a homogeneous fluid

In the preceding memoir in explaining the theory of the apparatus which I take here as a model of the constitution of the particles I showed that the intensity and the position of the different systems of waves which it produced united in any plane of polarization whatever, are independent of the azimuth in which the apparatus is directed and only depend upon the mutual inclination of the two extreme planes of polarization We may then suppose all the particles of the fluid turned in such a manner that their principal sections are parallel to each other then, if one of these particles is considered as comprised between two others its plane of entrance is at right angles to the plane of exit of the one which precedes it, and thus causes to disappear the quarter of an undulation difference produced by the latter In the same manner its plane of exit is at right angles to the plane of entrance of the following particle, which destroys consequently the modification which it had communicated to the light Thus all the intermediate planes of entrance and of exit may be put out of view, reserving only the plane of entrance of the first particle and the plane of exit of the last It is then evident that the formula which I have calculated for the appa

ratus is applicable to the fluid. If, then, o and e represent the numbers of ordinary and extraordinary undulations in the fluid, and i the angle which the primitive plane of polarization makes with the principal section of the rhomboid of calcareous spar that serves to develope the colours, we obtain, as a general expression for the intensity of the luminous vibrations in the ordinary image,

$$F \sqrt{\frac{1}{2}+\frac{1}{2}\cos\left[2i-2\pi(e-o)\right]}, \quad \text{or } F \cos\left[i-\pi(e-o)\right],$$

F being the intensity of the incident pencil, and for the extraordinary image,

$$F \sin\left[i-\pi(e-o)\right].$$

These formulæ have been calculated for the case in which the axis of the crystalline lamina inserted between the two glass parallelopipeds was to the right of the first plane of double reflexion; they apply consequently to those fluids the particles of which have their principal section to the right of their plane of entrance. In the opposite case, the formulæ become

$F \cos\left[i+\pi(e-o)\right]$ for the ordinary image, and
$F \sin\left[i+\pi(e-o)\right]$ for the extraordinary image.

M. Biot observed that the angle through which the principal section of the rhomboid of calcareous spar must be turned, in order to cause the disappearance of the same kind of rays of the extraordinary image, was proportional to the length of fluid traversed. This remarkable law is an immediate consequence of the preceding formulæ. In reality, the kind of rays in question would cease to exist in the extraordinary image when we have $i \pm \pi(e-o)=o$, or $i = \pm\pi(e-o)$, the upper signs correspond to the case in which the particles have their principal section to the right of their plane of entrance, and the lower signs to the contrary case. But e and o are proportional to the distance traversed in the fluid, consequently the angle i must also be proportional to it.

If it be supposed that $e > o$, the first value for i will be positive and the second negative. The angles having been reckoned from left to right in the calculations, we must conclude from these values for i, that in the first case the light rotates from left to right, and in the second from right to left, using the language of M. Biot, which is the simplest mode of expressing the appearances of the phenomenon. If, on the contrary, we suppose

$e < o$, the light will revolve from left to right when the principal section of the particles is to the left of their plane of entrance and from right to left when this plane is to the left of the principal section

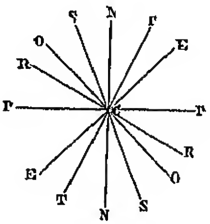
It is clear from this, that when polarized light traverses successively two fluids which cause the light to rotate in contrary directions, the effects produced by the one upon each kind of rays are subtracted from the effects produced by the other so that with homogeneous light the extraordinary image is made to disappear completely, by lengthening or shortening one of the tubes But it may happen with white light that this compensation is impossible, if, for instance, the variations of the double refraction of the different rays do not follow the same law in both fluids for then the relation of the lengths, which produce exact compensation for one species of rays, would not produce it for another

To complete the theory which I have just set forth there remain to be explained two phenomena described at the commencement of this memoir When polarized light has suffered at an azimuth of 45, the modification produced by double total reflexion before traversing the oil of turpentine, it no longer gives rise to colours and when it only undergoes this modification after passing out of the tube, the tints of the two images remain constant during the rotation of the rhomboid of calcareous spar with which they are observed and they only vary in intensity in passing into perfect whiteness, as those of the crystalline laminæ cut parallel to the axis

The cause of the first phenomenon is very simple the light then undergoes only one kind of refraction in the liquid In fact, we have seen that the rays polarized parallelly or perpendicularly to the principal section of a particle, after having suffered, on leaving it, the modification in question, can only undergo a single kind of refraction in the following particle The polarized light, thus modified, can only be refracted in one single manner in the oil of turpentine, and ought to produce, consequently but one single system of waves

I am now about to consider the case when the light only undergoes this modification on leaving the tube Let PP′ be the primitive plane of polarization We have seen that the action of the particles upon the luminous vibrations was always the same in whatever azimuth their axes were turned We

may consequently suppose all their principal sections inclined at an angle of 45° to the plane of primitive polarization, so that their planes of entrance or of exit coincide with that plane I shall suppose, for example, that they are the planes of entrance Having thus turned all the principal sections in the same direction, we may suppress all the planes of entrance and of exit, excepting the first and last The first coincides with P P′ by hypothesis, and the last, represented in the figure by N N′, is perpendicular to it Let R R′ be the plane in which the light is twice reflected in the glass parallelopiped, after having traversed the oil of turpentine, let, lastly, S S′ be the principal section of the rhomboid of calcareous spar with which the colours are produced I represent the angle P C R by i, and the angle P C S by s



The plane of entrance, coinciding with that of primitive polarization, does not modify the light By the double refraction of the particles it is divided into two systems of waves polarized, the one in the principal section O O′, the other in the perpendicular plane E E′ If F represents the velocity of the ethereal molecules in the vibrations of the incident pencil, their velocities, in the ordinary and extraordinary waves, will be

$$\sqrt{\frac{1}{2}}\,F_o \qquad \text{and} \qquad \sqrt{\frac{1}{2}}\,F_e$$

P O   P E′

o and e always representing the numbers of the ordinary and extraordinary undulations completed in the oil of turpentine by the kind of rays under consideration By the action of the plane of exit N N′, each of these pencils divides itself into two others, which gives in all the four following pencils

$$\frac{1}{2}F_{o+b} \qquad \frac{1}{2}F_{o} \qquad \frac{1}{2}F_{e+b} \qquad \frac{1}{2}F_e$$

P O N   P O P   P E′ N′   P E′ P

The double reflexion in the glass parallelopiped divides then each of these four pencils into two others, polarized, the one in the plane of reflexion R R′, the other in the perpendicular plane T T′ Lastly, by the action of the rhomboid of calcareous spar, each of these eight pencils is divided into two others, polarized

parallelly and perpendicularly to the principal section S S′. It is sufficient to consider those which concur in the formation of one of the images, the extraordinary image for example. Their intensities are represented by the following expressions:

$$\text{P O N R S} \qquad +\frac{1}{2}\sin i \;\cos(i-r)\; I_{+\frac{1}{2}}$$

$$\text{P O N T S} \qquad +\frac{1}{2}\cos i \;\sin(i-r)\; I'_{+\frac{1}{2}}$$

$$\text{P O P R S} \qquad +\frac{1}{2}\cos i \;\cos(i-r)\; I_{+\frac{1}{2}}$$

$$\text{P O P T}' \text{ S}' \qquad -\frac{1}{2}\sin i \;\sin(i-r)\; I$$

$$\text{P E}' \text{ N}' \text{ R}' \text{ S}' \qquad -\frac{1}{2}\sin i \;\cos(i-r)\; I'_{+\frac{1}{2}}$$

$$\text{P E}' \text{ N}' \text{ T}' \text{ S}' \qquad -\frac{1}{2}\cos i \;\sin(i-r)\; I_{+\frac{1}{2}}$$

$$\text{P L}' \text{ P R S} \qquad +\frac{1}{2}\cos i \;\cos(i-r)\; I_{+\frac{1}{2}}$$

$$\text{P E}' \text{ P T}' \text{ S}' \qquad -\frac{1}{2}\sin i \;\sin(i-r)\; I'$$

Adding the expressions which have the same characteristic, and observing that $\frac{1}{2}$ in the characteristic is equivalent to the minus sign, the eight pencils are reduced to four:

$$-\frac{1}{2}\sin i\,[\cos(i-r)+\sin(i-r)]\,I$$

$$+\frac{1}{2}\cos i\,[\cos(i-r)+\sin(i-r)]\,I'_{+\frac{1}{2}}$$

$$+\frac{1}{2}\sin i\,[\cos(i-r)-\sin(i-r)]\,I'$$

$$+\frac{1}{2}\cos i\,[\cos(i-r)-\sin(i-r)]\,I_{+\frac{1}{2}}$$

On inspecting these formulæ, it is seen at once that the image passes to white when $i-r=45°$, for then the two last pencils disappear; the intensity of the light becomes independent of the difference between e and o, and consequently is the same for every kind of rays. The colour of the image attains, on the contrary, its highest degree of vividness when $i-r$ is equal to zero or to 90°, that is to say, when the principal section of the

rhomboid of calcareous spar is parallel or perpendicular to the plane of double reflexion; in fact, the expressions in which the characteristic is a function of e become then equal to those in which the characteristic contains o.

It is easy to perceive also that the rotation of the rhomboid, that is to say the variations of i, ought not to alter the nature of the tint. In fact, if we consider the resultant of the two first systems of waves, the variations of i, affecting only the common factor $\cos(i-r)+\sin(i-r)$, cannot change the position of that wave, but only its intensity. For the same reason, these variations do not change the position of the wave resulting from the union of the two other pencils. Hence the interval between these two resultants, which alone determines the nature of the tint, suffers no change during the rotation of the rhomboid.

It is not the same with the variations of r, as they affect unequally the two first pencils, the one of which is multiplied by $\sin r$, and the other by $\cos r$, they cause the position of their resultant to change. They likewise change the position of the other resultant, and in a contrary direction, in consequence of the opposition of sign between the first and the third pencil. But this becomes still more evident on calculating the total resultant of these four systems of waves. The general expression for the intensity of its vibrations is found to be—

$$F\sqrt{\frac{1}{2}+\frac{1}{2}\left[\cos^2(i-r)-\sin^2(i-r)\right]\cos\left[2r-2\pi(e-o)\right]},$$

or

$$F\sqrt{\frac{1}{2}+\frac{1}{2}\cos 2(i-r)\cos\left[2r-2\pi(e-o)\right]}.$$

It is clear, from this formula, that the variations of i only affect the intensity of the tint*, whereas those of r change its nature. When r is equal to 45°, for instance, $\cos\left[2r-2\pi(e-o)\right]$ becomes $\cos 2\pi\left[\frac{1}{4}-(e-o)\right]$, and the colour of the image is that which corresponds to a change of a quarter of an undulation in

* The maximum intensity of the tint corresponds to $i=r$, as had been already observed by simple inspection of the constituent pencils. The formula then becomes

$$F\sqrt{\frac{1}{2}+\frac{1}{2}\cos\left[2r-2\pi(e-o)\right]},\ \text{or}\ F\ \cos\left[r-\pi(e-o)\right].$$

Thus the tint is precisely that which was observed before the interposition of the glass parallelopiped, with the same position of the rhomboid of calcareous spar.

the interval $e - o$ comprised between the two systems of waves. When r is equal to zero, on the contrary, the tint corresponds exactly to the interval $e - o$: it is this which may be called the fundamental tint. The formula then becomes—

$$F\sqrt{\frac{1}{2} + \frac{1}{2}\cos 2i \cdot \cos 2\pi(e - o)}$$

this is precisely the general expression for the intensity of the luminous rays in the ordinary image for the particular case of a crystalline lamina, the axis of which is placed in an azimuth of 45° with respect to the plane of primitive polarization.

If the double refraction exerted by oil of turpentine upon the different kinds of rays was sensibly constant, as in crystals, it would follow that we could always exactly compensate the effect which it produces upon polarized white light with a crystallized lamina of proper thickness, by turning the parallelopiped in such a manner as to make the plane of double reflexion parallel to the plane of primitive polarization. But we have seen that this is not the case, and that it follows from the changes of the fundamental tint, that the double refraction of the oil of turpentine varies on the contrary very much with the length of the luminous waves. We may even conceive that the law of these variations may be such as to render impossible an exact compensation in the case of white light.

To conceive clearly the necessary conditions of this compensation, instead of referring the intervals comprised between the two systems of waves in the oil of turpentine and in the crystalline lamina to the same unit of length, let us suppose them expressed for each kind of luminous undulation, in a function of the length of that undulation. It is clear that, if the differences between the numbers which express these relations for the tube filled with oil of turpentine are equal to the differences between the corresponding numbers of the crystalline lamina, exact compensation is possible: for it results from this hypothesis that the numbers of the crystalline lamina are equal to the numbers of the tube plus a common number, which is generally a fraction. Now we may suppress all the integer numbers, and consider only the remaining fraction, the only quantity which is opposed to the exact compensation. But, from the formula—

$$I\sqrt{\frac{1}{2} + \frac{1}{2}\cos 2(i - r)\cos[2r - 2\pi(e - o)]}$$

we perceive that it is always possible, by the value which is given to r, to introduce the fraction that is requisite into the parenthesis $2 i - 2 \pi (e - o)$, and to cause this last discordance to disappear. It is this last fraction that determines the azimuth into which the plane of double reflexion must be turned to obtain the complete disappearance of one of the images.

From some experiments of this nature, which I have not been able at present to conduct with all the precision of which they are capable, it appeared to me that the condition which I have just announced was visibly fulfilled in the oil of turpentine, for I observed the complete disappearance of one of the images, at least as far as I could judge.

The first experiment which I made is that which I have already mentioned at the beginning of this memoir. Having filled a tube $0^{m}{\cdot}50$ in length with oil of turpentine, I fixed at its posterior extremity a glass parallelopiped in which the emerging rays suffered double total reflexion in a plane parallel to that of primitive polarization, then, by placing between this parallelopiped and the rhomboid of calcareous spar a lamina of sulphate of lime, about $0^{mm}{\cdot}12$ in thickness, and inclining its axis to the right at an angle of 45° to the plane of double reflexion, I caused the extraordinary image, which was violet-red or purple of the third order, to disappear. A lamina of sulphate of lime, $0^{mm}{\cdot}12$ in thickness, does not quite correspond to this tint in the table of Newton; but, as it was necessary to incline this lamina a little perpendicularly to its axis to obtain complete disappearance, I estimated that the tube $0^{m}{\cdot}50$ in length ought to be compensated by a lamina of sulphate of lime, corresponding to the number 21 in the first column of Newton's table. If the rotation of the plane of polarization of the mean red rays, produced by a similar lamina comprised between two parallelopipeds placed at right angles to each other, is calculated, we find, by means of the formula—

$$i = - \pi (e - o),$$

for the entire arc, $309^{\circ}{\cdot}6$. But, from the succession of colours which oil of turpentine presents from zero to a length of $0^{m}{\cdot}50$, we have seen that there ought to be for this fluid one undulation less in the interval between the two systems of waves. Now, one undulation corresponds here to 180°; deducting 180° from $309^{\circ}{\cdot}6$, there remain $129^{\circ}{\cdot}6$, which, divided by 50, give $2^{\circ}{\cdot}59$ for the rotation of the red rays corresponding to 1 centimetre.

Making a similar calculation for the other kinds of rays, we obtain for the rotations which they suffer in traversing 1 centimetre of oil of turpentine, the following numbers

Orange rays	2.99
Yellow rays	3.36
Green rays	3.90
Blue rays	4.48
Indigo rays	4.96
Violet rays	5.49

Having fixed a lamina of sulphate of lime, 0^{m}.16 in thickness, upon a glass parallelopiped, I placed it at the extremity of an apparatus filled with oil of turpentine the length of which I could vary. By a double experiment I sought what length produced the most exact compensation and in what azimuth the plane of double reflexion of the parallelopiped must be placed to cause the complete disappearance of one of the images. The length I found to be 2.03, and the azimuth about 35° to the left of the plane of polarization: it was the ordinary image which disappeared. It follows, that to infer the rotation produced by this tube, we must first deduct 90° − 35° or 55°, from the rotation which is produced by the lamina 0^{m}.16 in thickness, which is 1145°.8 for the mean red rays. We must then, subtract an entire number of half circumferences depending likewise upon the difference in the succession of the tints produced by the lamina and by the oil of turpentine. My apparatus not permitting me to follow them from 0.50 to 2.03, I calculated this number from the preceding experiment being sure that I could not be half a circumference in error, and I saw that it was necessary to deduct three half circumferences or 540°. The rotation of the red rays produced by traversing 2.03 of oil of turpentine is therefore 550°.8, dividing this quantity by 203, we have for the rotation of the red rays in 1 centimetre 2°.71.* This result accords very closely with that obtained by M. Biot by the actual measurement of the angles, at least if they are the *mean* red rays which predominate in the light that he employed.

* Starting from these data we find that the ordinary and extraordinary red rays only differ in their velocity by $\frac{1}{1573000}$ and the ordinary and extraordinary violet rays by $\frac{1}{1170000}$ so that the double refraction of the red rays is to the double refraction of the violet rays as 1 to 1.34.

Making the same calculation for the other rays, we obtain the following angles —

Orange rays . .	3°07
Yellow rays . .	3 42
Green rays	3 91
Blue rays	4 44
Indigo rays . . .	4 87
Violet rays . .	5 35

These results differ sensibly from those deduced from the previous experiment, and the bases of the calculation are in fact sufficiently different, for if, by a proportion, setting out from the data of the second observation, we inquire what length of oil of turpentine ought to be exactly compensated by a lamina of sulphate of lime corresponding to the number 21 of the first column of Newton's table, we find it should be 0^m 548 instead of 0^m 50

Notwithstanding the difficulties which arise from the greater length of the apparatus in the second experiment, and which might be the causes of error, I am led to think that the results which have been deduced from it are more exact than the former, not only because the measurements and observations were made upon larger quantities, but also because I attended more to the precautions which are necessary to approach exactitude. Nevertheless, I do not consider even these last results as very exact, because the apparatus was not arranged in a sufficiently convenient manner for making such delicate observations with precision*. Before having the honour of presenting them to the Academy, I should have wished to have repeated the experiments with a better-arranged apparatus, and to have verified these angles by direct measurements of the rotation in homogeneous light, but other researches oblige me to relinquish these, at least for some time.

I have shown how it is possible to distinguish the different phænomena presented by oil of turpentine, in supposing that each of its particles possesses the power of double refraction,

* It appeared to me that the tints produced by the 2^m 03 of oil of turpentine were a little less feeble than those of the lamina 0^m 16 In traversing 2^m·60 of this essential oil, polarized light still presents an appreciable colorization, this appears to establish a slight difference between the phænomena and the hypothesis of complete compensation by the interposition of a lamina of sulphate of lime

and impresses on the luminous rays at their entrance and their exit, the same modification which they are subjected to by double total reflexion in the interior of transparent bodies The definition of these modifications in the present state of the theory is sufficiently complicated It is possible however that after all the hypothesis may be more simple than it appears It is at least certain that the phænomena cannot be more simply represented than by the general formula

$$F \cos [i \pm \pi (e - o)],$$

to which this hypothesis has led me It seems to me very probable consequently that this formula is really the expression of the resultant of all the various movements of the luminous waves in the oil of turpentine But it is possible that these elementary movements do not take place precisely in the manner that I have supposed However that may be, the theory which I have just advanced has the advantage of connecting the colorization of homogeneous fluids in polarized light with the same principles as those upon which the colorization of crystalline laminæ depend it indicates the points of contact in these phænomena, which differ so much in appearance and in this respect, it appears to me it may be of some utility to science

Article III.

Memoir on Metallic Reflexion. By M. J. Jamin.*

[From the *Annales de Chimie et de Physique* for March 1847.]

IN a remarkable memoir published in the Philosophical Transactions for April 1830, Sir David Brewster called the attention of scientific men to the phænomena presented by the reflexion of metals; and without endeavouring to determine the nature of the modifications produced on light by metals, he performed experiments which led him to the discovery of some isolated laws, of which he gave no theoretical explanation. Since that period, metallic reflexion has become the object of continued researches, some mathematical, of which we shall often have occasion to speak in the sequel; the others experimental, too few in number for the complete solution of the problem, often destitute of the necessary precision, and employing very complicated methods of measurement. It is with the intention of simplifying these methods and extending these researches, that I have undertaken the following experiments. Before proceeding to them I shall recapitulate the most simple and general laws discovered by Sir David Brewster.

1. If a ray of light, polarized in azimuths of 0° or 90°, be reflected from a metal any number of times, it always remains polarized in the same plane after reflexion.

2. Every ray, which before reflexion is polarized in any other azimuth, becomes partially depolarized after having undergone the action of the metal.

3. If we cause a beam of natural light to fall on a metallic mirror, it is not polarized by reflexion at any incidence, and when examined by a polariscope, presents the appearances of a partially polarized ray. Sir David Brewster has also remarked, and this is an important observation, that there exists a particular incidence for which the proportion of light polarized by reflexion is greater than for any other; this incidence has been called *the angle of maximum polarization.*

* The Editor has to acknowledge his obligations to Alfred W. Hobson, B.A., St John's College, Cambridge, who kindly undertook the translation of this paper.

4. When polarized light is reflected several times from parallel metallic mirrors at the incidence of maximum polarization, the polarization is restored after an even number of reflexions.

5. Finally, the reflected beam becomes again polarized after an even or uneven number of reflexions under a great number of incidences determined by laws which remain to be found.

Since a ray polarized in any plane before incidence may always be decomposed into two others polarized in azimuths of 0° or 90°, which, according to Sir David Brewster, do not change their azimuth by reflexion, the reflected ray will always be formed by the superposition of two rays polarized in those principal azimuths, and its state of vibration will be known if we have found out beforehand the modifications undergone by the component rays during reflexion. The first question therefore to be answered is this: What transformations occur during reflexion in rays polarized in the principal azimuths?

Now every polarized ray which undergoes any action without losing its polarization and without changing its azimuth can only be affected by changes of phase and variations of intensity; we have therefore to examine if these modifications occur, and according to what laws they are produced, for two rays polarized, one in the azimuth of 0°, and the other in the azimuth of 90°. This is the investigation we are about to commence, beginning with the determination of the intensities.

I. *Measure of the Intensity of Light reflected by Metals.*

If we cause rays polarized in the azimuths of 0° or of 90° to fall on a plate of glass, the intensities of the reflected beams will be represented by the following formulæ of Fresnel:—

$$J'^2 = \frac{\sin^2(i-r)}{\sin^2(i+r)} \qquad I'^2 = \frac{\tan^2(i-r)}{\tan^2(i+r)} \qquad (1.)$$

These formulæ, verified by MM. Arago and Brewster, are recognised at the present day by experimenters; they will serve us as a starting point for measuring the quantities of light I^2 and J^2 reflected by metals, in order to which it will suffice to compare I^2 and I'^2 on one side, and J^2 and J'^2 on the other.

To make this comparison, let us place in contact two plates, the one of glass, the other of metal, so that the two polished faces may be in the same plane, and the two plates form one reflecting surface, of which one portion is glass and the other metal; then

reflect from the middle of this double plate a ray polar the plane of incidence one half of the ray will be reflec the glass, the other by the metal, both will remain polar the azimuth of 0°, and will give only a single image in tr a doubly refracting prism, whose principal section coincid the primitive plane of polarization

But if we turn this prism through an angle (β), we sh tain one ordinary and one extraordinary image for each two portions of the beam reflected by the glass and by the hence there will be four images, whose intensities will be

	Metal	Glass
O	$J^2 \cos^2 \beta$	$J'^2 \cos^2 \beta$
E	$J^2 \sin^2 \beta$	$J'^2 \sin^2 \beta$

When (β) varies, the ordinary and extraordinary imag dergo inverse changes of intensity, and there is always a cular value of (β) which makes the ordinary image of the equal to the extraordinary one of the glass

We have in this case,

$$J^2 \cos \beta = J'^2 \sin \beta,$$

and replacing J'^2 by its value given by Fresnel's formula

$$J^2 = \tan^2 \beta \frac{\sin^2 (i-r)}{\sin^2 (i+r)}$$

If on the contrary, we seek for the value (β') which the extraordinary image of the metal equal to the ordina of the glass, we obtain

$$J^2 = \cot^2 \beta' \frac{\sin (i-r)}{\sin (i+r)}$$

Experiment will give us the values of (β) and (β'), which to be complementary, and we shall calculate J^2 by means formulæ (2) and (3)

It is moreover obvious that this method is applicable case where the light is polarized in the azimuth of 90° t muth of equal tints will be determined in the same way, shall obtain

$$I^2 = \tan^2 \beta \frac{\tan^2 (i-r)}{\tan^2 (i+r)}, \qquad I'^2 = \cot^2 \beta' \frac{\tan^2 (i-r)}{\tan^2 (i+r)},$$

only, in the neighbourhood of the angle of polarization fo there will no longer be any light reflected by this substan therefore no comparison possible, hence will result a some degrees in the experiment

Thus we shall polarize the light successively in the plane of incidence and the plane perpendicular to it, and in order to obtain the proportion of light reflected in each of these two cases by the metallic mirror, we shall turn the analyser until two images of contrary name, produced by the two substances, become equal. We shall find by two distinct observations, which ought to agree, the azimuths (β) and $90^\circ-\beta$, of the principal section, and the intensity of the light reflected by the metal will be equal to that reflected by the glass multiplied by the square of the tangent of (β).

This method, which in a theoretical point of view is of extreme simplicity, cannot lead to accurate results unless the index of refraction of glass is perfectly known, since the intensities I' and J'^2 of the light reflected by this substance are functions of the incidence and of the index of refraction. Now there are two methods of ascertaining this latter quantity: the first consists in seeking directly for the index by forming the glass into a prism; the second in determining the angle of polarization (i) of glass and putting $\tan(i)=n$. Unfortunately, the two methods have given results differing by a notable quantity; and in order to choose between the two, it must be remembered that the preceding formulæ cannot be used unless they are true in each particular case, and provided they give the intensity nothing for light reflected at the angle of maximum polarization, when the ray is polarized perpendicularly to the plane of incidence, which requires that we have $\tan(i)=n$. We must therefore employ for the determination of the index (n), a method which verifies formula (1.), I have adopted the following.

The two formulæ (1.) lead to a third, which gives us the value of the azimuth A' of the reflected light, when the incident ray is polarized at 45° to the plane of incidence; this formula is the following:

$$\tan A' = \frac{\cos(i+r)}{\cos(i-r)},$$

a relation evidently verified by the same value of (n) as the preceding, since it is a consequence of them; and instead of the value of the index of refraction which satisfies the former, we may determine that which agrees with the last. We obtain successively

$$\left.\begin{aligned}\tan A' &= \frac{\cos(i+r)}{\cos(i-r)} = \frac{1-\tan i\tan r}{1+\tan i\tan r}\\ \tan i\tan r &= \frac{1-\tan A'}{1+\tan A'} = \tan(45^\circ - A'),\\ \tan r &= \frac{\tan(45^\circ - A')}{\tan i}\end{aligned}\right\} \quad (5.)$$

Then, the azimuth of polarization of the incident light being 45°, the incidence being i°, we shall measure A', calculate (r) by means of the formula (5.) and (n) by the equation $n=\frac{\sin i}{\sin r}$. The value of (i) being arbitrary, we are at liberty to experiment under various incidences, and to obtain from each experiment values of (n) of which we shall take the mean. The following are some of the results:—

Incidences	Values of n
80	1.4909
70	1.4932
60	1.4896
50	1.4919
40	1.4900
30	1.4965
Mean	1.4925

This result differs only by three hundredths from that given by direct experiments for the index of refraction of glass. We shall adopt it in the calculation of formulæ (1.), and the success of our experiments will henceforth depend on the care with which the angles (i) and (β) are measured. I shall now enter into some details on this subject.

A horizontal circle having a copper stand, supports a tube blackened in its interior, fixed on the circle, constantly directed towards the centre, and furnished at its two extremities with cross wires for the purpose of fixing the direction of the incident ray. This tube carries a Nichol's prism which polarizes the light, and whose direction is determined by a graduated vertical circle placed on the tube. A second tube which receives the reflected ray is moveable round the circle, and its displacements are measured by means of a vernier; the reflected light is analysed by a doubly refracting prism placed at its exterior extremity, and the direction of the principal section of this prism is known by means of a second vertical circle fixed on this move-

able tube. At the centre of the horizontal circle is a table, on which the double plate is adjusted vertically in such a position, that the line of separation of the two substances rests on the exact centre of the apparatus: this table is moveable round the centre and an "alidade," which traverses the limb of the graduated circle, allows of the incidences being varied and measured.

The verticality of the double plate being an indispensable condition, it was at first endeavoured to be accomplished by known methods, and afterwards was verified by polarizing the light in the principal azimuths, and observing that the polarization remained rectilinear after reflexion from the metal, and that the azimuth was not altered by turning the reflecting surface through 180°. In addition, two series of observations have always been made, the reflecting surface being placed on the right and left of the observer alternately, in order to correct errors arising from a want of verticality in the double plate.

The incidences were measured, both by the deviation of the reflected ray and by the displacement of the plate: the angle (β) was ascertained with great precision: in fact, the ease with which the eye can judge of the equality of two lights of the same tint is well known; and I found that a little practice renders the sensibility of this organ truly remarkable. The results of experiments made under the same circumstances never differ by more than fifteen minutes; and if greater errors are committed, it is because the points used for distinguishing (*points de repère*) whether for the measure of incidences or for the position of the planes of polarization, are not always obtained with so great an accuracy. Let it be observed moreover that in each quadrant there are two angles (β) and $90^\circ-\beta$, which render the ordinary or extraordinary image of the metal equal to the extraordinary or ordinary image of the glass; each result therefore has been concluded from eight observations.

In all my experiments the light was supplied by a Carcel lamp, placed in a closed box at the focus of a lens which rendered the rays parallel, so that the operations were conducted in the most complete darkness: the light employed was very intense, and always precisely the same; it was made sensibly homogeneous by a red glass of great thickness chosen with much care, and which, whilst permitting the transmission of a sufficient number of rays to render the observations easy, diminished the

intensity sufficiently to allow of the perfect polarization by Nichol's prism

My experiments, performed with plates of steel and mirror metal well polished, are given in the following tables it will be remarked, that the intensities of the reflected light, polarized in the plane of incidence, vary little, and that they diminish progressively from the incidence of 90° to that of 0°

If, on the contrary, the light is polarized in the azimuth of 90°, the intensities diminish from the grazing incidence (*rasante*) up to the angle of maximum polarization, and afterwards increase up to the normal incidence

Steel —*Square root of the intensities of light reflected in the plane of incidence* $i_1=76$ $\varepsilon=57\ 53$

Incidences	Angle observed β	Square root of the intensities: Observed	Calculated	Difference
90	18 2	[illegible]	0 977	−0 020
80	52 9	0 915	[illegible]	−0 000
75	56 15	0 916	[illegible]	+0 011
70	59 10	[illegible]	0 910	+0 005
65	61 56	0 898	0 892	+0 006
60	61 52	0 897	0 871	+0 023
55	66 15	0 869	0 856	+0 013
50	67 57	0 828	0 842	−0 014
45	69 37	0 818	0 827	−0 009
40	71 7	0 780	0 815	−0 035
35	72 10	0 800	0 801	−0 001
30	73 3	0 790	0 795	−0 005
25	73 56	0 791	0 787	+0 001
20	74 26	0 780	0 781	−0 001

Steel —*Square root of the intensities of light reflected in the plane perpendicular to the plane of incidence*

85	45 12	0 719	0 709	+0 010
80	48 21	0 547	0 583	−0 037
75	60 00	0 566	0 563	+0 003
70	69 15	0 545	0 569	−0 021
65	79 11	0 627	0 599	+0 028
60	86 10	0 630	0 630	0 000
55				
50	85 4	0 666	0 681	−0 015
45	82 22	0 689	0 701	−0 012
40	80 32	0 688	0 717	−0 029
35	79 10	0 741	0 730	+0 011
30	78 10	0 760	0 742	+0 018
25	77 20	0 769	0 751	+0 018
20	76 36	0 770	0 758	+0 012

Mirror metal.—*Square root of the intensities of light reflected in the plane of incidence.* $i_1=75\ 50$, $\epsilon=6\ 1$

Incidence	Angles of polarization β	Square roots of intensities		Differences
		Observed	Calculated	
86	47 38	0.958	0.981	−0.016
84	48 53	0.[illegible]	0.970	−0.017
82	50 13	0.937	0.96[illegible]	−0.03
80	52 33	0.9[illegible]	0.961	−0.002
78	53 17	0.944	0.954	−0.010
76	55 3	0.950	0.948	+0.002
74	56 50	0.940	0.934	+0.006
72	57 58	0.926	0.932	−0.006
70	58 51	0.869	0.92	−0.056
68	60 13	0.906	0.919	−0.013
66	62 10	0.930	0.912	+0.018
64	63 35	0.940	0.905	+0.035
62	64 10	0.911	0.900	+0.011
60	64 41	0.890	0.894	−0.004
58	65 16	0.902	0.888	+0.014
56	66 8	0.850	0.88	−0.032
54	66 53	0.850	0.870	−0.017
52	68 16	0.877	0.879	+0.00
50	69 [illegible]	0.880	0.866	+0.014
48	69 10	0.869	0.861	+0.008
46	70 23	0.869	0.857	+0.012
44	71 8	0.873	0.852	+0.021
42	71 53	0.841	0.848	−0.007
40	72 00	0.842	0.844	−0.01
38	72 10	0.833	0.840	−0.007
36	73 4	0.823	0.836	−0.013
34	73 5	0.835	0.833	+0.002
32	73 48	0.850	0.830	+0.020
30	74 5	0.84	0.827	+0.018
28	75 18	0.837	0.824	+0.013
26	74 5	0.851	0.821	+0.033
24	7[illegible] 27	0.858	0.810	+0.013
22	75 12	0.857	0.816	+0.011
20	75 15	0.8[illegible]8	0.811	+0.014

Mirror-metal.—*Square root of the intensities of light reflected perpendicularly to the plane of incidence.*

Incidences	Angle observed β	Square root of the intensities: Observed	Square root of the intensities: Calculated	Differences
86°	46° 36′	0.754	0.800	0.046
84	47 33	0.715	0.736	−0.021
82	50 58	0.697	0.683	+0.014
80	53 18	0.655	0.654	+0.001
78	56 32	0.631	0.633	−0.002
76	60 6	0.623	0.626	−0.003
74	64 17	0.666	0.626	+0.040
72	69 18	0.678	0.630	+0.048
70	73 18	0.688	0.637	+0.051
68	76 3	0.666	0.646	+0.020
66	79 14	0.654	0.659	−0.005
64	82 21	0.729	0.666	+0.063
62	84 24	0.701	0.677	+0.024
50	85 59	0.819	0.730	+0.089
48	85 11	0.760	0.737	+0.023
46	83 52	0.801	0.744	+0.057
44	82 15	0.723	0.749	−0.026
42	82 00	0.747	0.755	−0.008
40	81 46	0.793	0.761	+0.032
38	80 23	0.764	0.765	−0.001
36	80 34	0.791	0.770	+0.021
34	80 12	0.824	0.774	+0.050
32	79 56	0.860	0.778	+0.082
30	79 7	0.828	0.781	+0.047

In the fourth column of the preceding tables are given the numbers as calculated, between which and those furnished by experiment there is a satisfactory agreement: these numbers are given by formulæ due to M. Cauchy. This geometer, guided by the experiments of Sir David Brewster, has treated the problem of metallic reflexion theoretically, and we shall soon see that he has completely solved it. As his labours on this subject have not been published as a whole, we think fit to give here an abstract of the theoretical principles on which they are founded, and to state the formulæ at which he has arrived.

When light passes from vacant space into a homogeneous body, there exists between the lengths of the incident and refracted waves, a ratio which has been named the *index of refraction*, and which is constant when the body is homogeneous and not crystallized. If this body is transparent, the index of refraction is its sole characteristic, and the knowledge of this constant is sufficient to calculate in every case the action exerted by the

substance on light but if the body, remaining homogeneous, becomes opake, this datum is insufficient and the modification undergone by the ray is complicated by a new influence Bodies in fact, never being absolutely opake, give rise to refracted waves when they are struck by light, only these waves traverse but a very slight thickness we may therefore admit that they are rapidly enfeebled, so as to become insensible at a very small distance compared with the length of an undulation and by representing this diminution of energy by a second characteristic, the coefficient of extinction M Cauchy seems to have simply translated into a principle that which has been shown to us by experience, and to start from a most reasonable foundation

Thus, the formulæ which represent the reflexion and refraction of light in transparent bodies depend on one constant only, the index of refraction, and for opake bodies on two given quantities, viz the index of refraction and the coefficient of extinction

In order to deduce from observation the two constants which represent the action of any metal it will suffice — 1st, to determine the angle (i_1) of maximum polarization this is the first thing given 2nd, to find out the value, at this incidence of the ratio $\left(\frac{I}{J}\right)$ of the square roots of the reflected intensities of light polarized in the plane of incidence and the plane perpendicular to it and to calculate the angle whose tangent is equal to this ratio this angle which we shall call A, is the second given quantity

The following are the formulæ of M Cauchy —J and I represent the intensities of the reflected light, polarized in the plane of incidence and in the plane perpendicular to it, that of the incident ray being equal to unity

$$I^2=\tan(\phi-45^\circ), \qquad J^2=\tan(\chi-45^\circ), \qquad (6)$$

ϕ and χ are given by the formulæ

$$\left.\begin{aligned}\cot\phi&=\cos(2\varepsilon-u)\sin\left(2\,\mathrm{arc}\tan\frac{U}{\theta\cos i}\right),\\ \cot\chi&=\cos u\sin\left(2\,\mathrm{arc}\tan\frac{\cos i}{U}\right)\end{aligned}\right\} \qquad (7)$$

(i) represents the angle of incidence (θ) and (ε) are two constants, U and (u) variables which are calculated as functions of (i) (θ) and (ε) by the following equations

$$\left.\begin{aligned}\cot(2u-\varepsilon)&=\cot\varepsilon\cos\left(2\arctan\frac{\sin i}{\theta}\right),\\ \theta^2\sin 2\varepsilon&=U^2\sin 2u\end{aligned}\right\}\qquad(8.)$$

The constants (θ) and (ε) are determined as follows:—at the angle of maximum polarization, the variables U and (u) have the particular values

$$u=2A,\qquad U=\sin i_1\tan i_1,$$

replace (u) and U by these particular values in formulæ (8.), and (ε) and (θ) are found from them: these quantities being once determined, the formulæ (8.) will give the values of (u) and U for each incidence, equations (7.) give (ϕ) and (χ), and (6.), I and J^2.

In applying these formulæ, it is perceived that $\left(\frac{1}{\theta}\right)$ is always so small that we may neglect $\left(\frac{1}{\theta^2}\right)$ in the calculations: we have constantly satisfied ourselves with this degree of approximation, after having convinced ourselves that the errors committed were less than those of experiment.

Whatever care be used in executing the experiments, it appears to me impossible to obtain a more complete agreement between theory and calculation than is exhibited by our tables. The determinations are, in fact, liable to several sources of error, of which some are very serious, and cannot be entirely avoided, and which the least negligence would render enormous: and, moreover, the theoretical formulæ are calculated by means of two constants, furnished to us by experiment, and which are necessarily tainted with errors which alter all our results: it is therefore difficult to aspire after a more perfect experimental verification than that exhibited by our tables.

II. *Measure of the Difference of Phase.*

We have now to occupy ourselves with the second transformation operated on light by metallic reflexion; I refer to the displacement of the nodes of vibration.

I have occupied myself with a particular case of this question, and my experiments presented to the Academy of Sciences on the 13th of August 1846, prove,—1st, that a ray polarized perpendicularly to the plane of incidence is always retarded with regard to a beam polarized in the azimuth of 0°; 2nd, that the differ-

ence of phase is nothing at an incidence of 0°; that it increases progressively up to the grazing (*rasante*) incidence of 90°, for which it becomes equal to a semi-undulation; and that at the angle of maximum polarization it takes the value $\left(\frac{\lambda}{4}\right)$.

This law of the variation of phase results from experiments made on metallic oxides by a process inapplicable to metals; but as these oxides and metals act on light in the same way, according to the experiments of Sir David Brewster, it is incontestable that the difference of phase produced by the reflexion of metals will vary in the same direction between the limits of the incidences; we shall assume therefore, that for metals, the difference of phase between the reflected rays polarized in the principal azimuths is nothing for a normal incidence, and that it increases progressively at the same time as the inclination of the ray to the surface: this generalisation of a fact verified in a particular case is moreover conformable to the result obtained by M. de Sénarmont.

Starting from this law, I shall by a new method find the value of the difference of phase for particular incidences: this method possesses the advantage of employing nothing intermediate in order to modify the phase, and will for this reason be free from the objections to which the processes hitherto used are liable. The following is my mode of proceeding.

When we direct a beam polarized in any plane upon a metallic mirror, we may always consider it as formed by two rays of the same phase polarized in azimuths of 0° and 90°, azimuths which are not altered by the reflexion. If they be again reflected any number of times from mirrors of the same substance parallel to the former, the angle and the plane of incidence remaining the same, they will undergo each time the same action on the part of the metal; and after 2, 3, 4 . . . *m* reflexions they will have differences of phases equal to 2, 3, 4 . . . *m* times that produced in them by a single reflexion. If then we can find the first, it will be sufficient to divide them by the number of reflexions to obtain the second: this determination will be very easy in certain particular cases.

We know, in fact, by the experiments of Sir David Brewster, that after having been reflected several times by a metal, the ray has acquired a polarization, generally elliptical, but which becomes rectilinear for certain particular values of the angle of incidence

these values vary with the number of reflexions, and experiment shows, that of them there is one for two reflexions, two for three reflexions, and in general a number equal to the number of reflexions minus one. Sir David Brewster does not seem to have remarked this relation between the number of reflexions and that of the angles of renewed polarization. It is a very simple consequence of the manner in which the difference of phase varies, and the reader will shortly be able to recognise it; for the moment we content ourselves with pointing out the use to be made of this fact.

In order that two rays polarized at right angles, whose phases differ, may, on uniting, constitute a polarized beam, it is necessary that the differences between their phases be equal to

$$\frac{\lambda}{2}, \text{ or } 2\,\frac{\lambda}{2}, \text{ or } 3\,\frac{\lambda}{2}, \ldots$$

Therefore, if the polarization has again become rectilinear after a certain number of reflexions effected at the same incidence by the same metal, it is because the difference of phase of the two rectangular rays has become equal to a multiple of a semi-undulation, and the whole question is reduced to finding this multiple. Now this is very easy. We know, in fact, that after a single reflexion, the difference of phase goes on increasing from the incidence of 0° when it is nothing, up to that of 90°; therefore, for the angle nearest to 0°, which will restore the polarization after (m) reflexions, the difference of phase will be the smallest multiple $\left(\frac{\lambda}{2}\right)$; for that which comes after, $2\,\frac{\lambda}{2}$, and so on to that nearest to 90°, where it will be $(m-1)\,\frac{\lambda}{2}$. Therefore we shall have for a single reflexion at the same angles, the following values of the difference of phase:

$$\frac{1}{m}\,\frac{\lambda}{2},\quad \frac{2}{m}\cdot\frac{\lambda}{2},\ \frac{3}{m}\cdot\frac{\lambda}{2}\cdot \quad \ldots\ \frac{m-1}{m}\cdot\frac{\lambda}{2}.$$

The differences of phases will be expressed as a function of $\frac{\lambda}{2}$ by a fraction $\frac{n}{m}$, (n) taking all integer values from (1) up to $(m-1)$, m representing the number of reflexions.

It follows from this, that (m) and (n) varying, the same value of the fraction will be reproduced frequently for different numbers of reflexions: thus, after 2, 4, 6, 8 reflexions, we shall have

the values $\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$ of the difference of phase and therefore the corresponding angles of restored polarization ought to be sensibly equal. We shall thus obtain numerous verifications.

We perceive that it is sufficient to measure the incidence of restored polarization; as to the difference of phase, it is not measured, but is known when the reflected ray has again become polarized and the number of reflexions has been counted. It must also be remarked that the azimuth of polarization of the incident ray is any whatever; the observed incidences do not change when it varies, and the polarizing prism of Nichol may be placed as we please. If it be considered that it is always difficult to measure with precision the azimuth of the incident ray and that generally the slightest variation in its value alters the results which are measured, some importance will be attached to a process which leaves this quantity indeterminate, which requires as an indispensable condition only the parallelism of the plates, and which measures only one thing, the angle of incidence of restored polarization. This practical simplicity will conduct us to results of great accuracy.

To obtain multiple reflexions, it is sufficient to place two mirrors of the substance to be examined parallel and opposite to each other; light is to be made to fall on one of them, which will be reflected from the second, come back on the first, &c. The number of observable reflexions will evidently depend only on the distance of the plates, which ought to be variable at pleasure. The following is the arrangement which appeared to me most commodious:—The two mirrors are fixed with wax on two plates of brass, parallel and vertical; the first is fixed, the second is put in motion by a micrometer screw, which transports it parallel to itself. We may satisfy ourselves of the parallelism of the mirrors by bringing them into contact, and noticing if all the edges accurately coincide. This little apparatus is placed on the centre of the graduated circle, of which I have already spoken; it is placed so that the polished surface of the fixed mirror passes through the centre of the circle. After having been reflected several times between the two mirrors, the ray escapes into the air; but then its direction prolonged no longer passes through the centre of the circle, and cannot traverse the moveable tube in the direction of its axis. To remedy this inconvenience, I caused the tube to have a horizontal movement

of rotation round its support; a direction may then be given to it, in each case agreeing with that of the ray definitively deflected. If the plates are sufficiently separated, we perceive the images arising from one or two reflexions; and when the mirrors are brought near to each other, these images disappear we see successively those which arise from reflexions in greater number, and can easily count them.

The polarization is never perfectly restored when the incident light is white. The inequality of action excited by a metal on the different simple rays of the spectrum renders the images coloured, and we can only observe the incidence for which the extraordinary image has the minimum of brightness; but it is observed that this minimum corresponds exactly to the intermediate tint between the deep blue and dull purple. I contented myself in the experiments on the silver plate with observing this intermediate tint (*teinte de passage*), and taking for the angle of restored polarization that for which this tint is a minimum in the extraordinary image Experiment moreover shows that it varies in tint so rapidly with the direction of the principal section of the analyser, and that it undergoes for the incidence sought, so great a diminution of intensity, that the results lose nothing of their explicitness, even when the number of reflexions is very great I have besides made observations with a red glass on mirrors of steel, copper and zinc, the results are represented in the following tables: it will be observed that the differences of phases follow exactly the law of variation which we have already recognised in the oxides, and which has been previously announced.

Silver Plate.—*Table of differences of phases*

$i_1 = 71\ 10 \quad A = 36$

—	[illegible]		Differences of phase		Differences
	Observed	Mean	Observed	Calculated	
[illegible]	81 30	81 30	0 813	0 829	+0 001
[illegible]	83 50	83 50	0 800	0 801	−0 009
[illegible]	83 50				
[illegible]	81 37	81 37	0 7[illegible]0	0 716	+0 001
[illegible]	81 30	81 20	0 714	0 736	−0 022
[illegible]	81 10				
$\frac{7}{10}$	80 20	80 20	0 700	0 709	−0 009
[illegible]	79 00	79 [illegible]2	0 6[illegible]	0 674	−0 008
[illegible]	79 00				
[illegible]	79 10				
[illegible]	77 38	77 38	0 626	0 637	−0 011
[illegible]	77 00	76 12	0 600	0 611	−0 011
[illegible]	76 25				
[illegible]	75 57	7[illegible] [illegible]7	0 572	0 5[illegible]	−0 023
[illegible]	74 15	74 15	0 575	0 567	+0 008
[illegible]	74 0	74 [illegible]	0 [illegible]15	0 [illegible]5	−0 007
[illegible]	72 10	72 00	0 500	0 [illegible]00	
[illegible]	72 00				
[illegible]	71 2[illegible]				
[illegible]	72 15				
[illegible]	72 15				
[illegible]	72 00				
[illegible]	70 30	70 30	0 464	0 476	−0 0[illegible]2
[illegible]	69 15	69 1[illegible]	0 444	0 451	−0 007
[illegible]	69 00	69 00	0 4[illegible]9	0 417	−0 018
[illegible]	67 25	67 25	0 416	0 423	−0 007
[illegible]	66 38	66 20	0 400	0 402	−0 002
[illegible]	66 20				
[illegible]	64 10	64 10	0 375	0 375	
[illegible]	64 00	64 00	0 363	0 362	+0 001
[illegible]	63 00	6[illegible] 31	0 333	0 334	−0 001
[illegible]	62 20				
[illegible]	62 20				
[illegible]	6[illegible] [illegible]5				
[illegible]	60 10	60 10	0 300	0 307	−0 007

Silver Plate (Table continued).

$\frac{n}{m}$	Incidences of restored position		Differences of phases		Differences
	Observed	Mean	Observed	Calculated	
$\frac{2}{7}$	59° 35′	59° 35′	0 286	0 298	−0 012
$\frac{3}{11}$	57 10	57 40	0 272	0 277	−0 005
$\frac{1}{4}$	55 20				
$\frac{2}{8}$	55 15	55 26	0 250	0 250	
$\frac{3}{12}$	55 15				
$\frac{2}{9}$	53 30	53 30	0 222	0 221	−0·002
$\frac{1}{5}$	50 30	50 37	0 200	0 200	
$\frac{2}{10}$	50 45				
$\frac{2}{11}$	48 00	48 00	0 181	0 177	+0 004
$\frac{1}{6}$	46 35	46 36	0 180	0 165	+0 015
$\frac{2}{12}$	46 38				
$\frac{1}{7}$	43 50	43 50	0 143	0 143	
$\frac{1}{8}$	41 15	41 15	0 125	0 125	
$\frac{1}{9}$	39 10	39 10	0 111	0 112	−0 001
$\frac{1}{10}$	37 10	37 10	0 100	0 100	
$\frac{1}{11}$	35 40	35 40	0 091	0 091	
$\frac{1}{12}$	35 15	34 15	0·080	0 082	−0 002

Steel, $i^1 = 76$ $\varepsilon = 57°{\cdot}53$.—*Differences of phases.*

Incidences of restored polarization	Differences of phases		Differences
	Observed	Calculated	
84° 00′	0 800	0 796	+0 004
83 20	0 750	0 753	−0 003
80 46	0 666	0 641	+0 025
79 00	0 600	0 596	+0 004
76 00	0 500	0 500	
73 00	0 429	0 419	+0 010
71 50	0 400	0 392	+0 008
70 39	0 375	0 365	+0 010
68 16	0 333	0 320	+0 013
65 25	0 286	0 271	+0 015
63 38	0 250	0 250	
61 39	0 222	0 226	−0 004
58 37	0 200	0 194	+0 006
55 00	0 180	0 162	+0 018
51 00	0 143	0 133	+0 010
49 57	0 125	0 127	−0 002
46 21	0 111	0 105	+0 006
45 27	0 100	0 100	
41 53	0 091	0 083	+0 008
41 13	0 080	0 080	
38 59	0 071	0 071	

Not only does M Cauchy's theory make known the intensities

of the reflected light, it also shows that two rays of the same phase before incidence polarized in azimuths of 0° and of 90°, after undergoing the action of the metal, have a difference of phase (δ) variable with the incidence, and expressed by the formula

$$\tan \delta = \tan 2\,\omega \sin u, \qquad (9)$$

(ω) is found by the equation of condition

$$\tan \omega = \frac{U \cos i}{\sin i}$$

It is by means of this formula that the numbers calculated in the preceding tables have been obtained and the almost perfect identity of the theoretical and experimental results can leave no doubt as to the accuracy of the formulæ of the skilful geometer In order to show still more clearly that the agreement is as complete as possible, we shall observe that in the table relating to silver wherever the fractions $\left(\frac{n}{m}\right)$ have equal values, the corresponding incidences of restored polarization differ among each other by very small quantities often insignificant, and always less than thirty minutes These differences afford us, so to speak, a measure of the errors liable to be committed in the determination of the angles and if I add that the numbers in the table are the result of three series of experiments, performed by varying each time the azimuth of the incident ray, the conviction will follow that this limit to error is rarely attained on the other hand, an error of thirty minutes in the determination of the angle produces one of only $\frac{1}{100}$ in the difference of phase so that we may conclude that $\frac{1}{100}$ is the probable limit of error in determining the difference of phase

Now, if in the foregoing tables the column of differences be examined, it will be found that in more than fifty observations there are only three which give a difference of 0 03, eleven amount to 0 01, and amongst the rest many are identical, even to the thousandth part the difference between calculation and observation is therefore limited to the errors recognised as possible in experiment

At the time when I made these experiments I was not aware of the formulæ of M Cauchy and in presenting my results to the Academy of Sciences, I had sought to represent them by an empirical formula, which, although differing essentially from that of M Cauchy, gives the numerical results sensibly the same As it is very simple and may be employed usefully in an approximative calculation, I shall give it here

Put $\tan i = n$ and $\sin i = r$

then calculate the equation

$$\tan A' = \frac{\cos(1+r)}{\cos(1-r)}$$

The expression $(90^\circ - 2A')$ represents the difference of phase, or (δ). This formula applies exactly to silver and steel: it also represents, with a very satisfactory nearness, the following experiments performed on two plates of zinc, on which had been impressed a different polish in the two series of trials to which they were subjected, which has changed all the results numerically without altering their law.

First Series.—*Zinc*, $i_1 = 77$.

Incidences	Differences of phases		Differences
	Observed	Calculated	
87° 5′	0·800	0·865	−0·065
84 10	0·750	0·740	+0·010
82 7	0·666	0·661	+0·005
80 7	0·600	0·592	+0·008
77 00	0·500	0·500	
72 34	0·400	0·397	+0·003
69 00	0·333	0·332	+0·001
66 00	0·286	0·288	−0·002
62 45	0·250	0·249	+0·001
61 55	0·222	0·237	−0·015
58 30	0·200	0·201	−0·001
55 9	0·180	0·172	+0·008
52 15	0·143	0·149	−0·006
49 57	0·125	0·134	−0·009
47 10	0·111	0·117	−0·006

Second Series.—*Zinc*, $i_1 = 79·13$

87 00	0·833	0·829	+0·004
86 40	0·800	0·813	−0·013
86 00	0·750	0·778	−0·028
85 00	0·714	0·727	−0·013
82 30	0·666	0·647	+0·019
81 40	0·572	0·584	−0·012
82 20	0·600	0·611	−0·011
82 15	0·626	0·608	+0·018
79 13	0·500	0·500	
76 10	0·429	0·433	−0·004
76 00	0·444	0·412	+0·032
75 00	0·400	0·390	+0·010
73 5	0·375	0·349	+0·026
71 40	0·333	0·325	+0·008
69 35	0·286	0·288	−0·002
69 5	0·300	0·291	+0·009
66 48	0·250	0·250	
66 7	0·222	0·241	−0·019
60 19	0·200	0·215	−0·015
58 28	0·180	0·166	+0·014
56 15	0·143	0·149	−0·006
52 40	0·125	0·128	−0·003
51 15	0·111	0·117	−0·006
48 47	0·100	0·101	−0·001

III *Analysis of Light polarized elliptically*

We have already remarked that light, in being reflected from metal could only suffer alterations of its amplitudes and displacements of the nodes of vibration The formulæ of M Cauchy which represent with great accuracy the laws of these modifications, comprise all the principles of metallic reflexion we are therefore allowed to leave to the calculus the task of foreseeing the phænomena which remain to be studied, if they were not interesting in themselves, and if it were not very important to verify the theory even in its consequences With this view, we shall begin by causing to be reflected a single time from metal a beam polarized in any plane whatever

It is known, from the experiments of Sir David Brewster, that light ceases to be polarized when it has undergone the action of metal, and, according to this theory this depolarization arises from this,—that the vibrations of the ethereal molecules are performed in an ellipse We shall endeavour to verify this consequence experimentally

In order to define completely an elliptical oscillatory movement, the most simple plan is to determine the direction of the axes and the ratio of their lengths this we can always effect by the calculus, but it can also be done by experiment To show this, we shall now prove,—

1st That if an elliptical beam be made to fall on a doubly refracting prism, whose principal section is parallel to one of the axes of the trajectory it is decomposed into two rays, whose phases differ by a quarter of an undulation, and of which one has the greatest possible intensity, the other the least,

2nd That if the principal section of the prism is inclined at 45 to the direction of the axes of the ellipse, the intensities of the two images are equal

Let $(90^\circ - a)$ be the azimuth of polarization of the incident ray, we may replace this ray by two vibrations directed in the principal azimuths, and whose amplitudes are $\sin a$ and $\cos a$

By reflexion, these vibrations will be modified in their phase and amplitude and taking account only of the difference of their phases we shall have the following equations for expressing the co ordinates of the vibrating molecules after reflexion

$$x = \mathrm{I} \cos a \cos 2\pi \frac{t}{\mathrm{T}}, \text{ vibration in the plane of incidence,}$$

$y = \text{J} \sin \alpha \cos \left(2\pi \frac{t}{\text{T}} + \delta\right)$, vibration perpendicular to plane of incidence. To abbreviate, we shall put

$$\frac{\text{I} \cos \alpha}{\text{J} \sin \alpha} = \cot \alpha,$$

and there results, neglecting a constant factor,

$$\left.\begin{aligned} x &= \cos \alpha \cos 2\pi \frac{t}{\text{T}}, \\ y &= \sin \alpha \cos \left(2\pi \frac{t}{\text{T}} + \delta\right) \end{aligned}\right\} \quad \ldots\ldots \quad (10.)$$

The elimination of the time between these two equations will give the equation to the trajectory, which is an ellipse whose equation is

$$\frac{y^2}{\sin^2 \alpha} + \frac{x^2}{\cos^2 \alpha} - \frac{2 \cos \delta}{\sin \alpha \cos \alpha} x y = \sin^2 \delta.$$

To obtain at the same time the direction and length of the axes of the ellipse, we have only to replace the co-ordinate axes by another system, making an angle (ω) with that to which the equation is referred, and to take the condition that the coefficient of ($x y$) may disappear; we shall then have the equation of the ellipse

$$(\sin^2 \alpha \sin^2 \omega + \cos^2 \alpha \cos^2 \omega + 2 \sin \alpha \cos \alpha \sin \omega \cos \omega \cos \delta)\, y^2$$
$$+ (\cos^2 \alpha \sin^2 \omega + \sin^2 \alpha \cos^2 \omega - 2 \sin \alpha \cos \alpha \sin \omega \cos \omega \cos \delta)\, x^2$$
$$= \&c.,$$

and the equation of condition

$$\tan 2\omega = \tan 2\alpha \cos \delta. \quad \ldots\ldots \quad (11.)$$

This latter gives us the direction of the two axes at the same time; and replacing (ω) by its value in the coefficients of y^2 and x^2, we shall obtain numbers proportional, the first to the axis of (x), and the second to the axis of (y).

We shall put

$$\left.\begin{aligned} \text{A}^2 &= \sin^2 \alpha \sin^2 \omega + \cos^2 \alpha \cos^2 \omega + \frac{1}{2} \sin^2 \alpha \sin^2 \omega \cos \delta \ldots \text{axis of } x \\ \text{B}^2 &= \sin^2 \alpha \cos^2 \omega + \cos^2 \alpha \sin^2 \omega - \frac{1}{2} \sin 2\alpha \sin 2\omega \cos \delta \ldots \text{axis of } y \end{aligned}\right\} (12.)$$

Let us now direct this elliptically polarized ray, or which comes to the same thing, the two rectangular vibrations (10.) upon a doubly refracting prism, making an angle (ω) with the plane of incidence $o\,x$ we shall have, calling (x') the vibration

in the direction of the principal section, (y') that in the direction perpendicular,

$$x' = y \sin \omega + x \cos \omega,$$
$$y' = y \cos \omega - x \sin \omega$$

These two vibrations may be written

$$x' = \mathrm{A}' \cos\left(2\pi \frac{t}{\mathrm{T}} + \delta'\right),$$
$$y' = \mathrm{B}' \cos\left(2\pi \frac{t}{\mathrm{T}} + \delta''\right),$$

and A′ B′, δ′, δ″ will be obtained according to Fresnel's rule These quantities will be

$$\left.\begin{aligned} \mathrm{A}'^2 &= \sin^2 \alpha \sin^2 \omega + \cos^2 \alpha \cos^2 \omega \\ &\quad + \frac{1}{2} \sin 2\alpha \sin 2\omega \cos \delta \qquad \text{vibration in axis of } x, \\ \mathrm{B}'^2 &= \sin^2 \alpha \cos^2 \omega + \cos^2 \alpha \sin^2 \omega \\ &\quad - \frac{1}{2} \sin 2\alpha \sin 2\omega \cos \delta \qquad \text{vibration in axis of } y \end{aligned}\right\}, \quad (13)$$

$$\tan \delta' = \frac{\sin \alpha \sin \omega \sin \delta}{\cos \alpha \cos \omega + \sin \alpha \sin \omega \cos \delta}$$

$$\tan \delta'' = \frac{\sin \alpha \cos \omega \sin \delta}{-\sin \omega \cos \alpha + \sin \alpha \cos \omega \cos \delta}$$

These latter formulæ serve to calculate the difference of phase of the two rays they give

$$\tan(\delta' - \delta'') = \frac{\sin \delta \sin 2\alpha}{\sin 2\omega \cos 2\alpha - \sin 2\alpha \cos 2\omega \cos \delta} \qquad (14)$$

If we wish to find the direction for which the images are maxima and minima, we must differentiate formulæ (13) with regard to (ω), they give

$$-\cos 2\alpha \sin 2\omega + \sin 2\alpha \cos 2\omega \cos \delta,$$
$$\cos 2\alpha \sin 2\omega - \sin 2\alpha \cos 2\omega \cos \delta$$

These two differentials being equal, but of contrary sign, we conclude that one of the images is a maximum when the other is a minimum, and *vice versâ* and this will hold for the direction found by putting the differentials = zero, which gives

$$\tan 2\omega = \tan 2\alpha \cos \delta$$

a relation identical with that which gives the direction of the axes of the ellipse Therefore,

1st. One of the images will be a maximum, the other a minimum, if we place the principal section of the analysing prism in the direction of one of the axes of the ellipse.

It is to be also remarked that the formulæ (12.) and (13.) give, for A^2 and A'^2 on one side and for B^2 and B'^2 on the other, equal values, therefore

2nd. The intensity of the vibration in the direction of the axes of the ellipse is proportional to the square of their lengths; whence it results that, if the principal section of the prism coincides with the major axis of the ellipse, the vibration in the direction of this axis, that is to say the vibration of the extraordinary ray, will be a maximum, the ordinary ray being a minimum.

If we replace in formula (14.) the angle (ω) by the value which gives the direction of the axes, we find

$$\tan(\delta' - \delta'') = \infty \text{ or } \delta' - \delta'' = 90^\circ,$$

that is to say, that

3rd. Every elliptical vibration may be decomposed into two rays, polarized in the direction of two axes, whose intensities are proportional to the squares of the lengths of these axes, and whose phases differ by a quarter of an undulation.

Lastly, if we seek the condition which must be satisfied by the angle (ω) in order that the intensities of the two images may be equal, we must put

$$A'^2 - B'^2 = 0;$$

which gives

$$\left.\begin{aligned} \cot 2\omega' &= \tan 2\alpha \cos\delta = \tan 2\omega, \\ 2\omega' &= 90^\circ \pm 2\omega, \\ \omega' &= 45^\circ \pm \omega \end{aligned}\right\} \quad . \quad (15.)$$

Thus,

4th. The two images are equal when the direction of the principal section is inclined at 45° to that of the axes of the ellipse

These results may now be transformed into experiments. In fact, to obtain the position of the axes of the ellipse, it will suffice to find the direction of the principal section which gives to one of the images the greatest, and to the other the least intensity; and if we wish to obtain the ratio of the lengths of the axes, we must measure the ratio of the intensities of these images.

The first of these questions being the only one with which I have occupied myself, I shall explain how the requisite precision

may be given to the experiments. It is clear that in every case where the ellipse is not sensibly a straight line the difference between the maximum and minimum will not be very sensible, and therefore the direction of the axes will be difficult to find. But we may replace this determination by another, by calling to mind that the analyser being inclined at 45° to the direction of the axes, the two images are equal: we shall then determine this latter direction; and by increasing or diminishing the angle found by 45°, we shall have the position of the two axes.

But, in order to obtain certain results, it is absolutely necessary to operate with a light rendered homogeneous by a red glass well chosen: otherwise the two images would always have different tints, and the process would lose all its accuracy. Let it be observed also that there are four directions inclined at 45° to the axes, and that for each incidence we may determine the azimuths of equal tints (ω), $90° + \omega$, $180° + \omega$, $270° + \omega$. These directions being once known, those of the axes will be found by increasing or diminishing them by 45°.

In the following tables we have always indicated the azimuth for which the extraordinary image is a minimum, which is that of the minor axis of the ellipse: this would be the azimuth of the plane of polarization if the ellipse degenerated into a straight line.

On the other hand the direction of the axes of the ellipse is given theoretically by the formula

$$\tan 2\omega = \tan 2\alpha \cos \delta,$$

calling to mind the equation of condition

$$\tan \alpha = \frac{J}{I} \tan a.$$

These two formulæ allow of the angle (ω) being calculated as a function of a, δ, I and J for each particular incidence, and experiment may be compared with calculation.

I have made three series of observations on mirror metal in polarizing light in the azimuths 20° 15', 16° and 71° 25'. The observations repeated several times have always given numbers agreeing with each other, and the mean results are completely in accordance with the theory, as the following tables show.

Mirror-metal.—*Azimuth of the minor axis of the ellipse of oscillation of a molecule of æther after reflexion.*

Incidences	Polarized light in the azimuth of 20° 15′			Polarized light in the azimuth of 16°			Polarized light in the azimuth of 71° 25′		
	Azimuth of the small axis of the ellipse		Differ	Azimuth of the small axis of the ellipse		Differ	Azimuth of the small axis of the ellipse		Differ
	Observed	Calculated		Observed	Calculated		Observed	Calculated	
86	+15° 11′	+15° 9′	+0° 2′	+39° 24′	+39° 29′	−0° 5′	−20° 26′	−20° 45′	−0° 19′
84	+12 8	+12 12	−0 4	+36 13	+35 45	+0 28	−20 0	−20 0	
82	+ 9 32	+ 9 6	+0 26	+32 31	+30 54	+0 37	−17 16	−18 2	−0 16
80	+ 6 41	+ 5 54	+0 47	+24 49	+23 28	+1 21	−14 5	−13 30	+0 20
78	+ 3 28	+ 2 51	+0 37	+14 54	+13 33	+1 21	− 8 16	− 7 34	+1 12
76	+ 0 12	+ 0 13	−0 1	+ 1 39	+ 1 45	−0 6	− 1 19	− 0 31	+0 45
74	− 1 57	− 2 14	−0 17	−10 49	−10 46	+0 3	+ 5 44	+ 5 45	−0 1
72	− 4 12	− 4 28	−0 16	−19 54	−19 5	+0 49	+ 9 59	+10 21	−0 25
70	− 6 27	− 6 12	+0 15	−26 14	−25 8	+1 6	+13 23	+13 56	−0 33
68	− 8 11	− 7 58	+0 13	−29 36	−28 57	+0 39	+15 39	+16 14	−0 35
66	− 9 23	− 9 19	+0 4	−32 6	−32 32	−0 26	+18 3	+17 57	+0 6
64	−10 23	−10 40	−0 17	−34 6	−33 49	+0 17	+18 55	+18 45	+0 10
62	−11 39	−11 46	−0 7	−35 40	−35 23	+0 17	+19 24	+19 28	−0 4
60	−12 43	−12 42	+0 1	−36 49	−36 47	+0 2	+19 46	+19 54	−0 8
58	−13 30	−13 32	−0 2	−37 48	−38 9	−0 21	+19 50	+20 10	−0 20
56	−14 2	−14 27	−0 25	−38 53	−39 2	−0 9	+20 00	+20 15	−0 15
54	−14 48	−14 55	−0 7	−39 31	−39 26	+0 5	+20 0	+20 25	−0 25
52	−15 5	−15 31	−0 26	−39 49	−40 7	−0 18			
50	−15 49	−16 5	−0 16	−40 15	−40 41	−0 26			
48	−16 12	−16 30	−0 18	−40 45	−41 16	−0 31			
46	−16 35	−16 55	−0 20	−41 4	−41 45	−0 41			
44	−17 21	−17 17	+0 4	−41 30	−42 11	−0 41			
42	−17 45	−17 39	+0 6	−42 5	−42 35	−0 30			
40	−18 2	−17 57	+0 5	−42 36	−42 57	−0 21			
38	−18 28	−18 14	+0 9	−42 52	−43 17	−0 25			
36	−18 42	−18 28	+0 14	−43 13	−43 36	−0 23			
34	−18 57	−18 42	+0 15	−43 33	−43 53	−0 20			
32	−19 14	−18 54	+0 20	−43 55	−44 9	−0 14			
30	−19 46	−19 1	+0 45	−44 21	−44 23	−0 2			

Not only is it interesting to know the direction of the axes of the ellipse, inasmuch as it furnishes us with a verification of theoretical formulæ, but the determination alone of this unknown quantity will give us the ratio of the intensities $\frac{J^2}{I^2}$ of the rays reflected in the principal azimuths.

Let us call to mind that the azimuth of the principal section, for which the two images are equal, is given by the formula

$$\tan 2\omega' = -\frac{\cot 2\alpha}{\cos \delta}. \qquad (16.)$$

Developing $\cot 2\alpha$ and replacing (α) by its value, we obtain

$$2\cot\delta\tan 2\omega' = \frac{1}{\frac{I}{J}\cot a} - \frac{I}{J}\cot a \tag{17}$$

This formula contains two unknown quantities, δ and $\frac{I}{J}$ which experiment does not make known and if we wished to employ it to determine one of the unknowns, it would be necessary to know or to eliminate the other But we are able, by varying the experiment a little, to bring this equation into a much simpler form, and independent of the unknown quantity (δ)

We observe in fact, that of the two angles (a) and (ω') one is arbitrary Up to the present time we have polarized the light in an azimuth $(90 - a)$, which we were at liberty to choose at pleasure we caused the doubly refracting prism to be turned so as to render the two images equal, and we measured the azimuth (ω') We may now do the contrary, that is to say first place the doubly refracting prism in an azimuth (ω'), constant for all the experiments, but any whatever turn the polarizing prism of Nichol, and measure at each incidence the azimuth of polarization $(90 - a)$ for which the two images are equal Amongst all the values which I might give to (ω') I choose $\omega' = 0$ that is to say I place the principal section of the doubly refracting prism in the plane of incidence The formula becomes then

$$0 = \frac{I}{J}\cot a - \frac{1}{\frac{I}{J}\cot a}$$

or

$$\frac{J}{I} = \tan(90^\circ - a)$$

The difference of phase is then eliminated, and we arrive at this remarkably simple result —

The ratio of the square roots of the intensities of reflected rays polarized in the plane of incidence and the plane perpendicular, is equal to the tangent of the azimuth of the polarization of the incident ray, for which the two images are equal

This method does not yield in accuracy to any of those which we have described already it does not employ, in fact, any intermediate body, requires only a single reflexion allows of the use of a simple light, which removes the liability of error arising from the unequal refrangibility of the rays constituting white

light; and finally, it determines the angle sought (a), not by measuring the azimuth of polarization of a ray, which is always very uncertain, but the azimuth for which two tints are equal, which is infinitely more exact and more sensible.

Mirror-metal.—*Ratio of the square roots of the intensities of the reflected light* $\left(\frac{J}{I}\right)$.

Incidences	Angles observed 90—a	Report $\left(\frac{J}{I}\right)$		Differences
		Observed	Calculated	
88°	50° 20′	1 206	1 230	—0 024
84	52 37	1 357	1 327	+0 030
82	54 13	1 413	1 419	—0 006
80	55 41	1 465	1 476	—0 011
78	56 1	1 483	1 507	—0 024
76	56 10	1 520	1 515	+0 005
74	56 15	1 497	1 502	—0 005
72	55 37	1 461	1 463	—0 002
70	55 23	1 448	1 451	—0 003
68	54 50	1 419	1 421	—0 002
66	54 22	1 395	1 402	—0 007
64	53 15	1 364	1 357	+0 007
62	53 22	1 344	1 329	+0 015
60	52 24	1 298	1 301	—0 003
58	52 00	1 280	1 275	+0 005
56	51 36	1 261	1 236	+0 025
54	50 45	1 224	1 228	—0 004
52	50 20	1 206	1 206	
50	49 52	1 186	1 187	—0 001
48	49 29	1 170	1 169	+0 001
46	49 5	1 154	1 152	+0 002
44	48 48	1 142	1 150	—0 008
42	48 20	1 123	1 123	
40	48 10	1 117	1 110	+0 007
38	47 35	1 094	1 097	—0 003
36	47 22	1 088	1 086	+0 002
34	47 6	1 076	1 076	
32	47 00	1 072	1 066	+0 006
30	46 48	1 065	1 058	+0 007

I shall terminate this chapter by some remarks on the published labours of M de Sénarmont (*Annales de Chimie et de Physique*, 2e série, tome lxxiii. page 337)*.

This experimenter caused to be reflected from metal a ray polarized in any azimuth whatever, he received it afterwards on a plate of mica of such a thickness, that the two principal rays acquired in traversing it, a difference of route equal to a quarter

* In the Memoir of M de Sénarmont not one of the names of his predecessors in this inquiry is mentioned except that of Malus!—Ed.

of an undulation, and he placed the principal section of this plate in a direction (ω) which restores the rectilinear polarization. It is hence clear that the experiment amounts to this:—

The ray elliptically polarized by the metal decomposes itself into two beams polarized in planes parallel and perpendicular to the principal section of the thin plate; the calculation of the intensities and of the phases of these rays has already been previously effected: the difference of route is expressed by the formula

$$\tan(\delta' - \delta'') = \frac{\sin\delta \sin 2\alpha}{\sin 2\omega \cos 2\alpha - \sin 2\alpha \cos 2\omega \cos\delta} \qquad (11.)$$

In traversing the thin plate, these two rays acquire, in consequence of the thickness traversed, a new difference of phase equal to a quarter of an undulation, or to 90°, which is added to the former, or subtracted from it. Now, in order that the polarization may be restored it is necessary that the sum obtained should equal 0° or 180°, which cannot take place unless ($\delta' - \delta''$) is itself equal to ± 90: this determination amounts then to the investigation of a direction for which the two rectangular rays into which the ellipse resolves itself, have a difference of route equal to a quarter of an undulation; this direction is that of one of the axes of the ellipse: it is obtained by putting

$$\tan(\delta' - \delta'') = \infty, \text{ whence } \tan 2\omega = \tan 2\alpha \cos\delta.$$

To obtain the intensities of the rectangular rays in the direction which we have found, it will suffice to calculate A'^2 and B'^2 in the formulæ (13.), replacing (ω) by its value: and these intensities will be proportional to the lengths of the axes.

In the experiments of M. de Sénarmont, the difference of phase being reduced to zero and the polarization being restored by the superposition of two rectangular rays whose intensities are A'^2 and B'^2, the azimuth of restored polarization is given by the formula

$$\tan\beta = \frac{A'}{B'}.$$

Thus the experiments of M. de Sénarmont measure two azimuths:—

1st. The azimuth of the principal section of the plate of mica, that is the direction of one of the axes of the ellipse.

2nd. The azimuth of restored polarization, and the tangent of this angle expresses the ratio of the lengths of the axes of the ellipse of oscillation.

It has appeared to me useful to point out the theoretical meaning of these two determinations, which define completely the elliptical movement of the ethereal particles after metallic reflexion It would have been still more interesting to compare the theory with experiments, unfortunately, these do not appear to be sufficiently accurate, practical difficulties, which M de Senarmont has himself recognised, impairing the observations and rendering them often impossible

IV *Phænomena presented by multiplied reflexions*

Although I have already spoken of multiplied reflexions whilst treating of difference of phase, it remains nevertheless to be shown, that all the circumstances of these experiments are easily foreseen and calculated this I shall do, commencing with the case in which the reflecting surfaces are parallel

It will be remembered that several reflexions, even or uneven in number, are capable at certain incidences of restoring plane polarization, it will also be recollected that if the incident ray is polarized in a certain azimuth, to the left of the plane of incidence, for example, the reflected ray regains its polarization, sometimes to the right, sometimes to the left of this plane, and lastly, it is also known that the azimuth of the restored ray is always less than that of the incident ray There are then, as will be seen, three points to be examined in this phænomenon, namely,

1st The incidence for which the polarization is restored,

2nd The direction of the azimuth of the restored ray,

3rd The absolute value of this azimuth

We shall pass these successively under review

1st We may always calculate the angles for which, after a single reflexion, the differences of phases are

$$0,\ \frac{\pi}{m},\ \frac{2\pi}{m},\ \frac{3\pi}{m},\ \quad \frac{(m-1)\pi}{m},\ \frac{m\pi}{m}$$

In fact, the formulæ for the difference of phases being

$$\tan\delta = \tan 2\omega \sin u, \quad \tan\omega = \frac{U\cos i}{\sin^2 i},$$

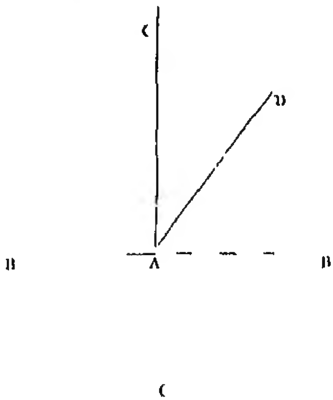
if we replace successively (δ) by the $(m+1)$ preceding values in these equations, they will make known the $(m+1)$ values of (i), of which the first is 0°, the last 90°, for which the difference of phase is equal to the preceding quantities By reflecting

light (m) times at these incidences, the differences of phase will be multiplied by (m) and will become

$$0, \ \pi, \ 2\pi, \ 3\pi, \ \ldots \ m\pi$$

There will therefore be ($m+1$) incidences if 0° and 90° be taken into the account, or ($m-1$) between these limits, for which the polarization will be restored, and which we can calculate from the theoretical formulæ. We observe further, that there cannot be more of them; for in order that the polarization may again become plane, it is necessary and it is sufficient that the difference of phase be equal to a multiple of (π); since it varies from 0 to π between the limiting incidences for a single reflexion, it will be comprised between 0 and $m\pi$ after (m) reflexions, and between these numbers ($m-1$) multiples, only, of (π) can be found. There will therefore be only ($m-1$) angles of restored polarization between 0° and 90°. These theoretical consequences are exactly in accordance with facts, the first point is therefore completely disposed of.

2nd. It will not be difficult to foresee the direction of the azimuths of polarization.



Suppose the vibration of the incident ray to be performed in the direction A D; it will be decomposed into two vibrations in the two principal planes A B, A C. After (m) reflexions, when the polarization will be restored, the phases of the two components will differ by a multiple of a semi undulation; if this multiple is even, the difference is a whole number of undulations; the vibrations are in the same condition as if it were nothing, they agree as before reflexion, and their resultant will lie in the

angle C A B; in this case, the restored ray will be polarized on the same side of the plane of incidence as the incident ray.

If the multiple is uneven, the component vibrations have a final difference of route equal to a semi-undulation, they are discordant, and the restored vibration will take place in the angle B′ A′ C′, or C A B′ The direction of its polarization therefore will be changed.

Thus, for angles which, after (m) reflexions, will give between the principal rays differences of phase equal to

$$0,\ \frac{2\lambda}{2},\ \frac{4\lambda}{2},\ \frac{6\lambda}{2},\ldots\ldots$$

the restored ray will be polarized to the right of the plane of incidence, if the incident ray was polarized to the right.

But if the differences are

$$\frac{\lambda}{2},\ \frac{3\lambda}{2},\ \frac{5\lambda}{2},\ldots\ldots$$

the plane of restored polarization will be the left of the plane of incidence.

Therefore, for the angle nearest to 0°, the polarization will be restored to the left, for that which comes next, to the right; and so on alternately to the last

It is as well to remark, that these two data, the angle which restores the polarization, and the direction of the azimuth of the restored ray, depend absolutely only on the difference of phase of the rays polarized in the principal planes. We shall soon see that the azimuth of the restored ray depends only on their intensity.

3rd. The incident ray being still polarized in the azimuth of $(90^{\circ}-a)$, it is decomposed into two others whose amplitudes are $\cos a$ and $\sin a$; after the first reflexion they become

$I \cos a$, vibration in the plane of incidence,

$J \sin a$, vibration perpendicular.

A second reflexion will cause them to undergo proportional changes; they will become

$$I^2 \cos a, \quad J^2 \sin a.$$

They will be finally, after (m) reflexions,

$$I^m \cos a, \text{ and } J^m \sin a.$$

And if the polarization is restored at a certain incidence, the cotangent of the azimuth of vibration, or which comes to the same thing, the tangent of the azimuth of restored vibration, will

be expressed by the ratio of the vibration in the plane of incidence to the vibration perpendicular, and we shall have

$$\cot x = \frac{I}{J}\frac{\cos a}{\sin a} = \left(\frac{I}{J}\right) \cot a$$

Thus in order to obtain after (m) reflexions the tangent of restored polarization at a given angle, we should have to calculate for this incidence the ratio $\frac{I}{J}$, raise this ratio to the mth power, and multiply it by the tangent of the azimuth of the incident ray.

Let us revert to a remark already made: the incidence of restored polarization depends only on the difference of phase; its azimuth, only on the ratio of the intensities. Too much importance cannot be attached to this phenomenon of restored polarization, which is the result of two modifications in the light, a change of phase and a modification of amplitude, in which there are two things to be measured, an incidence and an azimuth, which are separate functions of the difference of phase and of the ratio of the intensities of the principal rays; so that the observation of the incidences has served us to determine the phases, and that of the azimuths might, if we had not other means, have led to the discovery of the ratio of the intensities. This phenomenon therefore suffices for the determination of all the elements of metallic reflexion.

The experiments of Sir David Brewster verify the consequences of the theory for the particular case where the incidence is that of maximum polarization; on this subject I shall refer the reader to the memoir of M. de Sénarmont. I have thought it right to make new experiments on a metal hitherto not tried, namely copper, to determine at the same time the incidences and the azimuths of restored polarization for all the numbers of reflexion possible, and to calculate theoretically the results. Although one may be sure beforehand of finding experiment and calculation agree, this last verification was not without its use.

Copper.—*Angles and azimuths of polarization restored by multiplied reflexions.*

(The azimuth of the incident ray is 45°.)

Number of reflexions	Angle of restored polarization		Differences	Azimuth of restored polarization		Differences
	Observed	Calculated		Observed	Calculated	
6	83° 33′	84° 14′	−0° 41′	−29° 40′	−29° 57′	−0° 17′
4	81 0	80 52	+0 8	−30 3	−31 36	−1 33
6	77 40	77 33	+0 7	+22 35	+22 6	+0 29
3	78 0		+0 27	+32 20	+32 30	−0 10
10	74 25	74 42	−0 17	+11 5	+12 1	−0 56
2	70 9	70 0	+0 9	−34 0	−34 15	+0 15
4	70 54		+0 54	+22 30	+23 16	−0 46
6	70 0			−16 0	−15 45	+0 15
8	69 40		−0 20	+11 35	+10 28	+1 7
10	70 0			− 6 35	− 7 0	−0 25
10	64 40	64 28	+0 12	+13 30	+13 42	−0 12
6	60 10	60 5	+0 5	+24 45	+25 3	−0 18
3	60 40		+0 35	−33 15	−34 21	−1 6
10	57 40	57 37	+0 3	−16 0	−16 57	−0 57
4	55 5	55 21	−0 16	−31 30	−32 54	−1 24
8	55 18		−0 6	+22 35	+22 42	−0 7
10	49 40	48 40	+1 0	+23 30	+23 29	+0 1
6	45 0	44 56	+0 4	−34 0	−33 43	+0 17
8	42 10	43 39	−1 29	−30 15	−30 27	−0 12

The last experiments which we have to examine are those in which the two planes of incidence make with each other a determinate angle (ω). Sir David Brewster caused the light to be reflected from the first surface at a determinate and constant angle, then he sought by experiment for the incidence at which it was necessary to reflect the light from the second plate in order to restore the polarization: these are the experiments which Sir David Brewster has represented by an empirical construction, highly ingenious no doubt, but without any theoretical explanation (the complements of the angles of incidence on the second surface were made equal to the radii vectores of an ellipse): it will not be useless to show that in this latter case also, the theory is perfectly accordant with the facts.

The calculation made, pages 86 and following, applies here without its being necessary to change anything in it. An incident ray polarized in the azimuth $(90°-a)$ was reflected at the first metallic plate at a given incidence; it produces, after reflexion, two beams polarized in the principal azimuths, represented by the formulæ

$$= \cos \alpha \cos 2\pi \frac{t}{T}$$

$$y = \sin \alpha \cos \left(2\pi \frac{t}{T} + \delta\right)$$

These two beams fall on the second surface whose plane of incidence makes an angle (ω) with the first, they give rise to others polarized in the principal planes of the new plate, and whose vibrations are represented by

$$x' = A' \cos \left(2\pi \frac{t}{T} + \delta'\right)$$

$$y' = B' \cos \left(2\pi \frac{t}{T} + \delta''\right),$$

and the difference of phase of these rays will be expressed before the reflexion at the second plate by the formula

$$\tan(\delta' - \delta'') = \frac{\sin \delta \sin 2\alpha}{\sin 2\omega \cos 2\alpha - \sin 2\alpha \cos 2\omega \cos \delta} \qquad (11.)$$

We have thus decomposed the ray which has been reflected a first time into two beams polarized in the principal planes of the second reflecting surface: they have, before the reflexion, a difference of route ($\delta'-\delta''$), and by the act of the second reflexion, they acquire a new difference of phase (δ''') which is added to the former and gives a total $\delta'-\delta''+\delta'''$. In order that the ray may then be rectilinearly polarized, it is necessary and it is sufficient that $\delta'-\delta''+\delta'''=\pi$, which gives $\delta'''=\pi-(\delta'-\delta'')$. We may calculate by formulæ (9.) what is the angle of incidence on the second surface which is capable of producing this difference of phase, and it will remain to compare experiment with the calculation.

This comparison has been made for two tables extracted from the memoir of Sir David Brewster, pages 304 *et seq.* (Philosophical Transactions 1830.). In the first, relative to silver, the incidence on the first surface is 80°. The angles of the two planes of incidence are written in the first column, and in the following are placed the complements of the incidences which restore the plane polarization. The differences between calculation and observation will be found to be very slight, if we pay attention to the difficulty which must be met with in measuring exactly the azimuths and incidences in such complicated experiments.

Experiments of Sir David Brewster on Silver

Incidence, 80° on the first surface

Angle of the two planes of incidence	Differences of phase of the principal rays to the second incidence	Complement of incidences which restore, by a second reflexion, the rectilineal polarization — Observed	Complement of incidences which restore, by a second reflexion, the rectilineal polarization — Calculated	Differences
+90° 00′	51° 19′	10° 00′	9° 24′	+0° 36′
78 45	57 51	10 00	9 53	+0 7
67 30	66 26	11 32	11 37	−0 05
56 15	79 56	14 20	14 33	−0 13
45 00	96 7	18 20	18 37	−0 17
33 45	110 40	21 13	23 1	−1 48
22 30	120 30	25 20	26 32	−1 12
11 15	125 25	26 55	28 29	−1 34
0 00	125 41	28 2	28 37	−0 35
−11 15	122 90	24 40	26 30	−1 50
−22 30	113 34	21 00	23 59	−2 59
33 15	100 4	16 40	19 44	−3 4
45 00	83 53	14 35	15 28	−0 53
56 15	69 19	11 10	12 12	−1 2
67 30	59 30	10 00	11 15	−1 13
78 15	54 35	10 00	9 15	+0 45
90 00	54 19	10 00	9 24	+0 36

Incidence, 68° on the first surface

Angle of the two planes of incidence	Differences of phase of the principal rays to the second incidence	Observed	Calculated	Differences
+ 0 00	71 38	13 00	12 42	+0 18
+11 15	74 54	14 00	13 21	+0 36
+22 30	80 31	15 15	14 41	+0 34
+33 15	87 53	16 00	15 58	+0 02
+45 00	95 28	17 00	18 27	−1 27
+56 15	102 22	19 00	20 25	−1 25
+67 30	106 49	20 00	21 42	−1 42
+78 45	108 57	20 00	22 00	−2 00
+90 00	108 20	20 00	22 15	−2 15
−78 45	107 33	18 00	21 14	−3 14
−67 30	105 60	16 30	19 30	−3 00
−56 15	99 28	15 30	17 32	−2 2
−45 00	92 60	14 30	15 38	−1 8
−33 45	84 31	14 00	14 00	
−22 30	77 37	13 30	13 2	+0 28
−11 15	73 10	13 00	12 37	+0 23
− 0 00	71 3	13 00	12 42	+0 18

I have proposed to myself, in this memoir, not only to make known the experiments which belong to myself in particular, but also to recapitulate those due to experimenters who have preceded me in these researches, and to show that, thanks to the mathematical investigations of M Cauchy, the question of metallic reflexion is at the present time completely settled There remain yet numerous experimental researches to be made,

and, if they are easier they are not the less important There must be sought, for each metal, the values of the constants and the manner in which they vary with the circumstances which modify the polish, density, and molecular state of the body, must be determined further the different simple colours of the spectrum should be employed, and the laws of the inequality of action produced in them by metals, investigated

Conclusions

The memoir which I present has for its object to determine,—

1st The intensity of the light reflected from polished metals, when the incident ray is polarized in the azimuths of 0° and 90°

2nd The ratio of these intensities, by a different process

3rd The difference of phase of these rays, after reflexion

4th To show that the results of experiment are perfectly represented by the mathematical formulæ of M Cauchy

5th To investigate, after reflexion from a metallic mirror, the direction of the axes of the ellipse in which the molecules of æther vibrate, when the incident ray has been polarized in any azimuth whatever

6th To determine, by calculation and experiment, the incidences for which the polarization becomes again rectilinear after a certain number of reflexions from parallel plates

7th To find the azimuths of restored polarization under all incidences

8th To investigate the value of the angles of restored polarization when the two planes of incidence are inclined to each other, and the incidences on the two mirrors are unequal

Article IV.

Researches on the Electricity of Induction. By H. W. Dove.*

[Memoir read before the Academy of Sciences of Berlin.]

Introduction.

THE following researches were undertaken with a view to ascertain the effect produced by the division of a massive bar of iron into bundles of wire, and by the different modes of magnetizing the same upon the electric currents that are induced by it in the wire which surrounds it. Bachhoffner and Sturgeon † have shown that the shocks on opening a galvanic circuit is much more increased by the insertion of bundles of iron wires into the spiral coil forming part of the circuit, than by the insertion of iron in the form of a solid bar. The manner in which the extra current is produced can only be investigated by its physiological action and by the brilliancy of the spark which appears when connexion is broken in the completing wire. Besides, three actions concur to produce the latter, namely, the spark of the primary galvanic current, its augmentation by the action of the spiral coils of the completing wire upon each other, and the effect arising from the magnetism becoming evanescent in the inserted iron. In the production of the physiological effects the two latter causes alone concur, for a galvanic circuit which is closed by a short straight wire communicates no shock on breaking. By examining the secondary current instead of the extra current, *i. e.* by allowing the spiral coil terminating the galvanic circuit and surrounding the bundle of iron wire to act upon another wire not in contact with but parallel to it, I was enabled to investigate the resulting current by other rheometrical means besides sensation and the vividness of the spark. All that now remained to be done was to retain in the result the action of the *evanescent* magnetism *alone*, and this I obtained by constructing a *differential inductor*, in which *two equal* spiral coils interposed in the circuit acted upon *two equal* secondary coils, which being connected crosswise together, *completely neutralized* each

* The Editor is indebted to Dr. E. Ronalds for the translation and to Prof. Wheatstone for the revision of this memoir.

† Annals of Electricity, i. p. 181.

other's action. The disturbance of equilibrium on the introduction of iron into one of the previously empty coils, must therefore be the effect of this iron *alone*. The results which I obtained when the primary current was that of a *galvanic circuit* or of a *thermo-electric circuit* I presented to the Academy on the 24th of October 1839. They were of such a nature as to make it appear desirable that the experiments should be extended to other modes of magnetizing the iron than by the current of a galvanic circuit. The results obtained by the *discharge of an electric battery* were laid before the Academy on the 28th of October 1841; those obtained by *approaching the iron to a magnet* on the 18th of April 1842. By means of this last method of magnetizing, similar experiments could be instituted upon the *primary extra current*. The increased physiological action of secondary currents of higher orders by means of bundles of iron wires, was lastly the subject of a memoir laid before the Academy on the 11th of August, 1842. All these researches, forming together a complete whole, are here collected, with the consent of the Academy, into one memoir, in which that order has been followed which appeared to show the subject in the clearest light; the order in which they were first published will appear by the reports of the Academy.[1]

It is well known that electric currents of different origin

[1] Upon [illegible] induced currents of equal intensity [illegible] completely [illegible] and upon the application of mutually [illegible] 1838, pp. 21 and [illegible].

Upon the relation [illegible] and what [illegible] malleable iron to hard and soft steel with reference to the phenomena of induction which they produce. 1839, p. 7.

Upon magneto-electrical currents which, when in equilibrium as regards the galvanometer, produce powerful shocks upon the human body, and on the contrary, when their physiological action is mutually compensated, powerfully affect the magnetic needle. 1839, p. 1[illegible].

Upon the induced currents excited in magnetizing iron by means of frictional electricity. 1841, p. [illegible].

Upon the extra current at the commencement and close of a primary current. 1842, p. [illegible].

Upon the induced electric currents caused by the approach of massive iron or of a bundle of iron wire to a steel magnet. 1842, p. [illegible].

Upon the electric currents which the evanescent magnetism of electro-magnetized rods of iron or bundles of wires induces when the current which magnetizes them is produced:—

1. By the approach of a [illegible] wire to a steel magnet.
2. By the approach of soft iron to a steel magnet.
3. By the combination of both agencies by means of Saxton's machine. August 1842.

Upon the action of iron rods and bundles of wires upon induced currents of higher orders. August 1842.

cannot be determined to be equal when they cause the needle of the same galvanometer to deviate in the same degree, for according to Ohm's theory, the intensity of a current is equal to the electromotive force which produces it, divided by the resistance to conduction of all the parts through which the current passes, therefore, in the case where the resistance to conduction, a part of which only is due to the wire of the galvanometer, is unequal, a like amount of deviation in the needle of the galvanometer must prove an inequality to exist in the electromotive force. This inequality must become apparent if the resistance of both currents is increased or diminished in the same ratio In this manner it is explained, for instance, why a thermo-circuit and a galvanic circuit, both equally affecting a galvanometer, have a very different effect when a fluid is interposed in the connecting circuit The same reason explains why a voltaic pile and a galvanic battery, which have a like action upon the galvanometer, produce very different effects upon the human body or in a decomposing cell. But if the resistance to conduction is the same for both currents, if, for example, they both pass through the same conductor, and produce the same deviation in the galvanometer, then an equal amount of electromotive force must be ascribed to both If these currents produce different effects in cases where the resistance to conduction, on being changed in an equal degree in both, still remains the same, this difference can no longer be attributed to a difference in the electromotive force, but some other cause must be sought to explain it.

If an electric current is understood to be the equalization of an electric antagonism, however this may have been caused, then there are two things to be considered,—the original *force* of this antagonism, and the *time* which is required completely to equalize it. Differences in the action of two currents, which have been produced by the equalization of an equal amount of electric antagonism, must therefore be ascribed to the difference of time in which this equalization is effected.

If the *magnetic, chemical, physiological* and *calorific* effects of an electric current depended equally upon its *power* and *duration,* then two currents, known to be equal in one of those respects, should likewise be equal in the other three. This however is not the case.

With regard to the relation between the galvanometric effect

of a current and its chemical action, we may consider it proved by the experiments of Pouillet*, Jacobi†, and Weber‡, which directly confirm Faraday's law for constant electrolytic action, that with electric currents produced by galvanic means the decomposition of water in a given time is proportional to the constant power of these currents as it is indicated by the multiplier§ during that time. From two currents therefore which are known to be galvanometrically equal, we may expect an equal amount of chemical action.

In the phænomena of induction, up to the present time the cause of an increased physiological action has been ascribed to a larger quantity of electricity in motion which produced it, and hence it has been indirectly assumed, that in magneto-electric currents the physiological action is proportional to the deflection of the needle of the galvanometer, and to the volumes of the gases in the voltameters. With the electricity of the electric machine a difference has long been observed in this respect; for the discharge of a Leyden jar, which will communicate a powerful shock to the human body, is not capable of deflecting the magnetic needle, and only acquires the power of doing so when a wet string is interposed in the connecting circuit, and its resistance to conduction is thus increased. When this is done the physiological action decreases in a remarkable manner, whilst the dazzling white light of the spark is changed to a yellowish red colour. The shock is as completely prevented when the one coating of the jar is held in the hand and a point which is luminous with a bluish light in the dark is gradually approached to the other, or, as Lord Mahon states, when the jar is discharged by a piece of ivory. In this case of *gradual* discharge by the approach of a point there occurs, as Colladon first showed||, an action upon the magnetic needle. Suppose the electricity of both coatings were divided between two electrometers, of which the leaves of the one diverged as many degrees positively as those of the other diverged negatively, then if both were connected by a conductor when the leaves *slowly* collapsed,

* *Compt. Rendus*, v. p. 785.
† *Bulletin Scientifique de l'Académie de St. Pétersbourg*, 1839, p. 3[illegible]1.
‡ *Resultate des Magnet. Vereins*, 1840, p. [illegible].
§ Whether it be a multiplier compass or a tangent compass, a coil of wire wound round in the manner of a bifilar magnetometer and directed by the terrestrial magnetism.
|| *Annales de Chimie et de Physique*, xxxiii. p. [illegible].

a magnetic needle placed near the conductor would be deflected, but a human body causing the neutralization would not suffer any shock; if the leaves collapsed *suddenly*, on the contrary, a shock would be felt, whilst the magnetic needle would remain unmoved.

To these differences between the physiological and galvanometric action of the same quantity of electricity, according as it circulates more or less quickly through a conductor which have here been established, others may be added. In the electric currents produced by the motion of a closed conductor in the proximity of a magnet, the *power* is *directly* proportional to the *velocity*, the *duration* is *inversely* as the *velocity*; the tendency of a needle to move when placed in any fixed direction with respect to the coils of the multiplier during the continuation of the current is therefore entirely independent of the velocity, as Gauss has already shown*. The physiological impression is however not a product of the power and the duration, but is chiefly determined by the former: it increases therefore with the velocity of the motion, without affording to the person suffering the shock, by the shorter duration of a painful sensation, a full compensation for its increased acuteness.

Similar determinations to those relating to the physiological action of the current are likewise available for its property of *magnetizing hardened steel*; for if a Leyden jar be gradually discharged by a point or a rod of ivory, according to Seebeck's observation†, the magnetism produced in a steel needle by the connecting wire is either quite imperceptible or much less intense than when the discharge is effected in the usual manner by a discharger ending in a knob.

When therefore, of two currents produced in the same conductor, one of which has been induced by an electro-magnetized *bar of iron*, the other by an electro-magnetized *bundle of wires*, and both causing the *same* deviation in the galvanometer, the latter exhibits *more powerful physiological* action and *more vivid* sparks than the former, and at the same time magnetizes steel *more intensely*, we may conclude that in the latter an equal quantity of electricity is passing in a *shorter* time than in the former, and, on the contrary, the *physiological* and *magnetizing* action of both currents being *equal*, that that one which affects

* Schumacher's *Astronomisches Jahrbuch* 1836, p. 1[illegible].

† *Magn. d. galv. Kette*, p. 15. *Abhandlungen der Berliner Akademie*, 1821.

the *galvanometer* least must possess an increased velocity in proportion to its *decrease* of power.

I shall now first illustrate the apparatus which was made use of in the experiments by a simple figure, and then proceed to the more accurate description of it.

1. *Principle of the Differential Inductor.*

1. When an electric current is excited in two similar wires *a b* and *c d* (Plate I. fig. 1), which are connected with each other by a wire *b c*, this current will produce on being discontinued a secondary current in the same direction as the first in two wires $\alpha\,\beta$ and $\gamma\,\delta$ placed parallel to the first two. If these wires however are connected crossways, *i. e.* (fig. 2) α with γ and β with δ, then the currents induced in $\alpha\,\beta$ and $\gamma\,\delta$ by the primary current *a b* will flow in contrary directions, and if they are equal will completely neutralize each other. But if at the side of *a b* a second closed wire *e f g h* (fig. 2) is placed, the current induced in it reacts upon *a b* and $\alpha\,\beta$, and acts in a *retarding* manner as it is passing in the *same* direction as the currents in *a b* and $\alpha\,\beta$, *i. e.* in a manner to weaken them for all the tests which, the quantity of electricity being the same, are less affected when it traverses the wire in a longer than when the same occurs in a shorter time, therefore *weakening* with reference to the *physiological action* and to the *magnetizing of steel*, whilst the *effect upon the galvanometer* and the property of *magnetizing soft iron* are *not* changed by it. The phænomena of induction with relation to these tests which arise from the presence of *e f g h* in the wire $\alpha\,\beta\,\gamma\,\delta$ after the equilibrium of the current is destroyed, will appear from this to be caused by a current passing in the direction from β to α, as the unretarded current induced in $\gamma\,\delta$ by *c d* preponderates over the current induced in $\alpha\,\beta$ by *a b*, but which is retarded by *e f g h*. These phænomena of induction must also be solely ascribed to the action of *e f g h* upon $\alpha\,\beta$, as the direct action of *a b* upon $\alpha\,\beta$ is not lessened by the presence of *e f g h*, as is obvious from the principle of multiplication applicable in the case of inductions with superimposed coils of wire.

2. If instead of the endless wire *e f g h* a rod of iron *s n* (fig. 2) is substituted at right angles to the plane of the wire, it will be magnetized by the primary current. The evanescent magnetism of this rod of iron when the primary current *a b* is discontinued induces in like manner a current in $\alpha\,\beta$, which is moreover in a

like direction to that excited in $\alpha\beta$ by the electrical current existing in $a\,b$ at the moment of its cessation. The equilibrium of the currents which previously existed in $\alpha\beta\gamma\delta$ is therefore also destroyed, but the resulting current, even for all methods of trial, will exhibit an opposite direction, namely from α towards β; for now the augmented $\alpha\beta$ will preponderate over $\gamma\delta$ which has not been augmented. Let us suppose, lastly, the electro-magnetized bar of iron $s\,n$ surrounded by a conducting wire $e\,f\,g\,h$, then in consequence of the evanescent magnetism in $s\,n$ a *larger* quantity of electricity will be put in motion in the wire $\alpha\beta$ than in the wire $\gamma\delta$; but as an electric current is simultaneously excited in $e\,f\,g\,h$ this quantity of electricity will move *more slowly* than the lesser quantity in the wire $\gamma\delta$.

Here *three* different cases are possible:

1. The augmented quantity of electricity may *increase* some particular action of the current more than the retardation of the current diminishes it.

2. The increased action caused by the augmentation of the quantity of electricity may be exactly compensated by the retardation of the current.

3. The retardation of the current may diminish some particular action more than the augmentation of the quantity of electricity increases it.

In the first case, the current will be directed from α towards β; in the second, the equilibrium of the currents will remain stationary; and in the third, the current will flow from β towards α. When the primary current which magnetizes the iron is that of a *galvanic circuit*, that of a *thermo-battery* or *thermo-circuit*, or the induced current of a *Saxton's machine*, the *first* case is always observed, when however the primary current is produced by the discharge of a *Leyden jar* or of an *electrical battery*, the *third* case, and under particular circumstances the *second* case happens; but in such a manner, that when the *first* case occurs for one *method of testing* the current, the *third* case may occur for *another method*, and *vice versa*. Lastly, the primary current of a Saxton's machine may be so modified by the extra current which it produces, that all three cases may be observed with it.

A bundle consisting of insulated iron wires is not capable of producing peripherical electrical currents surrounding the whole of the bundle. If however it is inclosed in a *conducting sheath*,

for instance in a *closed brass tube*, then the bundle of iron wires will represent the magnet *s n* and the sheath will represent the wire *e f g h*. In a solid bar of iron its surface must be considered the surrounding sheath *e f g h*. *s n* with its surrounding wire *e f g h* is such an electro magnet.

3. If by the side of *c d* a similar combination $s' n'$ and $e' f' g' h'$ be placed, then the equilibrium of the currents will be destroyed in a twofold manner; but from the direction of the resulting current it will be evident which of the two disturbing causes of equilibrium is the more powerful. If these are lessened, either by modifying the stronger *s n* or the stronger *e f g h*, the equilibrium which had been destroyed may again be restored. The apparatus then becomes a measuring instrument.

For the purpose of increasing the action, it is convenient to give the magnetizing wires *a b* and *c d*, as well as the wires $\alpha\,\beta$ and $\gamma\,\delta$ in which the inducing action is effected, the form of spirals, the latter being wound in an insulated manner round the former, whilst into the former are placed the bars of iron to be magnetized, as well as that part of the apparatus representing the conductor *e f g h*.

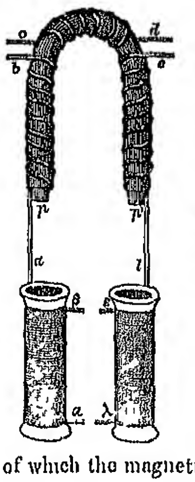
4. Some of the metallic rods placed within the spirals were *cylinders*, others *four sided prismatic bars*. The cylinders were of equal dimensions, namely 11 inches 7 lines long and $11\frac{1}{2}$ lines in diameter. There were thirteen of them, composed of *brass*, *tin*, *zinc*, *lead*, *hardened steel*, *gray cast iron* from a crucible furnace, *gray cast iron* from a cupola furnace with a hot blast, *gray cast iron* from a cupola furnace with a cold blast, *white cast iron* from a cupola furnace with cold blast, *white cast iron* from a crucible furnace, and two cylinders of very *soft wrought iron*; besides these, *gun barrels*, some *cut open lengthways*, others *uncut*, one *brass tube cut open* and another *entire*, a *tube of lead*, of *tin*, of *German silver*, of *nickel*, a *riveted* tube of *sheet iron cut open lengthways*,—all these had the same dimensions as the cylinders. The *wires composing the bundles* were of the same length as the cylinders. Amongst these were four sorts of *soft iron wire*, having diameters of $0'''\,70$, $1'''\,02$, $1'''\,16$, $2'''\,67$; the first sort was well varnished with shell lac. Bundles were also formed of *soft steel wire* $0'''\,57$ in diameter, of *hard steel wire* of $0'''\,87$, and of varnished *brass wire* of $0'''\,70$, of *copper wire* of $0'''\,75$, of *tin wire* of $1'''\,10$, of *lead wire* of $0'''\,80$, of *zinc wire* of $0'''\,60$ diameters; besides these, cylinders were constructed of

fine *iron borings* enclosed in glass tubes, and piles were composed of discs of *sheet steel*, of *tinned* and *untinned sheet-iron*; the discs were isolated by discs of paper; lastly, one cylinder was composed of discs of tinned sheet-iron with interposed pieces of silver coin: the diameter of these piles, consisting of several hundred separate discs, was 9 lines. The prismatic rods were composed of *nickel, antimony, bismuth, zinc, lead, copper, iron, brass*, 18 inches long and 5 lines broad. *Gold, silver, platina* and *iridium* were used in strips laid one upon the other.

5. Although the same magnetizing spirals $a\ b$ and $c\ d$, and the same induction spirals $\alpha\beta$ and $\gamma\delta$, may be used with different primary sources of electricity, yet it is preferable, when *galvanic* currents are used and a strong action is required, to give greater thickness to the connecting wire and a greater number of coils to the collateral wire than when *frictional electricity* is employed, and perfect insulation is then not so imperative as it is with the latter kind of electricity. If however the magnetization of the iron is effected in a direct manner by *approaching* it to a *steel magnet*, then the apparatus must be constructed in an essentially different manner. In the following experiments I made use of *four* different differential inductors; the description of the first three follows here, the last will be noticed in the sequel.

2. Differential Inductor for galvanic and thermo-electricity.

6. In the threads of two similarly cut screws of wood two spirals of copper wire $2\frac{1}{2}$ lines thick*, insulated by means of shell-lac,



* If it is required to use the apparatus here described as an electro-magnetic machine, in which induction takes place by means of an electro-magnetized horseshoe of iron, the arrangement depicted in the annexed figure may be employed. A cylindrical bar of soft iron $p\ p'$, bent into the form of a horseshoe, is wound round at the bend in the middle part of it by a thick spiral wire of copper $c\ d$, which is prevented from coming into contact with the iron by an intervening insulating substance. Upon the straight parallel limbs of the horseshoe, which are likewise covered with an insulating substance, two straight cylindrical spirals of the same wire $a\ b$ and $e\ l$ are placed, which being coiled in the same direction as $c\ d$, form, when b is joined to c and d to e, one continuous coil $a\ b\ c\ d\ e\ l$. The ends $a\ l$ of these two spirals proceed in a straight direction and parallel on the outer side of the limbs, so that they may not interfere when the keeper is applied to the poles $p\ p'$, nor prevent the induction-spirals $\alpha\beta$ and $\epsilon\lambda$, composed of long thin wire, from being drawn over the coils of thick wire of which the magnetizing spirals $b\ a$ and $e\ l$ are formed.

and passing round 29 times at an internal distance from each other of $18\frac{1}{2}$ lines formed, when in contact, the connecting wire of the galvanic circuit.

The iron cylinders and bundles of iron wires which are to be tried are placed within the cylindrical hollows of the wooden screws, which, electro magnetized by the copper wire induced a current in two superposed coils composed of wire half a line in thickness, wound round with silk and having each a length of 400 feet. The free ends of these transversely connected induction coils are joined by means of handles, the current passing through the body or by a galvanometer, and their reciprocal compensation determined in both cases.[*]

The equilibrium which is destroyed by introducing an iron cylinder into one of the spirals is restored by gradually inserting iron wires into the other spiral. In none of these experiments is induction produced by the insertion of the yet unmagnetic iron into the spirals which already form the connecting circuit of the battery, and on that account magnetize the affected iron, but by the iron already contained in the spirals becoming polarized and depolarized successively by alternately closing and breaking the galvanic circuit. All the currents of which we are here treating are of the kind called *momentary* currents. In the method of observation here pursued, as has been explained above,

[*] When the induction spirals are not of the same length as the magnetizing spirals upon which they are placed, and an induction spiral indicates an induced current of variable intensity according to the position in which it is placed upon a straight electro magnet, then an imperfectly attained compensation may be as easily remedied by altering the position of the induction spiral towards its magnetizing spiral as by diminishing the length of wire in the more powerful induction spiral. In order now to determine which part of an electro magnet had the most powerful inducing action, a covered copper wire was coiled into two spirals of 60 revolutions, which were connected by a long straight end. Into each of these spirals was inserted one of the poles of an electro magnet 22 inches in length and 11 lines thick, which was surrounded by a copper wire 2 [illegible] thick in 60 revolutions. When the compensation of the spirals had been determined by the galvanometer near to the ends of the electro magnet, one of the spirals was moved to a position nearer the middle, the other remaining unchanged, and the connection was broken between the electro magnet and the galvanic battery. Immediately great deviations were perceptible, and moreover in a contrary direction, when the spiral which had in the first instance been the more distant from the centre was made to assume the nearer position. The deviations were always traceable to the spiral which was most nearly approached to the centre, and they remained the same, whether under the direct influence of the electro magnetized iron horseshoe or whether that was made the keeper to another electro magnet, which by closing and opening the circuit was polarized and depolarized. The most advantageous position for an induction spiral is therefore the middle of an electro magnet.

the resulting current is only produced by the inserted iron, as the direct action of the connecting wire upon the secondary wire is completely compensated.

The galvanic batteries used in the experiments were sometimes small calorimotors of two coils and 4 inches high, at other times larger many-celled troughs united to form one battery 13 inches in width, with four interposed copper and amalgamated zinc plates. Afterwards constant batteries were employed with advantage, either Bunsen's carbo-zinc battery, or Grove's platino-zinc battery. The experiments with thermo-electricity were made with a thermo-battery consisting of eight bars of antimony and bismuth, which formed at their upper and lower extremity a chess-board of sixteen squares, each 8 lines in width, whilst the height of the bars was 3 inches 8 lines. The poles of this battery, which terminated in wide vessels containing mercury, were connected by the magnetizing spirals, and the unequal temperatures were produced by water cooled to 0° Celsius by means of snow, and by a suspended plate of red-hot iron. Afterwards a simple thermo-battery was used consisting of two bars of bismuth and antimony, $3'' \, 7'''$ in length and having a square section of $8''' \, 5$, soldered together, and warmed at the part where they were soldered by the flame of a spirit-lamp. All the connexions of the thick wires were made by means of cylindrical vessels containing mercury, with holes bored through them. All the connexions of the thin wires were effected by clamps doubly bored, the holes being at right angles to each other*.

3. *Differential Inductor for magneto-electricity.*

7. When the primary current was that of a Saxton's machine, instead of the inner spirals of thick copper wire, two spirals of thin wire $\frac{1}{2}'''$ thick and $400'$ in length were used, each an inch wide and a foot long. The outer spirals were the same as those used in the former differential inductor.

4. *Differential Inductor for frictional electricity.*

8. Upon two strong cylindrical glass tubes one foot long and

* If the holes in the clamps are at right angles to each other, by means of one screw the crossed wires can be clamped together. The holes being bored completely through the clamps, admit of the wires being connected not only at their ends, but by drawing any one of the wires the requisite distance through the clamp, the end of the one wire can be connected with the middle of the other

an inch in width (Plate I. fig. 3), are coiled two spirals of copper wire in the same direction, completely imbedded in shell lac and surrounded on the outside with paper. Each of the spirals forms 80 coils with 32 feet of wire. Of the wire clamps in which these spirals terminate, *a* is connected with the inner, and *d* with the outer coating of the insulated battery, after this has received a constant charge by means of a unit jar. As the clamps *b*, &c. are united by a cross wire, the two spirals *a b* and *c d* form together the connecting wire of the battery. The induction spirals, coiled in the same direction as the inner ones which they are to enclose, are wound upon tubes of pasteboard, and imbedded in shell lac, each wire having a length of 45 feet and 80 coils. The thickness of the wire of these spirals is the same as that of the wires of the connecting spirals, namely, half a line: both ends of each secondary spiral are on the same side (in the front of the figure); the longer end of each spiral which is bent back (β, γ), passes therefore along the external paper covering, enclosed in a glass tube, which is fixed by two silken bands, with the aid of small pieces of cork. Of the four ends of these spirals, two, α and γ, are connected by a cross wire, whilst the others, β and δ, either terminate in handles, as is represented in the figure, or are connected by a spiral containing an unmagnetized steel needle, by a galvanometer, an electro-magnet, an apparatus for decomposition, an electric air thermometer, or one of Breguet's metallic thermometers, an insulated preparation of the frog, a condenser, or an apparatus consisting of points with an insulated disc of resin between them, for the production of figures on the resin. Each of the connecting spirals, *a b* and *c d*, rests with its surrounding secondary spirals, $\alpha\beta$ and $\gamma\delta$, upon two glass feet $\frac{4}{5}$ of an inch in diameter, and well covered with shell lac: these branch out at the height of $8\frac{1}{2}$ inches into a fork composed of two glass rods, each of which is 3 inches in length, and these are fixed into brass caps by cement, at a distance of $1\frac{1}{2}$ inch from each other at the top, upon the vertical stems. Into the interior glass tubes the metallic cylinders and bundles of wire which are to be compared are introduced, as is shown in the figure, where the spiral *c d* incloses a solid cylinder, and the spiral *a b* a bundle of wire surrounded by a metallic tube. The apparatus was made by M. Kleiner with his usual care.

I now proceed to the experiments themselves.

I *Currents induced by the evanescent magnetism of electro magnetized rods of iron and bundles of wires, when the magnetizing current was that of a galvanic battery*

1 *Comparison of the galvanometric and physiological action*

9 If a *solid rod of iron* is placed in one of the spirals of the differential inductor, and a *bundle of wires* in the other, so that equilibrium is established as regards the galvanometer, these currents, which galvanometrically compensate each other, produce powerful shocks upon the human body when it is made to form part of the circuit By diminishing the number of wires, these shocks may be reduced to nothing, but then the currents, which physiologically compensate each other, cause a powerful deflection of the galvanometer needle in favour of the solid cylinder How great this difference is, may be seen by one of the series of experiments with wire of a line in thickness The number of wires requisite for compensation was as follows —

	For the galvanometer	For sensation
Forged iron	110 + x	15
Gray iron from the crucible furnace	92	24
Soft steel	91	9
Gray iron from the cupola furnace, with hot blast	45	18
White iron from the cupola furnace, with cold blast	43	8
White iron, crucible cast	41	10
Hard steel	28	7
Gray iron from the cupola furnace, with cold blast	27	11

With forged iron, the number of wires that could be placed in the wooden screw was insufficient for compensation in the galvanometer Without exception, therefore, the number of wires required to compensate a solid rod of iron is *greater* when the currents act upon the galvanometer than when they act upon the body, or in other words, when the currents are of *equal intensity as regards the galvanometer, the shock produced by the bundle of wires is greater than that produced by the solid bar of iron* To test this result in another manner, the following experiment was made —A differential galvanometer with two equal wires, each of which made 100 revolutions round its frame, was so connected with the induction spirals, which had

previously been separated, that the current of one spiral passed through the 100 revolutions of one wire of the galvanometer in an opposite direction, to the current of the other spiral which passed through the 100 revolutions of the other wire, when equilibrium had been established for the astatic needle between the solid cylinder and the bundle of wires, the force of the shocks in both the separated spirals was tested, and those produced by the bundle of wires were found to be decidedly stronger.

10. Although we have here direct proof (for the currents in the foregoing experiments always circulated in the same wire) that the non-existence of equilibrium for sensation, when the human body is interposed in that current which produces no deflection of the galvanometer, cannot be accounted for by an increase in the resistance to conduction, nevertheless, as a more rigid test, the following experiment was made. The induction spirals were increased in length to 300 feet, so that the currents having opposite directions, passed altogether through 400 feet of wire. Afterwards 2000 feet of wire, and again much greater lengths of wire were interposed, without in the least disturbing the equilibrium in the galvanometer. A great *increase of the resistance to conduction* was therefore without effect.

11. The results obtained for iron appeared also to be applicable to nickel. A four-sided rod of nickel, which was compensated as regarded sensation by iron wires, produced in the galvanometer a deflection in the direction of the current from the rod.

12. With regard to the *galvanometric* equilibrium, a remarkable phænomenon must be mentioned which indicates that the augmentation of the currents to the maximum of their intensity with the same mean power does not take place in the same time. Suppose the number of the wires to overpower the solid cylinder, so that the deflection of the needle is in the direction of the current produced by the wires, and that this excess is lessened by the gradual removal of wires, then the deflection is not observed to pass through the point of equilibrium, by gradually decreasing deviations, into one of an opposite direction, but the needle moves as if driven by a quick, short impulse, in the direction of the former deviation, then suddenly stops, and returns much more slowly in the direction of the other current. This *vibratory* motion is still observed when the second current has become the more powerful, so that the *short impulse* in the

one direction is followed by *a wider oscillation* in the opposite direction. Let $a\ c$ (Plate I. fig. 4) indicate the duration of the first current, and $a\ e$ that of the second; $a\ b\ c$ the curve of intensity of the first, $a\ d\ e$ that of the second; and if the superficies $a\ b\ c = a\ d\ e$, it is easily seen why the needle, which is only in equilibrium, at the point of section d first moves in the direction of the current $a\ b\ c$, and then in the direction of $a\ d\ e$; and that this may even continue for a certain time after the superficies $a\ d\ e$ has become larger than $a\ b\ c$. The vibration of the needle is more clearly perceptible when the exciting circuit is closed than when it is broken; but in both cases it is in the direction of the current from the wires.

13. With this phænomenon another stands in direct relation. For, as the needle of the galvanometer is set in motion by the difference of two currents, and this difference increases in proportion as both currents become stronger, the primary deviation will be augmented by an increase of power in the currents. If this difference arrives at a sensible magnitude, the second current finds the needle in a *less favorable position* with respect to the coils of the multiplier than the first did, and the former can for that reason *apparently* overpower the latter. This was observed several times, when calorimotors with very dry plates were used, after the slight vibratory motion had been obtained with previously moistened batteries. In this manner it can therefore be explained why more wires are required to compensate the solid cylinder with strong than with weak currents.*

14. Although the method of observation by means of two mutually compensating spirals is peculiarly adapted to point out the differences between two currents such as those which were excited, yet it is obvious that the numbers given above, placed in vertical rows, can only express a real numerical relation upon the supposition that the battery was *constant* in its action. The batteries which were then at my disposal did not warrant such a supposition. By the constant use of loose wires, also, the *insulation* of the iron wires was not at all times the same, for the shell lac varnish got rubbed off in places. I have therefore endeavoured to arrange the metals employed in a *galvanometric*

* What is here said applies of course only to a current induced by an electro-magnetized bundle of wires acting in opposition to one induced by a solid rod of iron, and not to currents produced by two solid rods of iron acting in opposition to each other. If equilibrium has once been established for these with weak currents, it is not altered by increasing the power of the currents.

series, making use of another sort of well varnished wire, 0·70 line in thickness, and employing one of Bunsen's carbo zinc batteries for the production of the magnetizing current. Of wires of this thickness 170 could be placed within the hollows, but they were more than compensated by the *cylinder of soft iron* by the *gun barrels*, and by the *slit tubes of sheet iron*, whilst a *welded tube of double sheet iron* just held them in equilibrium. The reason of the excess of power in the cut tubes above that of the welded tube, may be sought in the greater amount of external surface which their elasticity obliges the former to assume. By this mode of experiment the following series was obtained —

Substances	Number of wires 0‴ 70 thick required for galvanometric compensation
Cylinder of soft iron	170 + x
Gun barrel	170 + x
Slit tube of double sheet iron	170 + x
Welded tube of double sheet iron	170
Cylinder of soft steel	150
Cylinder of gray pig iron from a crucible furnace	140
Cylinder of gray pig iron from a cupola furnace, with hot blast	86
Cylinder of white pig iron, crucible cast Tube of sheet iron	84
Cylinder of white pig iron from cupola furnace, with cold blast	83
Cylinder of hard steel Cylinder of gray pig iron from cupola furnace, with cold blast	67
Four sided rod of nickel (4‴ 75 thick)	10
Tube of nickel Pile of iron discs separated by paper	4
Pile of steel discs separated by paper	2
Pile of tinned iron discs separated by paper Tube of German silver Cylinder of fine iron borings	1

15 The galvanometric arrangement of the different kinds of iron I determined in a more direct manner by placing one of the iron cylinders in one spiral, and counteracting its effects by the other eight successively placed in the other spiral, by the direction given to the needle of the galvanometer, it was ascertained

which cylinder had the more powerful action. With different species of iron the following series was obtained, with some exceptions, in the individual experimental series*:—

Soft iron.
Gray iron from the crucible furnace.
Soft steel.
Gray iron from the cupola furnace, with hot blast.
White iron.
Gray iron from the cupola furnace, with cold blast.
White iron from the cupola furnace, with cold blast.
Hard steel.

16. The determination of the exact number of wires which compensate the effects of a cylinder as regards sensation is, for another reason, difficult. For every degree of power in the battery, the number is smaller than that required for compensation in the galvanometer; but in the case of *weaker* currents, when the excess of the one over the other can no longer be felt as a shock, it becomes perceptible when the current is *stronger*. The latter sensation remains likewise for a length of time with apparently unchanged intensity, so that no perceptible change is produced by diminishing the number of wires.

17. I have therefore endeavoured to determine in another manner the physiological series for solid rods. If from two cylinders acting in opposition to each other a shock is felt, as the result of the one current being more powerful than the other, in order to ascertain from which rod the shock proceeded, we have only to draw out one of the rods from its magnetizing spiral, and whilst it is being drawn out, to break and close the circuit successively by turning round a rheotome. If the weaker rod is being removed, then the shocks become constantly more intense; if, on the contrary, the more powerfully acting rod is the one moved, the shocks become weaker until the rod has been drawn out a certain distance, when they cease altogether. If this distance is exceeded, shocks from the opposite current are obtained, which gradually increase in intensity, and the distance which a rod is drawn out from its spiral thus becomes a means of quan-

* It is hardly necessary to remark that the arrangement of such series is only intended to direct attention to the fact, that slight differences in the nature of cast iron and steel materially affect the inducing action of iron, and not positively to determine by the name of a substance the position which it holds in the series. A series of this kind would only be absolutely correct, if identical substances could be designated by the same name.

titatively determining the relative value of the two opposing currents Gray iron from the crucible furnace overpowered considerably both soft and hard steel Very hard white iron from the cupola furnace with cold blast comes very near in its action to soft steel, but very perceptibly exceeds that of hard steel The difference between malleable and cast iron was less than that between malleable iron and steel, and indeed with some kinds of cast iron the difference was so slight that it could not be accurately determined by the method of drawing out the bars

18 From these experiments and from those made with bundles of wires, it follows that the series obtained by the galvanometer for the different kinds of iron compared, is a different one to that obtained by physiological means

The *physiological* action is therefore dependent on the one hand upon the *mechanical discontinuity* of the mass, and on the other upon the *peculiar nature* of the iron Hence it follows, that wires of soft iron having a different diameter may compensate a cylinder of a particular kind of iron both as regards the magnetic needle and sensation at the same time This was the case, for instance, with twelve wires 2 67 lines in diameter, and a cylinder of gray iron from the crucible furnace The influence exerted by the peculiar nature of the kind of iron is also manifest from the following facts —When the cylinder is of hardened steel, no difference is perceptible between the induction shock produced by the polarization of the cylinder when the circuit is closed, and the shock produced by its depolarization when the circuit is broken, the difference between the shocks is perceptible if the cylinder is composed of soft iron, much more so with cast iron cylinders or bundles of wires, when the shock on breaking the circuit is more intense than that produced by closing it This difference depends more upon the nature of the iron than upon its mechanical discontinuity, for it was found to be greater with eleven soft iron wires than with fifteen hard steel wires, which, when opposed to each other, destroyed each other's physiological action

19 From all the experiments which have as yet been instituted, it appears that gray iron approaches the nearest in its inducing action to the bundle of wires, for its physiological action is proportionally greater than could have been expected from the intensity of the current determined by the galvanometer The inducing powers of gray pig iron lead therefore to the supposi

tion, that it is a substance in which the iron capable of being magnetized does not form a connected continuous whole, a result which accords with the chemical researches of M Karsten

1 *Effect of the inversion of the magnetic polarity upon the induced current*

20 In the foregoing phænomena an important circumstance has not yet been noticed, a neglect of which would render it impossible to compare different kinds of iron with each other,—I allude to the effect produced upon the induced current by the *inversion of the magnetic polarity* Malleable iron, steel, nickel, and cast iron retain always a greater or lesser portion of the magnetism which is momentarily excited in them, it becomes therefore important to know, when they are again electro magnetized, what relation the metallic bar which is already magnetic, bears towards the magnetizing spiral containing it, whether namely, the remagnetizing produces a like polarity to that already existing in the magnetic bar, or whether a contrary polarity ensues The influence which this exerts upon the phænomena of induction was ascertained in the following manner —When perfect equilibrium as regards the galvanometer had been established between two cylinders, the position of one of them was *inverted* in relation to its spiral, so that it was magnetized in an opposite direction on again closing the circuit The disturbance of equilibrium which hence arose always showed that, by *inversion*, the induced current became more powerful Previous magnetizing by touch gives analogous results to those produced by electro magnetizing If iron horseshoes are used which project with one limb into the spirals as far as their point of neutrality, they become converted into tri polar magnets Now, as an electrical current of sufficient intensity, when it exerts a magnetizing influence upon a bar of iron so as to cause polarity in a contrary direction to that already existing in the iron, annihilates immediately the polarity which the iron retained from the previous magnetization, and then produces the maximum polarization of which it is capable, the fact here adduced may be expressed by the following axiom —*That metal is possessed of the most powerful inducing action in which the greatest change of magnetic properties occurs** The indications of the

* In the construction of magneto electrical machines, it follows directly from this, that in the alternation of the currents there exists a peculiar principle for increasing their power

magnetic needle here accord with those of sensation with the harder kinds of cast iron the effect of inversion is so great, that with two cylinders which compensate each other, when the one is reversed a shock is the consequence A few examples will show how necessary it is to take into consideration this increase of power by the reversion of position

21 The inducing action of soft iron is superior to that of soft steel, and this again exceeds that of hard steel when the excitation continues uniform If no very great difference exists in the action of these latter, and into one spiral of the differential inductor the cylinder of soft steel is placed, whilst the other contains the cylinder of hard steel in a reversed position, the remarkable phænomenon is observed, that the galvanometer needle deviates in the same direction when the circuit is *closed* as when it is *broken* By inverting the polarity of the hardened steel on closing the circuit, the current which it excites becomes stronger than that caused by magnetizing the soft steel in the same direction as before On breaking the circuit, however, the soft steel cylinder loses more of the magnetism communicated to it than the hardened cylinder, and hence exerts a more powerful inducing action But as the direction of the current induced by the *evanescent* magnetism on breaking the circuit is opposed to the direction of the current induced by the magnetism resulting on closing the circuit, the current passes in both cases in a like direction through the connected induction spirals Similar relations were observed with cast iron, with this difference only, that a *repeated* closing and breaking of the circuit was requisite to produce this phænomenon, which, in the case of hardened steel, takes place when the circuit has been *once* closed for a short time, and hence it appears that white cast iron in particular offers a greater hindrance to the inversion of its polarity than steel

22 The series adduced at a former page would have been very different had not attention been paid to this principle of the increase of power For, whilst all kinds of cast iron, when the polarity is unchanged, exert a weaker action than malleable iron, yet on inversion the action of gray and white iron from the cupola furnace with cold blast is the more powerful, white iron, crucible cast, remains about the same, and gray iron from the crucible furnace and from the cupola furnace with hot blast are below these in power Thus soft and hard steel, on

the inversion of their polarity, exceed in power all kinds of cast iron; if however the polarity of these latter be reversed, they act more powerfully than soft and hard steel. So, when different sorts of cast iron are compared with each other, the strongest action is always in favour of the kind of cast iron the polarity of which has been reversed. Similar results were obtained with bundles of wires.

23. The foregoing experiments appear to me to explain a circumstance which has frequently been adduced in support of the view, that a *retardation* of the current *increases its physiological action*. The circumstance is this, that the shock is greater when the battery is discharged by sliding the wire than when it is effected by immediate separation. I find that the shock from the induction spiral is more powerful when the circuit is broken *very shortly* after it has been closed, and I explain it in this manner: the first current which is produced by closing the circuit is not completely gone when the second begins, as the production and disappearance of magnetism in iron requires a certain time. The second current therefore meets with a conductor which is being traversed by a current in an opposite direction, and probably this change of direction in the current takes place more quickly in this case than if the conductor had not been previously traversed by any current, as the tendency of the conductor to return to its natural unelectric state is aided by the action of the second opposing current. Sliding is nothing more than a quick, often repeated closing and breaking of the circuit, as may clearly be seen in the dark, and hence its increased physiological action.

2. *The action of the current in magnetizing steel compared with its action in magnetizing soft iron.*

24. One hundred feet of wire, surrounded with silk and well varnished, was coiled 200 times round a wooden frame, in which were placed the needles (darning needles) to be magnetized, at right angles to the magnetic meridian to which the wire-coils were parallel. The free ends of the induction spirals, which had been joined crossways, were connected by means of cups containing mercury with the ends of the wire of the frame, and in such a manner that when the galvanic circuit was closed this connection was not established, but always on the repeated breaking of the circuit. Hence the magnetizing action was *al*

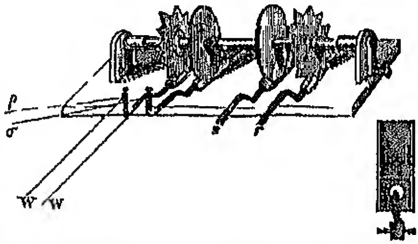
ways exerted *in the same manner*, and not alternately in opposite directions. The excess of power in the bundle of wires was so great, that even with seventy wires one line in thickness in one spiral, and the cylinder of soft iron in the other, the steel was magnetized in the direction of the current excited by the bundle of wires, although 110 wires did not neutralize the action of the cylinder in the galvanometer. When the induction spirals were connected in the same direction, and both contained bundles of wires, I was enabled to invert the polarity of a well hardened galvanometer needle. It is generally to be observed in experimenting with such currents, as is the case in galvanometric measurements with frictional electricity, that great changes occur in the length of the oscillations of a previously astatic needle.

25. A horseshoe of soft iron wound round with wire was now connected with the induction spirals. The current excited by a solid cylinder, as well as that caused by a bundle of wires, magnetized it in such a manner, that iron filings strewed upon it stood on end, and a magnetic needle placed near it was powerfully deflected. With opposing currents, the one of which was excited by a cylinder of soft iron, and the other by 100 wires, when the magnetizing spiral with the steel needle and the horseshoe of soft iron covered with silk were simultaneously placed in the closed circuit of the induced current, the soft iron horseshoe was magnetized in the direction of the first current, whilst the steel needle was magnetized in an opposite direction, *i. e.* in the direction caused by the wires. Now, as a continuous current is required to electro-magnetize soft iron, whilst steel is magnetized by the most sudden discharges, this experiment may be considered pretty conclusive in favour of the assumption which follows from all the other facts, that *in the current induced by a bundle of wires, a certain quantity of electricity is circulating in a shorter time than when that quantity is set in motion by a solid cylinder.*

3. *Sparks, heating effects, and chemical decomposition.*

26. When the same wire is traversed by two currents circulating in opposite directions, no spark is produced on disconnecting the wire when both currents neutralize each other. When therefore a massive cylinder and a bundle of wire oppose each other, and a spark is obtained at the differential inductor, by diminishing or increasing the number of wires the spark may

be made to disappear. In this manner, however, we should hardly be able to judge of very slight differences, as the current must have a certain amount of intensity in order to give rise to a spark when the circuit is broken. I therefore pursued a different mode of experiment. On a common axis of rotation three notched wheels were fixed, two of which are represented in the annexed woodcut. By the side of each notched wheel, and connected with it by a copper cylinder, is a disc, which dips constantly into a vessel containing mercury, whilst the notched wheel is alternately in and out of connexion with the mercury in a similar vessel: thus a previously existing metallic connexion is interrupted. This is the



rheotome or current interrupter which is now so frequently used for producing rapid consecutive disconnexion, which was invented in Germany before the year 1804, and is described in Aldini's *Traité du Galvanisme*, i. p. 202, pl. 6. figs. 2 and 5. As two such rheotomes joined in an alternate manner with each other, and intended to convert an alternating current into one in the same direction, have been called a commutator, I shall call three similarly arranged notched wheels, intended to disconnect simultaneously two equal induced currents traversing two totally unconnected wires, a disjunctor. When all the three notched wheels, which are moveable round their common axis by means of screw clamps, are fixed in the same position, the first $r\ s$ is connected by means of the mercury vessel i, which is transversely perforated with a cylindrical hole, with the galvanic battery, the second $\rho\ \sigma$ is connected with the induction spiral $\alpha\ \beta$, the third $\rho_1\ \sigma_1$ with the induction spiral $\gamma\ \delta$*.

We obtain, therefore, when the spirals are empty, two perfectly equal currents in distinct wires, their compensation having

* The disjunctor, consisting of three notched wheels, described above, can likewise be used for ascertaining what the effect is upon an induced current when it circulates in a closed wire for some time after its production. If the second and third wheels are somewhat altered in position, the disconnexion is not quite simultaneous, and we can then ascertain which of the disconnected wires exhibits the most powerful physiological or other action. For testing the intensity of the spark quicksilver is to be preferred; for other effects the disconnexion is better effected by pieces of interposed glass. This apparatus was constructed very carefully by M. Wagner.

been previously tested when these were connected The currents are also simultaneously interrupted, as the notches leave the mercury at the same moment If different rods of iron or bundles of wire are placed in the spirals, the sparks at each interruption, which before were alike, will be different In this manner it can be distinctly seen that a bundle of wire which had been previously completely compensated by a bar of soft iron as regards the galvanometer, produces after separation a much more vivid spark

27 In the same manner the heating effects can be measured by two electric thermometers by which each of the separated induction spirals is closed, and in like manner the chemical decomposition when connexion is made by two voltameters With respect to these however no measurements have been made, it has only been ascertained that the heating effects, as well as those of chemical decomposition of the empty spirals, are very much increased by placing iron rods and bundles of wires within them

4 *Experiments with iron tubes*

28 In some former experiments* I had shown that an electro magnetic spiral, which surrounded an iron tube of the dimensions of a gun barrel was not capable of magnetizing an iron cylinder placed within the tube and *vice versâ*, that a moveable magnet, or a fixed electro magnet placed within this tube, did not excite any phænomena of induction in the spiral which surrounded it† It follows therefore that, with relation to the phænomena which we are here examining, bundles of wire inclosed within gun barrels cannot increase the action of these, for they are as much protected by their iron case from the magnetizing action of the spiral which closes the circuit, as their inducing power itself is incapable of affecting the spiral of thin wire Sturgeon had previously made the observation, that wires in

* *Bullet de l Acad de St Petersb* viii ii p 20 and *Repert* i p 270

† Analogous phænomena may be observed with hollow iron tubes which surround straight steel magnets and which may be considered as keepers connecting the opposite poles over the whole periphery of the magnet A steel magnet fitting tightly into a hollow iron cylinder having the dimensions of a gun barrel or still thicker in metal exhibits no action on its external surface Placed in a spiral it scarcely induces any current suspended by silk both ends are attracted by both poles of a magnet held near it it does not rotate when exposed to the action of a rotating disc of copper It is therefore still more neutralized than a horseshoe magnet is by a straight keeper for this rotates slightly under these circumstances

closed within a tube of sheet iron did not increase its action If however the iron tube which separates the electro magnet from the induction spiral is of very thin metal, and has a considerable diameter, then the shocks are very perceptible both when the tube is entire or cut open lengthways* A solid electro-magnet also, one half of which is surrounded by an entire gun-barrel, the other half by a gun barrel cut open lengthways, does not destroy the equilibrium of two spirals which previously compensated each other with respect to the galvanometer, when one was enclosed by the entire and the other by the cut gun-barrel, whence it follows, that in this case the discontinuity of the tubes is not an indispensable condition But with bundles of wires the following phænomena are observed

29 When an entire iron tube, in its inducing action as regards the galvanometer, compensates the action of one that is cut lengthways, this compensation remains almost complete when any number of wires are placed into one or the other tube, *i e* with bundles of wires which are enclosed within entire and cut tubes, the inducing action as measured by the galvanometer is dependent entirely upon the enclosing iron It is however different as regards the physiological action In this case the action of the wires enclosed within the tube is nearly destroyed when the surrounding tube is *entire*, but not when the tube is *cut open*

30 The results thus obtained for hollow cylinders having the dimensions of gun barrels are somewhat modified when the tubes are made of sheet iron As regards the galvanometer, the wires exert an action through them, so that, when wires are placed within one of the cylinders, the galvanometric action of that cylinder is increased When the welded tubes were inserted one into the other, and the same was done with the cut tubes, and these latter were so placed towards each other that the sections should correspond, then the action of the wires placed within them was observed to be less than when they were enclosed in entire or cut tubes of simple sheet iron

* Upon one of the limbs of an electro magnet 28 inches in length, and surrounded with 65 coils of copper wire 2½ lines in thickness, was placed a coil of wire 4 inches 2 lines wide 500′ long and half a line thick, and the shocks of the current induced by this coil were tested when connection between the electro magnet and the galvanic current was broken The current remained almost quite as powerful when a cylinder of thin sheet iron 35 lines wide, first welded and then cut open lengthways, was interposed between the electro magnet and the induction spiral

With an increase of thickness therefore in the iron case, the action as measured by the galvanometer is lessened, and a mechanical division of the tubes by cutting has no very marked influence. But cutting the tubes even when the metal is thin produces an increase, although but small, in the physiological action. Lastly, the physiological action of the wires in a tube of sheet iron is proportionally small, but it is greater when the tubes are *cut open* than when they are *entire*. If for instance, *physiological* equilibrium has been established by any means between an *entire* and a *cut* tube, this will be destroyed when equally powerful bundles of wires are placed in both tubes, and the shock proceeds from the *cut* tube.

5. *Experiment with closed and unclosed conducting cases containing bundles of wire.*

31. Of two bundles of wires which compensated each other, one was placed without any case in the wooden tube of the magnetizing spiral, the other enclosed in a tube of cardboard, round which was wound in more than 200 coils of copper wire covered with silk, so that the coils surrounded the bundle of wire throughout its whole length. The projecting ends of this spiral, which, to distinguish it from the *magnetizing* spiral connected with the battery, and from the *induction spirals* surrounding the former, and connected by the human body or the galvanometer, we may call the *enclosing spiral*, could be connected by a clamp, or, to exhibit the secondary current induced in them, by a galvanometer. There were four such cardboard cases; the spiral in one of them was *right handed*, in the second *left handed*, in the third *half right handed, half left handed*; in the fourth it was made up of a wire folded and then twisted, and might therefore be considered as composed of two spirals wound in the same direction but unsymmetrically connected. The two last spirals were without action, both when their ends were or were not joined, not however the two first, whence it directly follows, that the effect produced by these must be ascribed to an electric current, which in the two latter was divided into two halves that mutually destroyed each other. Suppose the action of the bundle of iron wires replaced by that of an electro-dynamic solenoid, it is easy to perceive that its coils would run nearly parallel with the close coils of the enclosing spiral, whether the latter be wound in a like or in an

opposite direction. Judging from this it must be immaterial, for a certain given amount of polarity in the bundle of wires, what direction is given to the coils of the enclosing spiral; and this was borne out by the experiments. The results were the following:

32. If one of two bundles of wires, which compensate each other when both are unenclosed, is placed within a uniformly wound surrounding spiral with *connected* extremities, the galvanometric action of the latter differs from that of the unenclosed bundle, as does the action of a solid cylinder from that of a bundle of wires. For whilst *galvanometric* equilibrium is hardly disturbed, those characteristic *vibrations* of the needle occur which have already been mentioned, and the primary impulse always occurs in the direction dependent upon the unenclosed bundle of wires. The enclosing spiral, on the contrary, weakens in an extraordinary manner the physiological effect, so that a powerful shock is perceived arising from the unenclosed bundle of wires. An entire brass tube enclosing the bundle of wires presents analogous phænomena to those produced by a wire spiral with connected extremities; a brass tube cut longitudinally is however but little superior in power to a spiral with unconnected ends. The existence of the electric current which is on the point of being excited in such tubes can also be verified by the galvanometer when it is made to connect the cut edges of the tube.

33. The property of magnetizing hard steel is in relation with the physiological action of the current. Whilst seventy unenclosed wires, acting in opposition to the solid cylinder, magnetize the steel needle in the direction of the current produced by them, yet, when these wires are enclosed in an entire brass tube, the magnetic excitation takes place in the direction of the current produced by the solid cylinder. The longitudinally cut tube likewise diminished the magnetizing influence of the current upon steel, probably because, when it is filled with wires, which partially close the section, peripherical currents are produced, although of a more imperfect kind. When the copper spirals are exchanged for spirals of thin German silver wire, and the brass tube for one of German silver, the retarding action of the case is also diminished. When the bundles of wires are opposed to each other, the one in a *closed* and the other in an *open* tube, then in magnetizing steel the action of the open tube is more powerful than that of the closed tube.

6 *Experiments with piles of iron discs and with cylinders of iron filings*

34 As the division of an iron cylinder by longitudinal sections parallel to the axis hinders the formation of *peripherical electrical currents*, without interfering with the production of *magnetic polarity*, so, on the contrary the powerful development of magnetic polarity is prevented by cross sections at right angles to the axis, whilst the production of peripherical electrical currents is in no way retarded A column of iron discs arranged with discs of paper between them can therefore affect a galvanometer but slightly, on account of its small amount of magnetic polarity and from the facility with which electrical currents are excited in it, its physiological action must also be but slight, as the experiments have shown With a pile of iron filings, the excitation of electrical currents is at the same time hindered on account of the longitudinal separation, it will therefore exert a weak, though proportionally more powerful physiological action than the column of discs This is confirmed by the experiments in the former series (13)

35 The whole of the foregoing experiments lead to the result, that the metallic case surrounding the wire bundle (or, as is the case with a solid iron cylinder, the conducting metallic surface, combining all the single wires into one metallic whole) does not weaken the current induced by it, but only retards it, *i e* it spreads over a longer time the neutralization of the quantity of electricity set in motion by the evanescent magnetism in the enclosing wire, without decreasing the quantity itself This retardation has no influence upon the magnetic needle, which adds together the effects of the current, in which case it is quite immaterial how long this addition lasts The removal of the metallic case, or the frequent repetition of the interruption to metallic continuity, is to be compared to the accelerated motion produced by an inductor enclosing a magnet, which increases its physiological action without adding to the galvanometric effect

7 *Shock and sparks on breaking the circuit by means of spirals and electro magnets*

When the view deduced from the simultaneous considera

tion of the galvanometric and physiological effects, that the increase of the latter on breaking up a solid iron rod into a bundle of wires is to be ascribed to an acceleration of the current, not to an augmentation of the quantity of electricity set in motion, is to be applied to explain the phænomena of that department within which the physiological effects *only* can be submitted to an accurate investigation, and not the galvanometric effects, the application can only be warranted by a complete parallelism of the physiological phænomena in both departments. Now the physiological action of the extra current is already made known by the experiments of Sturgeon* and Magnus†, and is analogous to that of the secondary current which we have been examining, for the former has shown that the *shock on breaking* a galvanic circuit is stronger when, instead of a *solid iron cylinder*, a *bundle of iron wires* is introduced into the spiral forming the connecting wire; the latter, on the other hand, that the power of this shock is diminished when the bundle of wire is enclosed in an *entire conducting case*. I therefore only subjoin a few experiments, which show that it is not necessary for the metallic case to separate the bundle from the connecting wire, but that the same phænomena occur when this case surrounds externally the wire electro-magnet, and that spirals are just as effective as cases, whence it is rendered more obvious that the retarding cause is referable to an induced electric current.

36. Spirals of insulated copper wire were wound round bundles consisting of from twenty-five to fifty iron wires, and with the electro-magnet thus formed and others formed of solid iron, a galvanic circuit was closed by means of handles. On breaking the circuit a brilliant radiating spark appeared, and a considerable shock. The electro-magnets formed from the bundles of wires were now inserted in an entire brass tube. The shocks almost entirely disappeared, and the spark was very slight. The brass tube cut longitudinally, however, produced no change in the action of the electro-magnet; the spark retained its great brilliancy, and the shocks their former power.

The same results were obtained with the entire and cut gun-barrel when they surrounded the electro-magnetized bundle of wires, with this difference only, that in the entire gun-barrel a

* Annals of Electricity, i. p 481 † Pogg. *Ann* xlviii. p 95

very slight shock was perceived. The same takes place with tubes of sheet iron. Enclosing spirals of copper wire, which surround the spirals, have a similar influence in all these experiments to cut brass tubes when the ends of the spirals are unconnected, but they act on the contrary as entire tubes when their ends are connected.

Although the parallelism of the secondary current and the extra current cannot be traced further, yet it may be permitted to assume it for the galvanometric test, for as will be shown hereafter, complete correspondence can be proved to exist between both currents for the induced current of the Leyden jar.

II. *Currents induced by the evanescent magnetism of electro-magnetized rods of iron and bundles of wires, when the current magnetizing them is that of a thermo-battery or thermo-pile.*

37. If the poles of the thermo-battery described above (6) are united by a powerful electro-magnet with wire 2¼ lines in thickness, sparks are perceived on breaking the circuit, as when connexion is made by a flat spiral of sheet copper; at the same time the horseshoe attracts the keeper very decidedly. If the handles attached to the ends of the induction spirals which surround the electro-magnet are grasped in the hands previously moistened, a shock is perceived on breaking the circuit of pile. The shock of a bundle of wires disappears if it is enclosed in an entire brass tube. On the contrary, the *galvanometric* action is in both cases alike. The shock appears here likewise more intense when the break *quickly* follows the closing of the circuit.

38. The shock on breaking the circuit in a direct manner by means of a connecting wire of sheet copper forming a flat spiral, is perceptible when the discharge is effected by platinum spatulas through the tongue, and is very much increased by the insertion of bundles of wires. This last increase of power could not be perceived in the discharges of a battery with smaller elements, having the dimensions of one of Nobili's piles for measuring the conduction of heat.

The current induced by the connecting wire of a thermo-bat-

tery has the same properties therefore as that produced by the connecting wire of a galvanic battery.

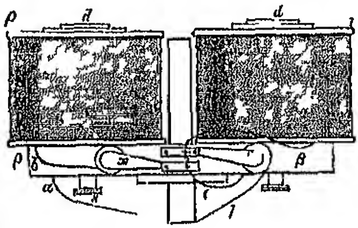
III. *Currents induced by the evanescent magnetism of electro-magnetized rods of iron and bundles of wires, when the current magnetizing them is produced—*

1. By the approach of an entire copper wire to a steel magnet.
2. By the approach of soft iron to a steel magnet.
3. By the combination of both methods of excitation in Saxton's machine.

39. Whilst a *magnet at rest* only exerts an influence upon what are called *magnetic metals*, exciting in them magnetic polarity by communication, the action of a magnet in motion is known to affect *all* the metals, producing in them electrical currents. Instead of moving a magnet mechanically, magnetism can also be produced and destroyed in a stationary bar of iron by the approach and removal of a steel magnet. If a powerful action is required, the wire in which the electric current is to be produced must surround with numerous coils the bar of iron which is to be magnetized. If the magnetizing is effected by the approach of a magnet to the iron rod, a mixed result is obtained produced by *two* excitations; for when the magnet approaches the iron, it approaches at the same time the coil of wire which surrounds it. The effect of this approach must on no account be overlooked, for I have obtained with a machine constructed on the principle of Saxton's machine, *the cylindrical coils of which however contained no iron,* such powerful shocks, when the hands were wetted and the keeper quickly rotated, that it was difficult, on account of the resulting cramp, to open my hands. But there is a means of neutralizing the effect produced by the approximation of the wire to the magnet. As a current in an opposite direction is produced in a spiral brought near the north pole of a magnet, to that which is produced in a similar spiral brought near to the south pole, it is only necessary to coil the wire surrounding the iron rod into another similar spiral, and to approximate this empty spiral in the same manner to the south pole as the one containing the iron nucleus is approximated to the north pole. The currents produced in the wire

by its approach to the magnet then destroy each other completely, and there remains only in the wire the effect produced by the magnetism of the iron nucleus. Instead of approaching two spirals coiled in a like direction to both poles of the magnet, two spirals, one right, the other left handed, can be connected crossways. Upon this principle I had the following apparatus constructed.

10. The annexed drawing represents the rotating keeper of a machine, constructed in other respects upon the principle of Saxton's by M. Oertling. It consists of a wooden disc upon a rotating axis, upon which are firmly fixed two hollow coils, the wire of each being 100 feet



in length, and one third of a line in thickness. Into these empty wire coils *ϱ ϱ* and *r r*, massive iron cylinders *d*, or bundles of wire *d′* 13‴ 6 in diameter, and 22‴ 5 in height can be placed, and by a screw *s* firmly fastened to the transverse wooden disc of the keeper. To be able to connect both wire coils in the same or alternate direction the ends of both coils must not be immediately connected with the interrupting arrangement of the keeper, but must remain free. A connexion of these ends by wire clamps is not however convenient, for, if they are not very tightly fastened, and the keeper rotates quickly, they are liable to be forcibly thrown off. I have therefore connected a contrivance to the keeper, which, as it is intended for compensating, may be called a *compensator*, and which, by means of two moveable arms *x x*, admits of both coils being connected in the same or in the alternate direction, and also of only one coil being active. In the first case, the arms rest in the figure upon + +, in the second upon — —, and in the third upon + —. When the arms rest in the position + + the connexion is then *p β a b a n*, when in the position — — then it is *p a β b a n*, and lastly, when they rest upon + — it is *p b a n*, in which case it is immaterial whether connexion takes place at the upper or lower plate. + — and — + are two small copper plates placed one above the other, *β* is clamped below the upper one, *α* below the lower one. The point of rotation of the arm *x′* leads by means of *p*

to the iron roller upon which the interrupting springs slide*; the point of rotation of the arm x leads by means of the whole coil of wire b a through n to the other.

When the compensator has the position + +, and the spirals are *empty*, a current is obtained induced by the approach of a closed conductor to a steel magnet. In the position of the compensator – – with empty spirals, an equilibrium of current is established for physical, chemical and physiological tests. If, however, one of the coils then contains a solid iron cylinder, a current is obtained, induced by the *sole* action of the magnetic polarity produced in the iron cylinder. The axis of the keeper must of course turn perfectly true upon a conical point, in concave conical hollows, because the masses now set in motion by the rotation are no longer symmetrically placed with relation to the axis of rotation. If, on the contrary, the compensator having the position + +, an iron cylinder is placed in *each* of the coils, then a current is obtained induced by the assumption of magnetic polarity by these cylinders, and by the approach of a closed copper wire to a steel magnet, producing therefore the most powerful action. As this arrangement is the same as that usually adopted in the construction of Saxton's machine, I have made use in these experiments of that instrument, as it is depicted (at fig. 7) and described in the sequel, § 70, in which the cross beam is also of iron, and the wire can be so connected that both coils are joined at their two ends in a kind of parallel connexion.

The current excited in this *triple* manner in the wire coils of the keeper was now circulated in the inner spirals of the third differential inductor, of which the inner and outer spirals were composed of wire 400′ in length. This apparatus was so sensitive when both inductions were combined, that a tube of thin sheet nickel produced a distinctly positive action, and the negatively disturbed equilibrium of the differential inductor could be traced by means of a bar of brass placed in one of the spirals, connexion being established by the mouth. The experiments gave the following results.

41. An *unenclosed* bundle of wire and one placed in a cu[t] tube very nearly compensated each other physiologically. If

* The more accurate description of these rollers, depicted in Plate I fig. 7 n_1 n_2, will be given afterwards, § 70

however, the unenclosed bundle acts in a contrary direction to an enclosed bundle powerful shocks are obtained In the *galvanometer* a *solid iron cylinder* more than overpowered 140 thin iron wires, whilst thirty six were sufficient to retain it in physiological equilibrium The *physiological series* for the following substances was ascertained by drawing out the rods which were opposed to each other The following is the series, beginning with the substance that excited the most powerful action —

Unenclosed iron bundle of wire
The same bundle in a longitudinally cut tube
Cut tube of tinned sheet iron
Entire tube of tinned sheet iron
Cut gun barrel
Entire gun barrel
Soft iron cylinder
Cylinder of white and gray pig iron
Soft steel
Hard steel
Tube of nickel and square bar of nickel
Bundle of iron wire in an entire tube of brass

This series as also the whole of the phænomena observed, are analogous to those which were obtained when the iron was magnetized by means of a galvanic battery

12 The two other modes of excitation of the current, on the one hand by means of an empty wire keeper, and coils of wire connected in the same direction, and on the other hand by means of compensating coils, in one of which was placed a bar of iron, gave analogous results, namely, a shock when an unenclosed bundle of wire was opposed to an enclosed bundle which compensated it galvanometrically

Uniform results are therefore obtained when the primary current is—

1 That of a galvanic battery
2 That of a thermo battery
3 That of a Saxton's machine
4 That excited by the approach of a closed conductor to a magnet
5 That excited by magnetizing iron in an enclosing wire by means of a steel magnet

The phænomena essentially differ from these when the primary current is that produced by the discharge of a Leyden jar.

IV. *Currents induced by iron which was magnetized by the discharge of an electrical battery*

43. If a battery*, to which a constant charge has been communicated by means of a unit jar, is discharged through the inner spirals of the differential inductor for frictional electricity (described § 8, fig. 3), the shock of the induced current in the same direction with the primary current is obtained from the secondary spirals, which are connected together in the same direction. This has been shown, independently of each other, both by Henry † and by Riess ‡.

This shock is modified when metallic substances are placed in the previously empty tubes. Whether the change which then occurs is due to an increase or decrease of power it is difficult to determine, when the modification is but slight, and other methods of testing it are requisite in order to arrive at a safe conclusion. When the secondary spirals are connected crossways, an equilibrium of the currents is established for all the methods of testing which are here applicable, and this is immediately destroyed by the insertion of a metal into one of the compensating spirals. But the current which then appears does not deflect the needle of the galvanometer; for, when the revolutions of the wire which are wound upon glass are insulated in the most careful manner from each other, by varnishing the wire already covered with silk, sparks will constantly pass from one coil to the other; the current tested with iodide of potassium exhibits no chemical decomposing power, nor does it *magnetize soft iron* in such a manner, that a magnetic needle placed by the side of it is deflected, or that iron filings stand on end when sprinkled over it. For ascertaining the direction of the current, therefore, there remains no other means than the process proposed by M. Riess, by which resinous figures are obtained§, or that by means of the

* Different batteries were employed in the experiments, some consisting of smaller, some of larger jars. the number of the jars was also changed, but not in the same series of experiments The results obtained from these different batteries were all in unison

† Transactions of the American Philosophical Society, vol. vi. p 17

‡ Poggendorff's *Annalen*, l p l § *Ibid* li p 353.

condenser*, and also a physiological test, to which I was led in the course of the experiments. The following are the results.

1. *Physiological and electroscopical effects of the induced current.*

44. The physiological action of the current induced in the secondary wire by the connecting wire of the battery is diminished by all unmagnetic metals, and so much the more the better the metal conducts. This decrease of power is therefore much less with antimony, bismuth and lead than with copper and brass. With previously compensated spirals the shock obtained is therefore so much the more powerful the better the metal conducts which is placed in one of them. The current tested by the condenser and by the resinous figures proceeds from the empty spiral; the resulting shock is therefore caused by the weakening influence of the metal upon the spiral in which it is inserted.

45. If, instead of a solid metallic cylinder or a metallic tube, a cardboard tube surrounded with a spiral wire made of copper and covered with silk is inserted into one of the connecting spirals, the equilibrium of the current remains unimpaired in the secondary spirals when their ends are not joined, but it is destroyed when their ends are connected. A spiral formed of a once doubled wire, which may be considered to consist of two like spirals united in an opposing direction, does not destroy, when the ends are *joined*, the equilibrium of the current in the secondary spirals: the effect of the first wire must therefore be attributed to an electrical current excited in it, the inactivity of the second to the mutual neutralization of the destructive influence by two equal electrical currents.

46. Such electrical currents must also exist in solid cylinders and entire tubes, for the effect of the former is diminished by a longitudinal division, *i. e.* by the conversion of the brass cylinder into a bundle of well insulated brass wires: the effect of the latter is also weakened by a longitudinal section. Bundles of brass wires exert a less obstructive action than an entire tube of

* These and a few others of the following experiments I instituted in common with M. Riess, who permitted me to make use of his apparatus for that purpose. The modes of proceeding proposed by [illegible] and Joule by means of a perforated card, and the passing of a spark with the wire ends projecting over each other, were made known at a later period.

much less mass of metal, the tube and the bundle being alike in external circumference. A simple method of testing whether a metallic bar placed within one of the tubes destroys the physiological equilibrium of the current in the secondary spirals by weakening the action of its spiral, is that of placing brass wires into the other empty tube, a certain number of which must eventually restore the equilibrium which had been disturbed

47. Forged iron, soft and hard steel, white and gray pig iron in the form of *solid cylinders* and *prismatic rods*, and also in the form of *entire tubes*, as *gun-barrels* and *welded tubes of sheet iron*, all weaken the *physiological* action of the induced current. The same is the case with piles of *discs* of steel, of forged iron and of tinned sheet iron, both when they are arranged with insulating and conducting discs between them. The current produced by forged iron and steel, tested by the condenser and the resinous figures, proceeded from the *empty* spiral. The weakening influence of forged iron, steel and pig iron, is however different; for with two opposing cylinders of different kinds of iron in the compensated spirals, vibratory motions are always observed on an insulated preparation of the frog.

48. The physiological action, on the contrary, is increased by longitudinally *cut gun-barrels*, and particularly by well *insulated bundles of iron wire*. A shock from the similarly connected secondary spirals that was perceptible in the joints of the hand, extended to the middle of the upper arm upon the insertion of two such bundles of wire, but it was so weakened by the insertion of two cylinders of wrought iron, that it could only be felt in the extremities of the hand. The current tested by the condenser and the resinous figures, with compensated spirals, proceeded from *that* spiral in which the bundle of wire was placed. Here then there is a marked distinction between the inducing action of iron depending upon the manner in which it is magnetized, whether by the current of a galvanic battery, or by that produced by the discharge of a Leyden jar. The inducing action of the spiral-shaped connecting wire of a galvanic battery upon a secondary wire is increased when iron in any shape is inserted into it, whilst the connecting wire of a Leyden jar has a less powerful inducing action upon a secondary wire when a solid iron rod is inserted in it, than when it is empty; and

on the contrary, the inducing action is greater when this iron is used in the form of a bundle of wire

49 A *bundle of insulated iron wire*, however, which is surrounded by an *entire tube of brass*, has the same action as a *solid iron cylinder*, *i e* it weakens the shock of its spiral, and a current is produced proceeding from the *empty* spiral The same is the case when it is surrounded by a *spiral of copper wire* coiled throughout in the same direction, and connected at the ends It also shows a weakening action, though this is but very slight, when this spiral is a bad conductor, composed of German silver for instance, and it is not impossible that, with a greater number of wires in its interior, and a thinner spiral wire, the action might be produced in a contrary direction A spiral formed of a *doubled* copper wire with connected ends, is likewise here without effect, for a bundle of wire enclosed within such a spiral retains in equilibrium an *unenclosed* bundle of wire in the other tube

50 A solid rod of nickel produces hardly a perceptible physiological action with compensated spirals The current produced by it, however, tested by the condenser and the resinous figures, proceeds from the spiral in which it is placed Solid nickel therefore, *increases* the inducing action, whilst solid iron *decreases it* The previously existing polarity of the nickel has likewise no effect upon it, for the direction of the current remains the same when an opposite position is given to the bar of nickel in relation to its spiral With varnished nickel wires we may therefore expect a still more marked increase of power

51 All the facts here established are independent of the relative position of the connecting spiral, the secondary spiral, and of the cylinder to each other, for they were obtained in the same manner when the battery was discharged through the *outer* spirals, and the induction tested upon the *inner* spirals

52 To ascertain whether a rod placed in one of the tubes *increases* the physiological action iron wires may be placed in the other tube until equilibrium is again established, whilst in the case of the inserted rod decreasing the action, the disturbed equilibrium must be restored by the insertion of wires of a non magnetic metal, such as brass *Thin* wires must be chosen for such testing experiments, for, as a single wire may be considered as a

cylinder, which, from what has been stated before, and particularly when it has a certain thickness, has a weakening effect, there will be, for wires of a definite thickness, a certain number which is quite inactive. Such an inactive combination of wires of the thickest kind of wire was actually very nearly obtained for a certain battery charge. This number must therefore be exceeded when wires are chosen for testing the increasing action of another substance, and the number must be ascertained by a preliminary experiment.

2. *Steel magnetized by the induced current.*

To avoid anomalies, *thick* needles were chosen, and the *length of the wire* remained *unchanged*, a *constant charge* was always communicated to the battery by means of a unit jar.

53. If the equilibrium of the current is destroyed, with compensating spirals, by the insertion of a conducting substance into one of the spirals, the polarity of a steel needle magnetized by the current in excess shows, that the current proceeds from the empty spiral when the inserted substance* is a foil of iridium, platinum, gold, silver, or a rod of copper, brass, tin, zinc, lead, or an alloy of 1 copper and 1 bismuth, of 3 copper and 1 bismuth, of 3 copper and 1 antimony, of 1 zinc and 1 bismuth, of copper, tin, lead, zinc and antimony, of lead and iron, of brass and iron, of bell-metal; lastly, strips of copper and antimony melted together crossways, of bell-metal and antimony, of antimony and bismuth. The equilibrium of the current remained undisturbed when this rod was composed of antimony or of bismuth, or of an alloy of 1 bismuth and 1 antimony, or of 3 bismuth and 1 antimony. On the contrary, the polarity was in the direction of the current proceeding from the filled spiral when it contained an *unenclosed bundle of wire*, or one *enclosed* in an *entire tube*, or a column of steel, iron, or tinned iron discs, a solid cylinder of forged iron, of soft or hard steel, of white or gray pig iron; and lastly, a rod or tube of nickel. A division of the mass of iron into wires increases in an extraordinary manner the magnetizing effect; for bundles of wire opposing cylinders

* The results which were obtained when that metal, which, in the form of a rod, was non magnetic, was inserted into the magnetizing spirals in the form of an insulated bundle of wire, will be given afterwards at § 62.

of forged iron, steel and pig iron in the other spiral, retain their more powerful action when the mass opposed to them is many times their own mass; fourteen insulated wires $0'''.70$ in diameter compensate exactly the cylinder of forged iron. If however the more powerful bundles of wire are enclosed in entire brass tubes, the same solid cylinders then overpower them in their magnetizing action.

In relation to the *magnetizing of steel needles* therefore, the phænomena are quite analogous, whether the magnetizing is effected by galvanic or by frictional electricity, and that difference which was observed in the physiological effects is no longer here perceptible, *i. e.* iron in whatever form it is used, or in whatever manner it may have been magnetized, increases the magnetizing action upon steel excited by the current induced in the secondary wire by the connecting wire; whilst, when the iron is magnetized by the discharge of a battery, it only increases the *physiological* action of the spiral when it is divided into wires, or is in the form of a longitudinally cut tube; on the contrary, it effects the same under any form when it is magnetized by the influence of a galvanic current.

3. *Calorific action of the induced current.*

The *heating influence* of the induced current is independent of its direction. It was therefore measured with a single pair of spirals, which was employed empty, and into which the substances to be tested could be inserted. An elevation of temperature points therefore directly to an increase of power in the current, a diminution thereof to a decrease. An electric air thermometer and a Breguet's metallic thermometer were employed for measuring the temperature.

51. When magnetism is produced by *frictional electricity*, the measured calorific effect of the current induced in the secondary wire by the connecting wire is *weakened* both by bundles of iron wire, by iron bars and by nickel; their action is therefore the same as that of unmagnetic metals, in which the same has been demonstrated by M. Riess. If however the primary magnetizing current is that of a galvanic battery, masses of iron and bundles of iron wires increase the calorific action of the induced current.

4. *Induction exerted by the connecting wire of a Leyden jar upon itself.*

55. This, to my knowledge, has never yet been experimentally proved. It may however easily be done in the following manner. —If *m n* (Plate I. fig. 5) represents the connecting wire of a Leyden jar, and *a b* the spiral portion of it, *c h h d* a secondary connection which, by means of the handles *h h*, is effected through the body, a shock is perceived at the moment a spark passes at *n*; this however is not the case when the secondary connexion is made as represented in fig. 6, even when the distance from *h* to *h* is precisely the same in both cases. In the former case the spiral portion of the connecting wire is closed by the body connecting *h* with *h*; in the latter it is not so. If the shock were produced by a division of the current, it must inevitably occur in both cases. As this is not the case, it must be the effect of a true induction. The power of the shock is increased very perceptibly by a bundle of wires. A cylinder of nickel 4 inches long and 1½ inch thick was now inserted, without enabling me to ascertain in which direction the change was effected, as the power of the shock rendered it comparatively small. The insertion of a solid iron cylinder, however, materially weakens the shock, as does also the insertion of a non magnetic metal. A closed secondary spiral surrounding the spiral portion of the connecting wire considerably decreases the power of the induction shock of the connecting wire, but very little however when it is composed of two pieces unsymmetrically joined. With the calorific test a decrease is observed at the secondary connexion on the insertion of iron in any form, but an increase in the power of magnetizing a steel needle. The induction of this extra current is therefore identical with that of the secondary current in separate wires.

5. *Results of the experiments with electro magnetized iron.*

56. If the results which have been obtained by magnetizing iron with electricity from different sources be collected into one general view, we find that—

a. Iron in the form of solid rods, of entire or longitudinally cut tubes, of insulated bundles of wires with or without con

ducting cases or in the form of piles of discs; moreover, forged iron, soft and hard steel, white and gray pig iron and nickel, when electro-magnetized by the current of a *galvanic battery*, a *thermo-battery*, a *Saxton's machine*, by the *approach of a closed wire to a magnet*, and lastly, by the *action of a piece of iron approaching a steel magnet upon a closed wire which surrounds it*, produces electric currents in a wire which surrounds it when this magnetism becomes evanescent.

b. The inducing action of the same mass of iron as a continuous whole is in general very different from the action of the same mass of iron when it is divided into wires: this difference however varies in kind, according to the mode by which the iron is electro-magnetized.

c. When the iron is magnetized by the connecting wire of a *galvanic battery*, a *thermo-battery*, or by a *magneto-electrical* current, in either of the three modifications distinguished above, the *galvanometric* action of the current produced by the evanescent magnetism on breaking the circuit, remains the same when the iron is broken up into bundles of wires, as does also the property of this current to magnetize soft iron: whilst on the other hand, its *physiological* action, the *sparks* which it gives rise to on being interrupted, and the magnetism produced by it in steel, are much more powerful. If the bundle of wires is surrounded by a conducting case, as an *entire* tube or a single coiled spiral with *connected* ends, its action is that of a solid bar of iron. If, on the contrary, the case is *not entire*, i. e. consists of a *longitudinally* cut tube or of a *simply coiled* spiral with *unconnected* ends, it acts nearly as powerfully as an unenclosed bundle. A spiral composed of a *doubled* coiled wire surrounding the bundle of wires, and having its ends *connected*, is as inactive as a *single coiled wire* with *unconnected* ends. If the mass of iron is divided by sections at right angles to its length into *discs*, the current induced by this pile of discs is very much weaker in its physiological action.

d. The differences which have just been noticed between iron rods and bundles of iron wires attain their maximum when they are magnetized by the discharge shock of a Leyden jar. A spiral wire with a nucleus of iron, for instance, induces a more powerful current in a secondary spiral surrounding it as regards the physiological, magnetizing, galvanometric, heating and chemical ac-

tions, than an empty spiral wire without an iron nucleus, when the galvanic current which magnetizes this iron ceases. The increase of physiological action on breaking up this iron nucleus into wires, and the greater degree of vividness of the sparks of the induced current, as well as the increasing intensity of the magnetism in a steel needle polarized by the current, are therefore due to an augmentation of the action *already exerted* by the solid iron. The inducing action of the empty spiral which is traversed by the momentary current produced by the discharge of a Leyden jar is *greater*, as regards the physiological and electroscopic actions of the secondary current, than when a solid iron nucleus is contained in it; it is *less powerful* however than that which is produced by the insertion of a bundle of iron wire, a longitudinally cut iron tube, or a solid rod of nickel. If the bundle of wire is surrounded with an entire case, the bundle which previously excited an augmenting action now acts as a solid rod, *i. e.* has a weakening effect. The heating effect of the secondary current is on the contrary diminished both by solid iron and bundles of wire, indeed by iron in every shape, as well as by unmagnetic metals: the capability of magnetizing steel is increased by iron and nickel in every form, but it is diminished by solid rods of unmagnetic metals.

e. If the connecting wire of the galvanic circuit or of the Leyden jar exerts an inducing action, not upon the secondary wire, but upon its own parallel coils, this extra current exhibits to all the tests that could be applied the same relations as the secondary current.

f. The influence of conducting cases is caused by an electrical current induced in them by the connecting wire, which can be shown to exist in them, by connecting the edges of the longitudinally cut cases by means of a galvanometer, or some other kind of rheoscope. The same is true of the ends of enclosing spiral wires, which, simply coiled, exhibit a current when their ends are connected by the galvanometer, but on the contrary, show no current when they are composed of a doubled wire, and then closed by the galvanometer. Tubes and enclosing spirals weaken the physiological action of the bundles of wire contained within them, so much the more the better the substance conducts of which they are composed. With solid iron rods the *surface* acts as a conducting case enclosing an insulated bundle

of wire. Hence it is explained why nickel in the form of a solid rod, magnetized by the discharge of a Leyden jar, has a more powerful inducing action than iron. It has the same action as a bundle of wire in a badly conducting case; iron acts as a bundle of wire in a case composed of a good conductor.

g. The current induced by an unenclosed bundle of wire attains sooner its maximum intensity than that induced by a solid iron rod, or by a bundle of wire contained in an entire case, when the quantity of electricity set in motion by both is the same; for, with two currents which compensate each other in the galvanometer, the needle assumes an oscillatory motion, first moving in the direction of the current from the bundle of wire, then in favour of that induced by the solid iron. For the same reason, equilibrium having been established in the galvanometer, the former current is more powerful than the latter in its physiological action, in the property which it possesses of magnetizing steel, and in the production of more vivid sparks.

h. Cast iron exerts a more energetic physiological action than could have been anticipated from its action on the galvanometer. It is therefore more allied in its inducing action to an insulated bundle of wire than to malleable iron.

i. All kinds of iron produce more powerful induction currents on repeated electro-magnetizing when they are magnetized alternately in opposite directions, than when this is always effected in the same direction. They all retain a portion of the magnetism excited in them, and therefore undergo a more powerful magnetic change when alternately magnetized in opposite directions, than when the same direction is always preserved.

6. *Some remarks relating to the theory of Ampere.*

57. Starting from the axiom that electric currents flowing in the same direction attract each other, whilst they repel each other when flowing in opposite directions, Ampere has shown that every magnetic action can be traced to the action of closed electrical currents. Ampere has gone a step further, and has proclaimed the *identity of electro-magnetic* and *magnetic* phænomena, and assumed, consequently, that an electric current circulates round every molecule of iron, which currents are in variable directions in unmagnetic iron, and assume a parallel direction under the influence of a magnet or of an electrical cur-

rent. This assumption has gained probability by the discovery of magneto-electricity, for every magnetic action of an electrical current, when it is produced by some other means than an electric current, gives rise to an electric current in an opposite direction to that current which would itself have produced it. The more numerous, however, the points of coincidence in both departments are, the more necessary it is to point out the phænomena which appear to be incompatible with their identity.

In the first place, as regards the occurrence of magnetic polarity by the influence of an electric current, it results always under such conditions as *never* give rise to electric currents. For an electric current excites in a conductor placed by its side another quickly subsiding electric current only when it *begins* and when it *ceases*, not however during its *continuance*. It produces magnetism, on the contrary, during the whole time of its continuance, in a piece of iron placed by the side of it, which attains its maximum in an appreciable space of time. The peripherical electric currents, supposed hypothetically by Ampère to surround the molecules of iron in order to explain this magnetism, differ therefore, on the supposition that they are now for the first time produced, from all known electric currents, inasmuch as they are produced during the continuance of an electric current, *i. e.* they occur under conditions where no other electric currents could be excited. This difficulty is avoided by the theory, on the ground, that *existing* currents circulating round the molecules of iron are only *directed* and not *produced* by the external current. But then the phænomenon that an electro-magnet returns to its unmagnetic state, when the primary magnetizing current ceases, is without analogy in the other departments. A polarized ray of light remains polarized when it is removed from the active agency of the reflecting or refracting substance; the oscillatory directions of the particles of æther which have become parallel remain parallel when they have once become so. For what reason then do the elementary currents which have become parallel cease to be parallel when the current which brought them into this parallel direction ceases to flow? for they themselves can have no tendency to digress from their parallel direction. The cause of this phænomenon, be it what it may, must nevertheless be, according to the

hypothesis of an electric nature. Wherefore then is it not connected with the power of conduction of the metals?

58. The inferences to be drawn from the experiments in this memoir, when viewed without any preconceived hypothetical notions, are that when iron is electro-magnetized, two phenomena result which are opposed to each other, namely, the excitation of electric currents, and the production of magnetic polarity. In the researches which have hitherto been made in this field of inquiry, the effect of the magnetic polarization always overpowered the obstructing influence of the electric currents produced at the same time: we obtained therefore, by preventing more or less the formation of these latter, merely an augmentation of the effect already produced by the magnetic polarization. The experiments instituted with the aid of frictional electricity showed, under the same circumstances, a complete reversion of this action into that of an opposite nature. This reversion, however, does not take place simultaneously for the physiological action of the induced currents, for their magnetizing properties and their calorific effects; so that the same experimental arrangement which renders more powerful one of these effects exerts at the same time a weakening influence upon another. Consequently all explanations which were advanced to explain *one* of these actions in its different modifications *alone* are set aside. Now as it does not appear advisable to call by the same name and consider identical two forces of nature, the one of which begins to act under conditions in which the other never occurs at all, and which, when they are both simultaneously active in the same body, so oppose each other, that sometimes one, sometimes the other predominates, I think it more appropriate to consider the magnetic polarization as not only an independent but as an opposing agency to the electric currents excited in iron.

The explanation of the phenomena which have here been observed would then be the following:—

59. The primary active electric current traversing a spirally coiled wire which surrounds the iron produces at the moment in which it commences electric currents in the iron; during its continuance it causes magnetic polarity, which is more tardily augmented than that current; and in the moment that it ceases an electric current is again produced. The second electric

current produced on the ceasing of the primary current, and having the same direction with it, acts in a contrary direction to that produced by the magnetism. If the magnetism has had time to develope itself during the longer continuance of the current, as is the case with galvanic magnetization, its action overpowers the opposing action of the electric current produced on the cessation of the primary current. All means therefore that are used to hinder the formation of electric currents, increase the action already excited by the solid iron. If however the primary current is of an exceedingly transient nature, as that caused by the discharge of an electric battery, and the magnetism has consequently not time to develope itself completely, then the electric current produced on the cessation of the primary current overpowers the action of the evanescent magnetism*. The dissolution of these electrical currents by breaking up the mass into wires, or the obstruction to their formation by a badly conducting mass, as nickel, completely reverses this action, for the excess which before this separation was in favour of the electric currents, is now brought in the first instance in favour of the evanescent magnetism. But the limit of equilibrium for both is not the same for the calorific, the physiological and the magnetizing actions, because the dependence of each of these upon the intensity of the evanescent magnetism will be different from their respective change by the opposing electric current; for the magnetizing action, the power of the evanescent magnetism will still remain predominant, when for the calorific effects the electric current is the more powerful, and the physiological phænomena fall on both sides of this line.

Ampère first considered a magnet as an iron rod which was peripherically surrounded by electric currents. As, however, according to Coulomb's view, we can only account for the distribution of magnetism in an iron bar by supposing it made up of linear magnetic elementary lamellæ arranged side by side, Ampère substituted for his first assumption an electro-dynamic solenoid, the most nearly approaching realization of which is an electro-magnetized bundle of wires. But to resolve the inducing

* At § 77 the same result is obtained by other means with magneto electric induction, namely, weakening the physiological action of a current by the insertion of massive iron, and increasing the same by bundles of iron wire.

action of a solid electro magnet into that of a bundle of wires, the latter must be surrounded by a conducting case A solid electro magnet would then be iron, in which besides the electric currents running parallel to each other round the individual particles, the whole is moreover surrounded by peripherical currents The electro magnetization of iron would then be an arrangement of already existing electric currents, and besides this, a production of new currents, and moreover of a different kind, as the action of the latter interferes with that of the former If however we are forced to distinguish the electric currents which can be shown in iron from those which are hypothetical, it would appear to be simpler to go a step further and proclaim *electricity* and *magnetism* to be two *distinct* forces of nature The question now arises, what phænomena of induction are presented by a bar of iron in which magnetism is evanescent, without the simultaneous excitation of electric currents in the iron, and what phænomena are presented by non magnetic metals in which the peripherical electric currents are destroyed by breaking them up into wires? The answer to these questions is the subject of the two following sections

V *Currents induced by the approach of solid iron and bundles of iron wire to a steel magnet*

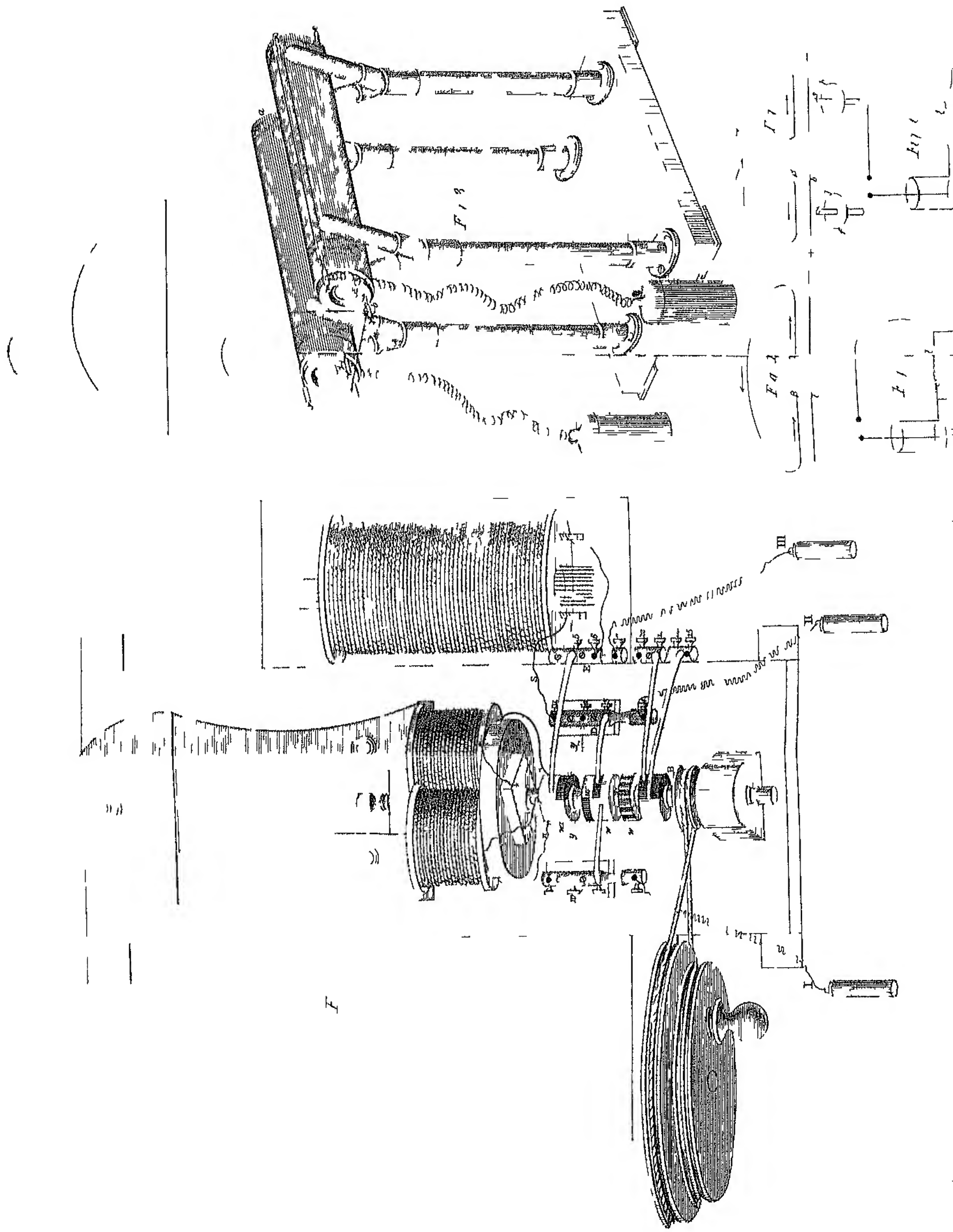
60 If, in the apparatus described at § 40, and constructed upon the principle of Saxton s machine, the compensator having the position — —, an equilibrium of current has been established, with empty spirals, for physical, chemical and physiological tests, then a disturbance of this equilibrium of current by the insertion of different substances into the cylinders, will show that the inserted substances have a different action and from the direction of the resulting current it can be ascertained which is the most powerful For this purpose solid iron cylinders and bundles of wires were employed The solid iron cylinders had a diameter of 13''' 6, and a height of 22''' 5 The bundles of wires were of the same dimensions, except as regards length the brass plate at the end of the surrounding case must be deducted, and from the diameter the thickness of the surrounding case of paper, wood or brass As the separation of the sliding spring must always be effected in the same manner in relation

to the middle of the cylinders, the iron wires must lie symmetrically in relation to the axis of the wire coil, they must therefore once for all be fixed. This is effected by wooden frames and brass holders, of each of which there were one cut open longitudinally, the other entire.

There were nine such pieces filled with wires, from 14 to 310 in number, the latter with a paper covering, the wires of which were held together by lac. All the iron wires were varnished to ensure more perfect insulation.

61. The final result of a very extensive series of experiments instituted with this apparatus was, that in relation to physiological action, heating an electrical thermometer, deflection of the galvanometer needle, magnetization of soft iron, chemical decomposition and production of sparks, the solid cylinders overpower the bundles of iron wires. If a bundle of wires opposes in the one coil a solid cylinder in the other, an addition to the number of wires constantly decreases the intensity of the shocks. The *experimentum crucis* in this department is this: two similar bundles of iron wires, the one in an entire, the other in a longitudinally cut tube, retain each other in complete physiological equilibrium when connexion is made by the handles with dry hands. The very slight action which is perceptible with wet hands arises from the current directly induced in the wire coils by approaching them to the steel magnet, exciting a secondary current in the enclosing case; it is therefore perceptible when no bundles of iron wires are inserted into the compensating wire coils, and is not to be compared to the powerful differences which are obtained with electro-magnetized bundles of wire which are unenclosed or inserted in cases. The currents induced by direct *magnetization* of the iron differ therefore from those excited by *electro-magnetization* of the iron by a want of those characteristic properties, which in these latter can be explained by the simultaneous excitation of electric currents in the iron.

[To be continued.]



SCIENTIFIC MEMOIRS.

VOL. V.—PART XVIII.

ARTICLE IV. continued.

Researches on the Electricity of Induction. By H. W. DOVE.

VI. *Magnetism of the so called unmagnetic metals.*

62. WHEN natural bodies are classified in relation to any physical agency, we soon find that the idea of antithesis by which the substances may be distinguished in that respect from each other, which at first presents itself, must be abandoned, for the action, which in certain bodies is very energetic, and in others appears to be entirely wanting, gradually diminishes throughout the series, so that the transition from one to the other is imperceptible. Thus between luminous and dark bodies phosphorescent substances intervene, between conductors of electricity and insulators, imperfect conductors, diathermanous substances pass gradually into athermanous, and conductors of heat into non-conductors. But the transition of the magnetic metals to the non-magnetic is so distinctly marked, that whilst all philosophers are agreed respecting the magnetic properties of the former, the possibility of magnetizing the latter has been as often maintained as it has been denied.

The process which, since the time of Brugmans, has always been adopted to prove the magnetism of other substances than iron and nickel, is, by endeavouring to direct and to move readily mobile substances by means of powerful magnets, or, *vice versâ*, to direct and move easily mobile magnets by those substances. The double magnetism of Haüy, and the frequent use of astatic double needles since the invention of Le Baillif's sideroscope, belong to the second method, whilst the first has merely been modified by the different experimentalists

according to the manner in which the substances were made moveable, namely, either by swimming upon water or quicksilver by means of pieces of cork, or by suspension on threads possessing very little torsion.

The method which I have pursued is however different. I have tested the relative magnetizability of the different metals by the electric currents induced by them in a spirally coiled conducting wire surrounding them, when the magnetism excited in them became evanescent. How far the results obtained in this manner agree with the observations of former natural philosophers, will be best seen after a short notice of their results has here been given.

63. According to Brugmans*, lead, tin, antimony, gold and silver possess no magnetic power; on the contrary, copper floating on water or mercury is slightly attracted, zinc more powerfully, as is also bismuth that has a white shining silver colour, whilst bismuth having a dark, nearly violet colour is repelled by both poles of the magnet. Cobalt exhibits a very weak attraction, and arsenic none at all; on the contrary, poles and a point of neutrality can be produced in brass. Lehmann† endeavoured at great length to prove that the magnetism of brass was attributable to iron mixed with it; whilst, on the contrary, Cavallo‡ came to a contrary conclusion as the result of his own experiments. Brugmans considers attraction by the magnet as a proof of the presence of associated iron.

Coulomb§ caused needles of gold, silver, lead, copper and tin, 7 millimetres long, and weighing 40 milligrammes, to oscillate between the opposite poles of a powerful magnet, and found the time requisite for four oscillations to be respectively 22″, 20″, 18″, 22″, 19″, whilst, when removed from the influence of the magnet, each required 44″ to complete four oscillations. On the repetition of Coulomb's experiments in the Royal Institution, Thomas Young obtained less marked results than Coulomb. Coulomb himself showed, by artificial combinations of iron filings with wax, how little iron was requisite to produce similar indications. Biot‖ considers the alternative, that these phæno-

* *Magnetismus, seu de affinitatibus magneticis observationes Academicæ*, 1778, 4

† *De cupro et orichalco magnetico*, Nov. Com Petr. xii. p. 368.

‡ Treatise on Magnetism, 1787, p 283.

§ *Journal de Physique*, liv. pp. 367, 454, 1802.

‖ *Précis Elémentaire de Physique*, sec. ed. ii. p. 78.

mena are either the effect of a real magnetism in the metals, or due to associated iron, as unnecessary, as they may be the results of another force. Lamé* expresses himself in the same manner with relation to the experiments of Coulomb, Becquerel and Lebaillif. Lebaillif† observed attraction with his sideroscope, when platinum, iron, nickel and cobalt were used; repulsion, on the contrary, with bismuth and antimony. Saigey‡ maintains, as the result of an extended series of experiments, that repulsion is the common property of all bodies suspended in the air, but that attraction is always due to the presence of iron. Ampère and De la Rive studied the action of a powerful magnet upon a disc of copper suspended so that it could move freely within a copper wire through which an electrical current circulated.

This electro-magnetized copper was affected by the poles of a powerful magnet in an analogous manner to electro-magnetized iron, according to a statement with which however I am imperfectly acquainted. Becquerel§, on the contrary, found no complete parallelism between the phænomena of a copper and iron needle, when both were suspended in the coils of a multiplier. His experiments agree with those of Muncke‖, who found that brass containing iron disposed itself in a more or less transverse position between the similar poles of two magnets. Seebeck¶ has proved the same property to exist in other substances besides those containing nickel. In these experiments the following metals exhibited transverse magnetic polarization:—

(1.) Copper wires from one half to four lines in thickness.

(2.) Platinum in the form of rods, foil, and as spongy platinum.

(3.) A cast rod of speiscobalt containing arsenic and nickel.

(4.) A strip of gold with 1 per cent. silver, copper and iron, and one purified with antimony.

(5.) Regulus of arsenic containing iron.

(6.) Alloys** consisting of 3 copper and 1 antimony, and of 1 copper and 1 antimony.

(7.) Alloys of 5 copper and 1 bismuth, 1 copper and 1 bismuth, and 1 copper and 3 bismuth.

* *Cours de Physique* ii. 110. † *Bulletin Univers.* I. viii. p. 87.
‡ *Ibid.* ix. p. 97. § *Annales de Chimie et de Physique*, xxv. p. 269.
‖ Poggendorff's *Annalen*, vi. p. 301.
¶ *Abhandlungen der Berliner Akademie*, 1827, p. 117.
** These alloys are the same as those mentioned at § 53.

On the contrary, the following exhibited no action:—

Mercury, bismuth, antimony, sulphuret of antimony, lead, tin, zinc, cadmium, pure silver, pure regulus of arsenic, an alloy of 4 antimony with 1 iron, and an alloy of copper and nickel.

An attraction between copper and the astatic needles has often been observed in the constructing of multipliers. Thus several years ago Professor Nervander of Helsingfors found during his sojourn in Berlin, amongst a great number of kinds of copper which he tested, only one rod of Japan copper belonging to me which did not attract the delicate needle of his multiplier. Again, Faraday* has found that cobalt and chromium, which have always been considered magnetic, are not magnetic when perfectly free from iron. As a high temperature weakens so materially the magnetic intensity of iron and nickel, it is possible that at a low temperature metals may be magnetic which are not so at common temperatures. But the following metals, tested by a very delicate double needle at temperatures of 60° to 70° F., were found to be unmagnetic:—

Arsenic, antimony, bismuth, cadmium, cobalt, chromium, copper, gold, lead, mercury, palladium, silver, tin and zinc.

Nevertheless, Pouillet maintains †, in the last edition of his work upon physics,—

(1.) That cobalt remains constantly magnetic, even at the most intense red heat;

(2.) That chromium loses its magnetism somewhat under a dark red heat;

(3.) That manganese is magnetic at a temperature of 20° to 25° C.

Lastly, M. Poggendorff‡ has recently made use of the phænomenon of deviation in a twofold direction, first discovered by him with Saxton's machine, for the purpose of showing the magnetizability of the metals, which up to this time have not been considered possessed of magnetic properties. But nickel, iron and steel are the only ones which gave positive results; even German silver was not magnetic.

64. In the fourth section (53) we have seen that the magnetic polarity excited in iron by the discharge of an electric battery

* London and Edinb. Phil. Mag. viii. p. 177.
† *Elemens de Physique*, 3rd edit. i. p. 381.
‡ Poggendorff's *Annalen*, xlv. p. 371.

on becoming evanescent produces an electric current in a secondary wire, which can always be proved by its magnetizing a steel needle. The polarity of this steel needle always remains the same when a magnetizable metal is inserted into one of the previously compensated spirals of the differential inductor, but it is weaker when the magnetizable metal is in the form of a solid rod or a pile of discs than when it is a bundle of insulated wires. The polarity of this needle is on the contrary reversed when the inserted metal is unmagnetic. In this case it is in favour of the current produced by the empty spiral.

In the electroscopic and physiological phenomena of the current induced by electro-magnetized iron and nickel, the remarkable fact was established, that the less powerfully magnetic nickel has an augmenting action whilst the more powerfully magnetic iron diminishes the effect, because the retarding electric currents cannot be so readily formed in the badly conducting nickel as in the better conducting iron; *i.e.* in relation to the electroscopic and physiological tests, solid iron acts as a non-magnetic metal, whilst it acts as a magnetic metal as regards the magnetization of the steel needle. Now we may readily infer that the so-called *non-magnetic* metals have the same action in relation to that property of the current which magnetizes steel, that iron has in relation to the electroscopic and physiological properties, *i.e.* that they *appear unmagnetic*, because the electric currents excited simultaneously with the magnetism obscure the action of the magnetic polarity, but that they really are *not unmagnetic*. It is therefore only necessary, in order that the latter action should predominate, to prevent the formation of electric currents, *i.e.* to break them up also into wires, and then test the direction of the induced current by magnetizing a steel needle. If the current proceeds from the spiral containing the bundle of wires, the metal is magnetic; if it proceeds on the contrary from the empty spiral, it is non-magnetic.

65. For preliminary experiments brass was chosen. In the form of a cylinder it diminished the intensity of the current from its spiral, for the resulting current proceeded from the empty spiral; when the brass wires which were inserted had a certain thickness, the equilibrium of the current was preserved; when thin, well varnished brass wires were used, the current on the

contrary proceeded from the filled spiral. In this form therefore the previously non-magnetic brass became magnetic.

These experiments were now extended to antimony, lead, bismuth, tin, zinc and mercury. The insulation of the mercury was effected by enclosing it in glass tubes sealed at both ends; the wires of the other metals were covered with shell-lac. The copper was free from iron, according to the analysis of M. Henry Rose. The lead contained a very slight trace of iron; tin, antimony and bismuth however more; the zinc, examined by Dr. Marchand, was chemically pure. I shall repeat these experiments with some metals which I have since obtained in a perfectly pure state.

The thickness of the wires was as follows:—Copper 0'''·75, tin 1'''·10, lead 0'''·80, zinc 0'''·60, brass 0'''·75, antimony 2'''·80, bismuth 2'''·80; the mercury was contained in common thermometer tubes. In the experiments the same kind of sewing-needles (darning needles) were always employed, and the electric battery always received the same charge by means of a unit jar. If the compensation of the empty spirals was not complete, it could always be effected by slightly altering the position of the interior spiral towards the outer, or it was established previously by the insertion of brass wires. The experiments showed a very appreciable amount of magnetism with copper, quite as much with tin, mercury, antimony and bismuth, less with zinc, and very little with lead. A tube of brass diminishes the action of its spiral,—acts therefore in an unmagnetic manner. A tube of German silver, as also drawn tubes of tin and of lead, had a powerful magnetic action, even more powerful than bundles of tin and lead wires. It is therefore probable that in drawing these soft metals into tubes they become covered with a thin film of iron.

The positive result obtained with mercury is for this reason of importance, that no possible admixture of iron could result from drawing. In a former paragraph (53) alloys of iron have been mentioned, which in the form of rods, tested in the same manner, show themselves unmagnetic; the admixture of iron cannot therefore, as such, determine the result. The magnetism of these metals however compared with that of iron is so very weak, that a single iron wire of the same thickness is capable of overpowering in its magnetizing action a whole bundle of wires

of the other metal It would however be premature to arrange the metals in a series at present

VII *Influence of the presence of iron on induced currents of higher orders*

A current induced by the connecting wire of a galvanic or an electric battery can, as Henry* has lately shown, be again used as a primary current, and thus induce a second current, this again a third, and so on Henry employed for the examination of these currents flat spiral coils, all his experiments are therefore only electro dynamical The question, whether the principle of increasing the power by means of bundles of wires was applicable to these currents, appeared to me worthy of investigation for two reasons, the one a practical one, because, if it should be the case, the experimenter would be enabled to examine these currents without such a mass of copper wire and copper ribbon as Henry made use of in his experiments, the other a theoretical one, it being important to know whether the same differences obtain, between the induced currents of higher orders, as those which have been observed in induced currents of the first order, when the primary current was either that of a galvanic or of an electric battery

1 *Currents of higher orders when the first induced current is electro dynamically induced*

66 The first induced current, which causes the production of currents of higher orders, may either be electro dynamically induced, or it may be a magneto electric current, and both methods of excitation admit again of several modifications It is well known that electro dynamic induction may take place in two ways, either by the *approach* of a closed wire to, or its *removal* from a continuous current (as, for instance the connecting wire of a galvanic or thermo battery), or secondly, when in one of two parallel wires which remain equally distant from each other an electric current is excited or ceases For the first mode of induction the following apparatus may be made use of —Suppose two circular currents intersecting each other in the manner that two large circles would intersect a ball, they will then tend to cause each other to revolve in one plane, according to the law

* Transactions of the American Philosophical Society vol vi p 1

of Ampère, that two currents cutting each other are mutually attracted when both flow from the angle formed by their intersection or flow towards it, but, on the contrary, they repel each other when the one flows towards, the other from that angle. Now, if one of these circles is a fixed wire-ring, in the coils of which the current of a galvanic battery is circulating, the other a closed wire-ring of somewhat larger diameter capable of revolving round the first, it will easily be perceived, that for every whole revolution of this ring round the first, two alternating currents of equal intensity will be induced. This ring, placed upon the axis of a Saxton's machine, forms a corresponding apparatus of induction, which is however of an entirely electro-dynamic nature*. In the absence of such an apparatus, I have employed the second mode of electro-dynamic induction only, in which an incipient current, or one just on the point of ceasing, acts upon a secondary wire at rest.

One of Grove's platino-zinc elements was closed by a spiral A of thick copper wire. A second spiral B of thin copper wire 400′ long surrounded the first spiral, and was itself connected with a third spiral C 400′ long, and of the same thickness of wire. This third spiral C was inserted and insulated in a fourth spiral D 400′ long, which could be closed by handles, or some other method of testing the current. The galvanic current circulating in the connecting wire A induced in the first instance a secondary current in B, which, traversing C, produced a current of the third order in D. The changes were now examined which took place in the current of the third order, when solid iron cylinders, or bundles of iron wires enclosed in entire or cut tubes, were inserted into the spiral C. The arrangement was made in the same manner when the primary current was that produced by the discharge of an electric battery. A flat copper spiral A imbedded in resin, $11\frac{1}{3}$ inches in diameter, and consisting of 31 coils of copper wire $53\frac{1}{2}$ feet long and $\frac{2}{7}$ line in thickness, placed upon an insulating glass foot, formed a part of the connecting circuit of the battery. Opposite to this, and only separated from it by a plate of glass or of mica, was a second flat spiral, the coils of which were exactly parallel to those of

* Instead of the apparatus here described, that proposed by Henry may be employed, in which the empty keeper of the Saxton's machine, instead of rotating before a magnet, rotates in front of two equal coils of wire, through which a galvanic current is circulating.

the first, and which, by means of a sliding board, could be brought to any requisite distance from it. This spiral $B_{/}$ was connected with the two inner cylindrical spirals *a b* and *c d* of the differential inductor for frictional electricity (fig. 3), which on the discharge of the battery induced in the outer spirals *α β* and *γ δ*, which were connected in a uniform manner, a current of the third order, the physiological action of which could be tested when *a b* and *c d* were empty, or when they contained iron. Lastly, that case was examined in which the primary current was that of a Saxton's machine. The wire coils of this instrument (fig. 7) were connected with a spiral $A_{//}$ 400′ long, which excited an inducing action upon a coil of wire $B_{//}$ 100′ long, into which it was inserted. This first outer spiral $B_{//}$ was connected with an inner one $C_{//}$ 400′ long, and corresponding completely with $A_{//}$, which excited an inducing action upon a second outer spiral $D_{//}$ which entirely corresponded to $B_{//}$. The current of this spiral $D_{//}$ was tested when $C_{//}$ contained iron, or when it was empty.

The result of these experiments was, that these second induced currents, or as Henry calls them, *the currents of the third order, behave as the currents of the second order, which give rise to them*, i. e. the currents whose primary current was excited by *machine electricity* were *weakened* by solid iron, and on the contrary, their power was *increased* by *bundles of iron wires*, whilst those induced by galvanic agency were increased in power by both, but by *bundles of wires* more than by *iron rods*. The same applies to the currents of the third order from Saxton's machine.

2. *Currents of higher orders when the first induced current is a magneto-electric current.*

67. If iron is already present in the connecting spiral A of the galvanic battery or of the Leyden jar, then the first induced current is not alone produced by electro-dynamic agency, but is chiefly excited by the evanescent magnetism of the electro-magnetized iron. If the magneto-electric portion of the current is to be employed alone as the primary current for the currents of higher orders, then the galvanic battery or the electric battery must be closed by a differential inductor, only one of the spirals of which must contain iron. The current then induced by it exerts an inducing action upon the adjacent spiral. If the

electro-dynamic and magneto-electric induction are combined, then currents of higher orders can be examined as if this combination had not been effected. For machine-electricity the following arrangement was made:—The battery was discharged by the inner spiral $a\ b$ of the differential inductor (fig. 3). The spiral enclosing this, $\alpha\ \beta$, was connected with the second inner spiral $c\ d$, and the spiral surrounding this, $\gamma\ \delta$, was connected with the flat spiral $A_{\prime\prime}$ described before (66), whilst on the spiral $B_{\prime\prime}$ parallel with this last, the shocks were tested, when solid iron rods or bundles of wires were inserted in $a\ b$ and $c\ d$. We thus obtain—

in $a\ b$ the primary current,
in $\alpha\beta$ and $c\ d$ the current of the second order,
in $\gamma\ \delta$ and $A_{\prime}$ the current of the third order,
in $B_{\prime}$ the current of the fourth order.

The currents of the fifth order, produced by smaller flat spirals $C_{\prime}\ D_{\prime\prime}$, could not be proved to exist physiologically. This however succeeded easily with Saxton's machine, or when the primary current was that of a galvanic battery, for although the inner spirals for higher orders had often only two lengths of wire, one over the other, yet they nevertheless acted powerfully upon fresh preparations of the frog, and were distinctly perceptible with the moistened hands. As each higher order requires two new spirals, it was often necessary in these experiments to employ spirals which had originally been intended for other purposes, and were often coiled in an unfavourable manner for the object here in view, the distance between the inner and outer spiral being often very considerable. The augmenting action of inserted bundles of iron wires was here exerted with the greatest energy, for by their means currents became very perceptible, when in the case of electro-dynamic induction no trace of any action was discernible. The weakening influence of enclosing tubes or closed surrounding spirals is therefore here exceedingly prominent. The needle of the galvanometer oscillated at last with the higher orders of galvanic induction and with the Saxton's machine, as if driven by the shortest possible impulse, and was not affected at all by the currents of the fifth order, which excited distinct physiological action. Probably these induced currents of the higher orders approach more and more to the momentary discharges of frictional electricity. Success however did not attend the endeavours to prove this empirically, *i. e.* to obtain a decrease of the

physiological action of a spiral by the insertion of solid iron although in cases where higher orders can be examined, it might perhaps be attained with a greater number of spirals coiled for this particular purpose As it is certain that the currents induced by electro magnetized bundles of wires, from the properties which they have been shown to possess generally, fill up by a number of intermediate grades the wide gap between continuous galvanic currents and the momentary currents of frictional electricity, so in all probability the secondary currents of higher orders will be the means of supplying the remaining omissions between those two extreme members of the series

The researches which have been detailed in this memoir have tended to point out the influence which is exerted by the presence of solid iron and insulated bundles of iron wires upon induced currents excited by primary currents from different sources, both when they were developed as adjacent currents in separate wires, or existed in the form of the so called extra currents in the coils themselves of the connecting wire, on disconnecting it with the source of electricity It now remains to be examined what influence this iron exerts on the initial counter current which a commencing primary current produces in its own coils As however nothing whatever is known of the physiological action of this current, the first requisite was to invent means to produce it in such a manner as would admit of rheometric measurements being applied to it The following sections contain the account of the researches undertaken for this purpose

VIII *Extra current at the commencement and close of a primary current, and its modifications by the presence of iron*

As an electric current, the intensity of which is increasing may be considered at any moment as consisting of two portions, the one of which the constant part remains unchanged, the other is that which is constantly being added, and again in a current the intensity of which is decreasing, the portion which is leaving it may be distinguished from the constant portion, then the law of induction, that a primary current induces at its commencement a current flowing in an opposite direction, and at its cessation, one in a like direction, that it induces no current at

all however during its continuance, may be expressed more generally in the following terms: a primary current induces, as long as its intensity is on the increase, a secondary current in an opposite direction; as long as it is decreasing, it induces one in a like direction. If the term *secondary current* be applied to the current induced by a primary current in a wire parallel to, but not connected with it, and *extra current* to the secondary current produced in a spiral connecting wire with or without an iron core, by the action of each separate coil upon that which lies next to it, if therefore this extra current is considered as a particular kind of secondary current, in which the same wire is the medium for the passage of the primary and the induced current, then the phænomena which have been discovered in the secondary current may with great probability be supposed to exist in relation to the extra current. But the spark produced on breaking the circuit of a galvanic battery is more intense when the circuit has been closed by a long spirally coiled wire, than when the same has been effected by a short straight wire, and powerful physiological effects appear, particularly when this spiral wire surrounds a piece of iron, which are not perceptible with short, straight connecting wires. Faraday, who bases upon these phænomena the existence of the extra current, conceives therefore (§ 1104) that corresponding effects will always ensue by means of a spiral and an electro-magnet when the electromotor is *closed*. These effects must cause in the first moment a resistance, therefore something that is opposed to the shocks and sparks. It is difficult to invent means for proving the existence of such *negative* effects. Faraday therefore endeavours to prove them by positive effects which are simultaneously produced in a secondary connexion. Now, as in more recent experiments in this department the real experimental difficulty is not done away with, namely, the prevention of the extra current being excited on breaking connexion, and as moreover no diminution of power at the end of the extra current corresponding to the increased intensity of the sparks and of the physiological action has been proved for the extra current supposed at the commencement, the following researches may be viewed as a filling-up of this gap, inasmuch as, by their aid, that which was required has been effected in so perspicuous a manner, that these experiments may

be directly admitted into the domain of common experimental demonstrations.

68. The primary current was produced by a Saxton's machine, represented at fig. 7, and constructed by M. Oertling, in which the interruption is effected by means of brass springs, which slide upon two iron cylinders inlaid with pieces of wood. The first of these cylinders, w_1, is fixed in an insulated manner upon the axis A B of the keeper, and holds the end of the wire n composing the coil of the keeper; the second, w, is directly connected with this axis and hence in conducting connexion with the other end of the coil p. The inlaid portions, composed of wood, of the cylinder w_1 w_2 and w_4, occupy half the circumference of this cylinder; that which is seen at a in the middle of the cylinder w however occupies only one sixth of its circumference, and diametrically opposite it is another corresponding to it for the production of alternating currents of equal intensity. One of the springs 1) or 5) slides continuously upon the first cylinder, as does 9) upon the second, the third 3) either in the same manner uninterruptedly, or it passes once* at an azimuth of 90° (*i. e.* in a rectangular position of the keeper, perpendicular to the line connecting the poles of the magnet) or twice at an azimuth of 90° and 270° over the insulating surface of inlaid wood. In the first case (which only occurs with galvanometric tests and chemical decompositions), the wire of the coil of the keeper which is constantly in metallic connexion is traversed by alternating currents, which pass into each other at the azimuth 0° and 180°, and on account of the symmetrical distribution of the whole, reach their maximum about the azimuth 90° and 270°.†

If the intermittent spring is interrupted once at 90°, then the secondary connexion which alone establishes the continuity in the handles I and II, either by means of the body or some other means of testing the current, receives the full intensity of the positive current; if it takes place twice during one whole revolution of the keeper, it receives two opposed currents in alternating succession; and if a voltameter be interposed, a mixture of oxygen and hydrogen at both electrodes. This alternation can be

* It is then fixed in a somewhat slanting position, so that it touches the edge of the inlaid wood nearest to the keeper.

† On rapid revolution somewhat beyond that, when the sparks as well as the shocks are most intense.

suspended by the use of two springs $y\ y$ in the shape of a Y, which with their two arms compass both cylinders at the same time, the one touching wood whilst the other touches metal, and thus upon the principle of the commutator transform alternating currents into currents of a like direction. The points of contact of the one spring are situated diametrically opposite to those of the other, the one y passing from the higher support 10), slides upon the lower surface of both cylinders, the other y passing from 2) slides upon the upper surface. This arrangement, applied for the purposes of chemical decomposition, eliminates the gases separately, and moreover in double the quantity they are produced by the usual arrangement, in which the opposing current is not reversed, but is suspended by interrupting the connexion.

The different combinations of the springs are accordingly the following:—In common experiments without the insertion of a spiral for the production of the extra current, 9) and 3) slide upon the cylinder w_2, as is depicted in fig. 7; upon the cylinder w_1, however, instead of the spring proceeding from 5), one that proceeds from the clamp 1), and moreover 1) and 9) continuously, 3) on the contrary intermittently. Alternating currents are however obtained when that which has hitherto been a secondary connexion becomes a chief connexion; currents in a like direction, when it is inclined obliquely, and slides on the once interrupted edge. The galvanometer, the apparatus for producing incandescence in platinum and charcoal, as also the human body, are inserted between 4) and 8). For uninterrupted currents in the same direction, $y\ y$ alone are used. The arrangement with an inserted spiral for alternating currents is represented at fig. 7. When the sparks of the secondary current are not to be examined, the springs 13) and 14) are left out. With currents of a like direction, the springs $y\ y$ are inserted alone in the clamps, whilst the apparatus for measuring the currents is inserted between I and III instead of between I and II. If the current is to be interrupted often during one revolution of the keeper, the spring 3) is made to slide upon the cylinder w_3.

The weight of the covered wire is 1220 grammes, the thickness of the uncovered wire is about $\frac{1}{3}'''$, its length 880′. The height of the cylindrical rolls of wire is $1\frac{3}{4}$ inch, their diameter $14'''$, that of the outer coil $2\frac{1}{2}''$. The front iron plate of the

keeper is 5″ long, 2″ broad, and ½″ in thickness Each of the four cylinders *w* has a diameter of 16‴, the magnet, consisting of four lamellæ, is 10″ long, the height of the four pieces together is 22″ The internal distance between the poles is 1″, the external 4¾″ The rotating wheel is at the side, and revolves obliquely to prevent the abrasion of the crossed cord, it can be drawn out from the base of the machine, by which means the requisite amount of tension can be given to the cord At each turn of the wheel the keeper revolves 8½ times The support extending from 8 to 11 on the left side is 5″ high, the supports on the right hand are only 2″ high, by which means the side view of the apparatus is better seen The distance of the rotating keeper from the magnet is regulated by the screws between which the axis turns The two wire coils surrounding the limbs of the keeper can be connected in a twofold manner, either so that the one forms a continuation of the other, or that both are connected at their two extremities, so as to form a so called parallel connexion 440′ in length The changes in the intensity of the resulting current which are produced when the wire is coiled in a particular manner, have lately been shown by M Lenz* For if L represents the resistance to conduction of one of the coils, Λ the resistance to conduction of the apparatus inserted for measuring the current, then with a parallel connexion there are two ways presented to the current induced in the wire coil at its exit, namely, the apparatus for measuring the current and the other wire coil, between which it divides itself in an inverse ratio to their resistance to conduction If A therefore represent the electromotive force of a coil of wire, then with a parallel connexion a current of the intensity $\frac{2\,A}{2\,\Lambda + L}$ will circulate through the measuring apparatus, if on the contrary, the connexion is continuous, a current will pass of the intensity $\frac{2\,A}{\Lambda + 2\,L}$ If therefore the apparatus for measuring the current offers as great a resistance to conduction as one of the electromotive coils of wire, *i e* if $\Lambda = L$, then the parallel connexion is quite as advantageous as the continuous, and there is no occasion in this case for any arrangement to effect both connexions As however the same machine has to be used with different kinds of apparatus for measuring the current, and it is not convenient to

* *Bulletin Scientifique de l Acad mie de St Pétersbourg* ix p 78

According to these transformations and notations, the system of the new equations, in which neither a_1 nor b_1 will any longer be found, will be,

$$\left.\begin{array}{l}\left.\begin{array}{l}(a_2-b_2x^{-2.3})(1-x^3)(1-x^2)(1-x)+(a_3-b_3x^{-3.3})(1-x^4)(1-x^3)(1-x^2)\ldots\\ \qquad+(a_i-b_ix^{-i.3})(1-x^{i+1})(1-x^i)(1-x^{i-1})\end{array}\right\}=2(2)_1\sqrt{-1},\\ \left.\begin{array}{l}(a_2x^2-b_2x^{-2.4})(1-x^3)(1-x^2)(1-x)+(a_3x^3-b_3x^{-3.4})(1-x^4)(1-x^3)(1-x^2)\ldots\\ \qquad+(a_ix^i-b_ix^{-i.4})(1-x^{i+1})(1-x^i)(1-x^{i-1})\end{array}\right\}=2(2)_2\sqrt{-1},\\ \left.\begin{array}{l}(a_2x^{2.2}-b_2x^{-2.5})(1-x^3)(1-x^2)(1-x)+(a_3x^{3.2}-b_3x^{-3.5})(1-x^4)(1-x^3)(1-x^2)\ldots\\ \qquad+(a_ix^{i.2}-b_ix^{-i.5})(1-x^{i+1})(1-x^i)(1-x^{i-1})\end{array}\right\}=2(2)_3\sqrt{-1},\\ \left.\begin{array}{l}(a_2x^{2.3}-b_2x^{-2.6})(1-x^3)(1-x^2)(1-x)+(a_3x^{3.3}-b_3x^{-3.6})(1-x^4)(1-x^3)(1-x^2)\ldots\\ \qquad+(a_ix^{i.3}-b_ix^{-i.6})(1-x^{i+1})(1-x^i)(1-x^{i-1})\end{array}\right\}=2(2)_4\sqrt{-1},\end{array}\right\}\quad \ldots \quad (13)$$

. . . .

The law by which these equations are deduced from the equations (10) is one of the simplest. Thus the factor

$$[a_i x^{i.\lambda}-b_i x^{-i(\lambda+3)}]$$

is the sum of the first and last term of the analogous factors existing in two of the equations (10) two rows distant from each other. Before and after the factor $(1-x^i)$ which already existed in the equations (10.), two are found in which the exponent of x is respectively superior or inferior by a unit to the index i.

7. We pass from the equations (13) to the equations deprived of the coefficients a_2 and b_2 by analogous considerations; that is to say, by forming the following combination,

$$M_1+M_3-M_2(x^2+x^{-2}),$$

of the corresponding members M_1, M_2 and M_3 of three of the equations (13) taken consecutively. Let us remark, that in virtue of the first of the equations (6.),

$$x^2+x^{-2}=2\cos 2\alpha,$$

and put

$$\left.\begin{aligned}(3)_1&=(2)_1+(2)_3-(2)_2\,2\cos 2\alpha\\(3)_2&=(2)_2+(2)_4-(2)_3\,2\cos 2\alpha,\\(3)_3&=(2)_3+(2)_5-(2)_4\,2\cos 2\alpha,\end{aligned}\right\}\qquad(14)$$

The equations sought will be the following, which will be deduced from the equations (13) just as those were deduced from the equations (10)

$$\left.\begin{aligned}(a_3\quad -b_3x^{-3.5})(1-x^5)\quad(1-x)\quad+(a_1\quad -b\,x^{-})(1-x^{+})\quad(1-x^{-})&=2(3)_1\sqrt{-1}\\(a_3x^3-b_3x^{-3.6})(1-x^5)\quad(1-x)\quad+(a_1x^{1}-b\,x^{-6})(1-x^{+})\quad(1-x^{-2})&=2(3)\sqrt{-1}\\(a_3x^3-b_3x^{-3.7})(1-x^5)\quad(1-x)\quad+(a\,x^{-}-b\,x^{-7})(1-x^{+})\quad(1-x^{-})&=2(3)_3\sqrt{-1}\end{aligned}\right\}\qquad(15)$$

and so forth we get rid of a_3 and b_3, a_4 and b_4 and arrive at two equations containing now only a and b these quantities will be thus determined

8 We may henceforth remark that the important condition which we laid down at the commencement is satisfied. For to arrive at the equations which only contain a and b_2, we have merely to form the sequence of the quantitie

$$\left.\begin{array}{llll}(1)_1 & (1)_2 & (1)_3 & (1)_i\\(2)_1 & (2)_2 & (2)_3 & (2)_i\\(3)_1 & (3) & (3)_3 & (3)_{i-}\\ & & & \\(i)_1 & (i)_2 & & \end{array}\right\}\qquad(16)$$

which are deduced from one another by the relations (9) (12) (14) and by analogous relations Now if we find that the values of a and b are not small enough for us to stop at these it would suffice to calculate in order to have regard to the superior value of the index i two new numerical values of the disturbing function namely R_{i+1} and R_{i+2}. By means of these values we should add to the right of the table (16) two new numbers in each horizontal line, and thus arrive at $(i+1)_1$ and $(i+1)_2$

which would make known a_{i+1} and b_{i-1}, without more calculations than if we had regard from the beginning to the index $(i+1)$.

9. We have just proved that we should easily arrive at the value of the coefficients affected by the greatest index. To seek the formulæ by means of which we shall attain to the coefficients affected by the preceding indices, it is necessary to write the three last systems of equations at which we ought to arrive.

The first of these systems, still containing the coefficients affected by the indices $i-2$, $i-1$ and i, will be

$$\left.\begin{array}{l}
\left.\begin{array}{l}
(a_{i-2} - b_{i-2}\,x^{-(i-2)(2i-5)})\,(1-x^{2i-5})\,.\,(1-x) \\
+(a_{i-1} - b_{i-1}\,x^{-(i-1)(2i-5)})\,(1-x^{2i-4})\,..\,(1-x^2) \\
+(a_i - b_i\,x^{-i(2i-5)})\,(1-x^{2i-3})\,..\,(1-x^3)
\end{array}\right\}=2(i-2)_1\sqrt{-1}, \\
\left.\begin{array}{l}
(a_{i-2}\,x^{i-2} - b_{i-2}\,x^{-(i-2)(2i-4)})\,(1-x^{2i-5})\,...\,(1-x) \\
+(a_{i-1}\,x^{i-1} - b_{i-1}\,x^{-(i-1)(2i-4)})\,(1-x^{2-4})\,(1-x^2) \\
+(a_i\,x^{i} - b_i\,x^{-i(2i-4)})\,(1-x^{2i-3})\,..\,(1-x^3)
\end{array}\right\}=2(i-2)_2\sqrt{-1}, \\
\left.\begin{array}{l}
(a_{i-2}\,x^{(i-2)2} - b_{i-2}\,x^{-(i-2)(2i-3)})\,(1-x^{2i-5})\,..\,(1-x) \\
+(a_{i-1}\,x^{(i-2)} - b_{i-1}\,x^{-(i-1)(2i-3)})\,(1-x^{2i-4})\,..\,(1-x^2) \\
+(a_i\,x^{i\,2} - b_i\,x^{-i(2i-3)})\,(1-x^{2i-3})\,.\,(1-x^3)
\end{array}\right\}=2(i-2)_3\sqrt{-1}, \\
\left.\begin{array}{l}
(a_{i-2}\,x^{(i-2)3} - b_{i-2}\,x^{-(i-2)(2i-2)})\,(1-x^{2i-5})\,(1-x) \\
+(a_{i-1}\,x^{(i-1)2} - b_{i-1}\,x^{-(i-1)(2i-2)})\,(1-x^{2i-4})\,.\,(1-x^2) \\
+(a_i\,x^{i\,3} - b_i\,x^{-i(2i-2)})\,(1-x^{2i-3})\,.\,(1-x^3)
\end{array}\right\}=2(i-2)_4\sqrt{-1}, \\
\left.\begin{array}{l}
(a_{i-2}\,x^{(i-2)4} - b_{i-2}\,x^{-(i-2)(2i-1)})\,(1-x^{2i-5})\,...\,(1-x) \\
+(a_{i-1}\,x^{(i-1)4} - b_{i-1}\,x^{-(i-1)(2i-1)})\,(1-x^{2i-4})\,.\,(1-x^2) \\
+(a_i\,x^{i\,4} - b_i\,x^{-i(2i-1)})\,(1-x^{2i-3})\,.\,(1-x^3)
\end{array}\right\}=2(i-2)_5\sqrt{-1}, \\
\left.\begin{array}{l}
a_{i-2}\,x^{(i-2)5} - b_{i-2}\,x^{-(i-2)(2i-0)})\,(1-x^{2i-5})\,..\,(1-x) \\
+(a_{i-1}\,x^{(i-1)5} - b_{i-1}\,x^{-(i-1)(2i-0)})\,(1-x^{2i-4})\,...\,(1-x^2) \\
+(a_i\,x^{i\,5} - b_i\,x^{-i(2i)})\,(1-x^{2i-3})\,(1-x^3)
\end{array}\right\}=2(i-2)_6\sqrt{-1}
\end{array}\right\}\quad \cdots\cdots \quad (17.)$$

The coefficient $(i-2)_k$ of the second member is furnished by the relation

$$(i-2)_k=(i-3)_k+(i-3)_{k+}\ -(i-3)_{k+1}\ 2\cos(i-3)\alpha \tag{18}$$

The second of these systems containing now only a_{-1} and b_{-1} a and b_i, will be

$$\left.\begin{array}{l}
\left.\begin{array}{llll}(a_{-1} & -b_{-1}x^{-\ -1)(\ -3)}) & (1-x^{-\ -3}) & (1-x) \\ & +(a\quad -b\,x^{-(\ -3)}) & (1-x^{-\ }) & (1-x^{-})\end{array}\right\}=2(i-1)_1\sqrt{-1} \\
\left.\begin{array}{llll}(a_{i-1}x^{i-} & -b_{-1}x^{-(-1)(\ -)}) & (1-x^{-\ -3}) & (1-x) \\ & +(a\,x^{i}\quad -b\,x^{-(\ -)}) & (1-x^{-\ }) & (1-x^{-})\end{array}\right\}=2(i-1)\ \sqrt{-1} \\
\left.\begin{array}{llll}(a_{i-1}x^{(-1)} & -b_{-1}x^{-\ -1)(\ -1)}) & (1-x^{-\ -3}) & (1-x) \\ & +(a\,x\quad -b\,x^{-(\ -1)}) & (1-x^{-\ }) & (1-x^{-})\end{array}\right\}=2(i-1)_3\sqrt{-1} \\
\left.\begin{array}{llll}(a_{-1}x^{(-1)3} & -b_{-}\ x^{-\ -1)}\ ) & (1-x^{-\ -3}) & (1-x) \\ & +(a\,x^{i\ 3}-b\,x^{--}\ ) & (1-x^{-\ }) & (1-x^{-})\end{array}\right\}=2(i-1)_4\sqrt{-1}
\end{array}\right\} \tag{19}$$

the coefficient $(i-1)_k$ of the second member being given by the relation

$$(i-1)_k=(i-2)_k+(i-2)_{i-2}-(i-2)_{i+}\ 2\cos(i-2)\alpha \tag{20}$$

Finally this last system containing now only a and b_i, will be

$$\left.\begin{array}{lll}(a\ -b\,x^{-(\ -1)}) & (1-x^{-\ -1}) & (1-x)=2(i)_1\sqrt{-1} \\ (a\,x-b\,x^{-}\qquad) & (1-x^{-\ -1}) & (1-x)=2(i)\ \sqrt{-1}\end{array}\right\} \tag{21}$$

the coefficient $(i)_k$ being given by the formula

$$(i)_k=(i-1)_k+(i-1)_{k+}\ -(i-1)_{k+1}\ 2\cos(i-1)\alpha \tag{22}$$

10 Let us first calculate by means of the equations (21) not the quantities a and b_i, but the coefficients A_i and B which are connected with them by the formulæ (7)

I will remark to this effect that in virtue of the formulæ (5) and (6),

$$\frac{1-x^{-k}}{x^k}=x^{-k}-x^k=-2\sqrt{-1}\sin k=\frac{2\sin k}{\sqrt{-1}}$$

and by means of this relation I shall transform the terms which compose the equations (17) (19) and (21) in the following manner

All these terms are comprised in the general form

$$[a_k x^{kp} - b_k x^{-k(p+2h+1)}]\,(1-x^{k+h}) \,.\, (1-x^k) \quad (1-x^{k-h}),$$

k and h being two entire and positive numbers starting from unity, and p capable of having all the entire and positive values, zero included. Observing that the sum of the exponents

$$(k+h)+(k+h-1)+.\quad +(k-h+1)+(k-h)$$

is equal to $k\,(2\,h+1)$, this general term may be written as follows:

$$[a_k x^{k(p-h+\frac{1}{2})} - b_k x^{-k(p+h+\frac{1}{2})}]$$

$$\times\,(x^{-\frac{k-h}{2}} - x^{\frac{k+h}{2}})\,(x^{-\frac{k+h-1}{2}} - x^{\frac{k+h-1}{2}}) \;..\,(x^{-\frac{k-h}{2}} - x^{\frac{k-h}{2}}),$$

or

$$\left.\begin{array}{l} [a_k x^{k(p+h-\frac{1}{2})} - b_k x^{-k(p+h-\frac{1}{2})}] \times \\ \times \left(\dfrac{2}{\sqrt{-1}}\right)^{2h+1} \sin\,(k+h)\dfrac{\alpha}{2}\;\; \sin\,(k+h-1)\dfrac{\alpha}{2}.\;\;.\sin\,(k-h)\dfrac{\alpha}{2} \end{array}\right\} \quad . \quad . \quad . \quad . \qquad (23)$$

11 By means of this formula (23.), and of suitable values given to k, h and p, the equations (21) will become

$$\left.\begin{array}{l} [a_i x^{i(i-\frac{1}{2})} - b_i x^{-i(i-\frac{1}{2})}] \times \\ \times \left(\dfrac{2}{\sqrt{-1}}\right)^{2i-1} \sin\,(2i-1)\dfrac{\alpha}{2}\;\; \sin\,(2\,i-2)\,\dfrac{\alpha}{2} \ldots \sin\dfrac{\alpha}{2} \end{array}\right\} = 2(i)_1 \sqrt{-1},$$

$$\left.\begin{array}{l} [a_i x^{i(i-\frac{1}{2})} - b_i x^{-i(i-\frac{1}{2})}] \\ \times \left(\dfrac{2}{\sqrt{-1}}\right)^{2i-1} \sin\,(2\,i-1)\dfrac{\alpha}{2}\;\; \sin\,(2\,i-2)\,\dfrac{\alpha}{2}\,.\;\;.\sin\dfrac{\alpha}{2} \end{array}\right\} = 2(i)_2 \sqrt{-1},$$

or, by putting generally for all the values of i

$$\left.\begin{aligned}
[i]_1 &= \frac{(i)_1\ (-1)^i}{2^{2i-1}\sin(2i-1)\frac{\alpha}{2}\sin(2i-2)\frac{\alpha}{2}\ \cdots\ \sin\frac{\alpha}{2}}, \\
[i]_2 &= \frac{(i)_{i+1}\ (-1)^i}{2^{2i-1}\sin(2i-1)\frac{\alpha}{2}\sin(2i-2)\frac{\alpha}{2}\ \cdots\ \sin\frac{\alpha}{2}},
\end{aligned}\right\} \quad (21)$$

we shall have

$$\left.\begin{aligned}
a_i x^{i(i-\frac{1}{2})} - b_i x^{-i(i-\frac{1}{2})} &= 2[i]_1 \\
a_i x^{i(i+\frac{1}{2})} - b_i x^{-i(i+\frac{1}{2})} &= 2[i]_2
\end{aligned}\right\} \quad (25)$$

These equations, on replacing a_i and b_i by their values in terms of A_i and B_i and on having again regard to the relations (6), will become

$$A_i\cos i(2i-1)\frac{\alpha}{2} - B_i\sin i(2i-1)\frac{\alpha}{2} = [i]_1$$

$$A_i\cos i(2i+1)\frac{\alpha}{2} - B_i\sin i(2i+1)\frac{\alpha}{2} = [i]_2,$$

and we shall thence deduce

$$\left.\begin{aligned}
A_i &= [i]_1\frac{\sin i(2i+1)\frac{\alpha}{2}}{\sin i\alpha} - [i]_2\frac{\sin i(2i-1)\frac{\alpha}{2}}{\sin i\alpha}, \\
B_i &= [i]_1\frac{\cos i(2i+1)\frac{\alpha}{2}}{\sin i\alpha} - [i]_2\frac{\cos i(2i-1)\frac{\alpha}{2}}{\sin i\alpha}
\end{aligned}\right\} \quad (26)$$

Thus then, $(i)_1$ and $(i)_2$ being determined by the formula (22), we shall deduce from it $[i]_1$ and $[i]_2$ by the formulæ (21), then A_i and B_i by the formulæ (26).

12. The calculation relative to the determination of A_{i-1} and of B_{i-1}, will be simpler by having recourse to the second and third formula of the system (19), rather than to the first and second. These equations will become, by transformations wholly similar to those which we have just developed,

$$\left.\begin{aligned}
&\left[a_{i-1}x^{(i-1)(i-\frac{1}{2})} - b_{i-1}x^{-(i-1)(i-\frac{1}{2})}\right] \\
&\quad\times\left(\frac{2}{\sqrt{-1}}\right)^{2i-3}\sin(2i-3)\frac{\alpha}{2}\ \cdots\ \sin\frac{\alpha}{2} \\
&+\left[a_i x^{i(i-\frac{1}{2})} - b_i x^{-i(i-\frac{1}{2})}\right] \\
&\quad\times\left(\frac{2}{\sqrt{-1}}\right)^{2i-3}\sin(2i-2)\frac{\alpha}{2}\ \cdots\ \sin 2\frac{\alpha}{2}
\end{aligned}\right\} = 2(i-1)_2\sqrt{-1},$$

$$\left.\begin{array}{l}
\left[a_{i-1}\, x^{(i-1)(i+\frac{1}{2})} - b_{i-1}\, x^{-(i-1)(i+\frac{1}{2})}\right] \\
\qquad \times \left(\dfrac{2}{\sqrt{-1}}\right)^{2i-3} \sin (2i-3)\dfrac{\alpha}{2} \ldots \sin \dfrac{\alpha}{2} \\
+ \left[a_i\, x^{i(i+\frac{1}{2})} - b_i\, x^{-i(i+\frac{1}{2})}\right] \\
\qquad \times \left(\dfrac{2}{\sqrt{-1}}\right)^{2i-3} \sin (2i-2)\dfrac{\alpha}{2} \ldots \sin 2\dfrac{\alpha}{2}
\end{array}\right\} = 2(i-1)_3 \sqrt{-1}.$$

It is from the care which we had in taking the second and third of the equations of the system (19.) that the first members of the equation (25.), which have already been calculated, are again found here as factors. If we have regard to these conditions and to the notation (24.), we shall be able to put for shortness,

$$\left.\begin{array}{l}
k'_{i-1} = [i-1]_2 - [i]_1 \dfrac{\sin (2i-2)\dfrac{\alpha}{2}}{\sin \dfrac{\alpha}{2}}, \\
k''_{i-1} = [i-1]_3 - [i]_2 \dfrac{\sin (2i-2)\dfrac{\alpha}{2}}{\sin \dfrac{\alpha}{2}}.
\end{array}\right\} \quad . \quad . \quad . \quad . \quad (27.)$$

These expressions reduce the preceding equations to the following:

$$\left.\begin{array}{l}
a_{i-1}\, x^{(i-1)(i-\frac{1}{2})} - b_{i-1}\, x^{-(i-1)(i-\frac{1}{2})} = 2k'_{i-1}, \\
a_{i-1}\, x^{(i-1)(i+\frac{1}{2})} - b_{i-1}\, x^{-(i-1)(i+\frac{1}{2})} = 2k''_{i-1}.
\end{array}\right\} \quad . \quad . \quad (28.)$$

Their form is the same as that of the equations (25.), and we shall deduce from them in a similar manner,

$$\left.\begin{array}{l}
A_{i-1} = k'_{i-1} \dfrac{\sin (i-1)(2i+1)\dfrac{\alpha}{2}}{\sin (i-1)\alpha} - k''_{i-1} \dfrac{\sin (i-1)(2i-1)\dfrac{\alpha}{2}}{\sin (i-1)\alpha}, \\
B_{i-1} = k'_{i-1} \dfrac{\cos (i-1)(2i+1)\dfrac{\alpha}{2}}{\sin (i-1)\alpha} - k''_{i-1} \dfrac{\cos (i-1)(2i-1)\dfrac{\alpha}{2}}{\sin (i-1)\alpha}.
\end{array}\right\} \quad . \quad (29.)$$

13 The determination of A_{i-} and B_{-} must be effected by means of the third and fourth formulæ of the system (17) These formulæ, transformed like the preceding become

$$\left.\begin{aligned}
&[a_{i-}\, x^{(-)(-\frac{1}{2})} - b_{-}\, x^{-(-)(-\frac{1}{2})}]\left(\frac{2}{\sqrt{-1}}\right)^{-5} \sin(2i-5)\frac{\alpha}{2}\ \sin\frac{\alpha}{2}\\
+&[a_{-}\, x^{(-1)(-\frac{1}{2})} - b_{-1}\, x^{-(-1)(-\frac{1}{2})}]\left(\frac{2}{\sqrt{-1}}\right)^{-5} \sin(2i-4)\frac{\alpha}{2}\ \sin 2\frac{\alpha}{2}\\
+&[a_{i}\, x^{i(-\frac{1}{2})} \qquad - b\, x^{-(-\frac{1}{2})} \qquad]\left(\frac{2}{\sqrt{-1}}\right)^{-5} \sin(2i-3)\frac{\alpha}{2}\ \sin 3\frac{\alpha}{2}
\end{aligned}\right\} = 2\,(i-2)_3\sqrt{-1}$$

$$\left.\begin{aligned}
&[a_{i-2}\, x^{(-)(+\frac{1}{2})} - b_{-}\, x^{-(-)(+\frac{1}{2})}]\left(\frac{2}{\sqrt{-1}}\right)^{-5} \sin(2i-5)\frac{\alpha}{2}\ \sin\frac{\alpha}{2}\\
+&[a_{-1}\, x^{(-1)(+\frac{1}{2})} - b_{-1}\, x^{-(-1)(+\frac{1}{2})}]\left(\frac{2}{\sqrt{-1}}\right)^{-5} \sin(2i-4)\frac{\alpha}{2}\ \sin 2\frac{\alpha}{2}\\
+&[a\, x^{i(-\frac{1}{2})} \qquad - b\, x^{-(+\frac{1}{2})} \qquad]\left(\frac{2}{\sqrt{-1}}\right)^{-} \sin(2i-3)\frac{\alpha}{2}\ \sin 3\frac{\alpha}{2}
\end{aligned}\right\} = 2\,(i-2)_4\sqrt{-1}$$

If in taking account of the relations (25) and (28) and of the notation (24) we put

$$\left.\begin{aligned}
k'_{-} &= [i-2]_3 - k'_{-1}\frac{\sin(2i-4)\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} - [i]_1\frac{\sin(2i-4)\frac{\alpha}{2}\sin(2i-3)\frac{\alpha}{2}}{\sin\frac{\alpha}{2}\sin 2\frac{\alpha}{2}}\\
k''_{-2} &= [i-2]_4 - k''_{-1}\frac{\sin(2i-4)\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} - [i]\frac{\sin(2i-4)\frac{\alpha}{2}\sin(2i-3)\frac{\alpha}{2}}{\sin\frac{\alpha}{2}\sin 2\frac{\alpha}{2}}
\end{aligned}\right\} \qquad (30)$$

the preceding equations will be written

$$a_{i-2}\, x^{(i-2)(i-\frac{1}{2})} - b_{i-2}\, x^{-(i-2)(i-\frac{1}{2})} = 2\, k'_{i-2},$$
$$a_{i-2}\, x^{(i-2)(i+\frac{1}{2})} - b_{i-2}\, x^{-(i-2)(i+\frac{1}{2})} = 2\, k''_{i-2},$$

and there will be deduced from them

$$\left.\begin{aligned} A_{i-2} &= k'_{i-2}\,\frac{\sin (i-2)(2i+1)\frac{\alpha}{2}}{\sin (i-2)\alpha} - k''_{i-2}\,\frac{\sin (i-2)(2i-1)\frac{\alpha}{2}}{\sin (i-2)\alpha}, \\ B_{i-2} &= k'_{i-2}\,\frac{\cos (i-2)(2i+1)\frac{\alpha}{2}}{\sin (i-2)\alpha} - k''_{i-2}\,\frac{\cos (i-2)(2i-1)\frac{\alpha}{2}}{\sin (i-2)\alpha} \end{aligned}\right\} \quad (31.)$$

And thus, rising from system to system, by these formulæ, the law of which is evident, we shall arrive by symmetrical calculations at the determination of all the coefficients A_i and B_i, A_{i-1} and B_{i-1} ... up to A_1 and B_1, A_1 and B_1 in particular will be given by the formulæ of the ranks i and $(i+1)$ of the system (10.)

Lastly, the first of the equations (4.) will give B_0 very simply

11. In short, the numerical operations which have to be effected to obtain a complete system of the values of A_i and B_i corresponding to the same value of the longitude l', will be as follows —

1° The $(2i+1)$ numerical values R_0, R_1, R_2, ... and R_{2i} of the disturbing function are determined

2° By means of the formulæ (9.) and (22.), the numerical values comprised in the table (16.) are determined

3° By means of the formulæ (21.), the $2i$ quantities $[i]_1$ and $[i]_2$, $[i-1]_2$ and $[i-1]_3$, $[i-2]_3$ and $[i-2]_4$ up to $[1]_i$ and $[1]_{i+1}$, which only requires some logarithms, are calculated

4° By means of the formulæ (27.), (30.) and those analogous to them, the $2(i-1)$ quantities k'_{i-1} and k''_{i-1}, k'_{i-2} and k''_{i-2}, ... up to k'_1 and k''_1 are calculated

5° The formulæ (26.), (29.), (31.), and those analogous to them, will give all the quantities A_i and B_i, and the first of the formulæ (4.) will give the quantity B_0

All these calculations are symmetrical, their nature admits of executing them with exactness. We may moreover simply control the quantities $(i)_1$, $(i)_2$, $(i)_3$ ... For if we add all the equations which the formula (22.) gives, when we suppose that the index k varies from n up to n', we shall have

$$\Sigma_n^{n'}(i+1)_i = \Sigma_n^{n'+2}(i)_i + \Sigma_{n+2}^{n'}(i)_i - 2\cos i\alpha\, \Sigma_{n+1}^{n'+1}(i)_k, \qquad (32.)$$

the sign Σ being relative to the different values of index k Th
relation will permit of verifying rapidly the whole series of th
coefficients $(i)_k$ or only a certain number among them taken con
secutively, which will easily disclose the errors that may ha
slipped in

Lastly it will be necessary for the first and second of th
equations (1) to agree in giving the same value of the constan
B_0

15 Let us return to the relations (3) and first to the fir
among them By the preceding calculations we shall be able t
determine the values of B_0 corresponding to the mean longi
tudes

$$l'=0, \qquad l'=\alpha, \qquad l'=2\alpha \quad , \qquad l'=2i\alpha,$$

which will furnish $(2i+1)$ relations to determine the constan
C of the disturbing function and the coefficients $(0, i')$ an
$[0, i']$ corresponding to the different values of i' from 1 up to
It is clear in fact that the index i being null, it suffices to attri
bute to i' positive values The relations to be solved being mor
over wholly similar to the relations (1), we have nothing to ad
on this point

16 It remains for us to solve the system of the two last of th
equations (3), as it is presented when we give to l' the value
0 1 2, , $2i$

Restricting ourselves to the system composed of the equa
tions furnished by one only of the equations (3), it would b
still more completely similar to the system (1) and it would b
treated in the same manner But we should be obliged to em
ploy thus twice as many numerical values of the function R a
when using the two equations (3) at a time We shall then a
tempt to employ them, and shall find in this the advantage
decomposing the equations which afford the coefficients $(i,$
and $[i\ i']$ for one value of i and for different values of i', positiv
or negative in two separate systems

I remark with this view, that in giving to i' the value $-i$, th
coefficients $(i\ i')$ and $[i, i']$ become of the order *zero* in relatio
to the excentricities and the inclinations that to have regard t
the coefficients which are of the order p in relation to these el
ments it will be necessary to give to i' all the values fro
$(-i-p)$ up to $(-i+p)$ And thus the system of the two equa
tions (3) will become for one value of l' equal to $n\alpha$

$$\left.\begin{array}{l}
\left.\begin{aligned}
&(i,\ -i\ )\cos(\ -i\ )n\alpha-[i,\ -i\ ]\sin(\ -i\ )n\alpha\\
&+(i,-i-1)\cos(-i-1)n\alpha-[i,-i-1]\sin(-i-1)n\alpha\\
&+(i,-i+1)\cos(-i+1)n\alpha-[i,-i+1]\sin(-i+1)n\alpha\\
&\quad\cdots\cdots\cdots\cdots\cdots\cdots\\
&+(i,-i-p)\cos(-i-p)n\alpha-[i,-i-p]\sin(-i-p)n\alpha\\
&+(i,-i+p)\cos(-i+p)n\alpha-[i,-i+p]\sin(-i+p)n\alpha
\end{aligned}\right\}=A_i^{(n)},\\[2ex]
\left.\begin{aligned}
&[i,\ -i\ ]\cos(\ -i\ )n\alpha+(i,\ -i\ )\sin(\ -i\ )n\alpha\\
&+[i,-i-1]\cos(-i-1)n\alpha+(i,-i-1)\sin(-i-1)n\alpha\\
&+[i,-i+1]\cos(-i+1)n\alpha+(i,-i+1)\sin(-i+1)n\alpha\\
&\quad\cdots\cdots\cdots\cdots\cdots\cdots\\
&+[i,-i-p]\cos(-i-p)n\alpha+(i,-i-p)\sin(-i-p)n\alpha\\
&+[i,-i+p]\cos(-i+p)n\alpha+(i,-i+p)\sin(-i+p)n\alpha
\end{aligned}\right\}=B_i^{(n)}.
\end{array}\right\}\quad(33.)$$

I now eliminate successively between these two equations $[i,\ -i]$, then $(i,-i)$, and I obtain the two relations:

$$\left.\begin{aligned}
(i,\ -i)&+\{(i,-i-1)+(i,-i+1)\}\cos n\alpha\\
&+\{[i,-i-1]-[i,-i+1]\}\sin n\alpha\\
&\quad\cdots\cdots\cdots\cdots\\
&+\{(i,-i-p)+(i,-i+p)\}\cos pn\alpha\\
&+\{[i,-i-p]-[i,-i+p]\}\sin pn\alpha
\end{aligned}\right\}=P_i^{(n)},\quad .\quad .\quad (34.)$$

$$\left.\begin{aligned}
[i,\ -i]&+\{(i,-i+1)-(i,-i-1)\}\sin n\alpha\\
&+\{[i,-i-1]+[i,-i+1]\}\cos n\alpha\\
&\quad\cdots\cdots\cdots\cdots\\
&+\{(i,-i+p)-(i,-i-p)\}\sin pn\alpha\\
&+\{[i,-i-p]+[i,-i+p]\}\cos pn\alpha
\end{aligned}\right\}=Q_i^{(n)},\quad .\quad .\quad (35.)$$

in which I suppose that $P_i^{(n)}$ and $Q_i^{(n)}$ have the following values:

$$\left.\begin{aligned}
P_i^{(n)}&=A_i^{(n)}\cos in\alpha-B_i^{(n)}\sin in\alpha,\\
Q_i^{(n)}&=A_i^{(n)}\sin in\alpha+B_i^{(n)}\cos in\alpha.
\end{aligned}\right\}\quad .\quad .\quad .\quad .\quad (36.)$$

Let us, to simplify the matter, put

$$(i,-i-p)+(i,-i+p)=x_p,$$
$$[i,-i-p]-[i,-i+p]=z_p;$$

the formula (34.) will become

$$\left.\begin{aligned}
(i,-i)&+x_1\cos n\alpha+x_2\cos 2n\alpha+\ldots+x_p\cos pn\alpha\\
&+z_1\sin n\alpha+z_2\sin 2n\alpha+\ldots+z_p\sin pn\alpha
\end{aligned}\right\}=P_i^{n},$$

and by attributing to n the different entire and positive values on starting from zero, we shall have a system of equations similar to the system (4.). We shall then deduce from them in the same manner the values of $(i,-i)$, x_p and z_p.

To have regard to the equations (35), I shall make

$$(i,-i+p)-(i,-i-p)=y_p,$$
$$[i,-i+p]+[i,-i-p]=t_p,$$

these equations will then be written

$$\left.\begin{array}{l}[i,-i]+y_1\sin n\alpha+y_2\sin 2n\alpha+\quad+y_p\sin pn\alpha\\ \qquad\qquad+t_1\cos n\alpha+t_2\cos 2n\alpha+\quad+t_p\cos pn\alpha\end{array}\right\}=Q_i^{(i)}$$

and we shall easily deduce from them the values of $[i,-i]$, y_p and t_p

After these determinations we shall have

$$(i,-i+p)=\tfrac{1}{2}x_p+\tfrac{1}{2}y_p,$$
$$(i,-i-p)=\tfrac{1}{2}x_p-\tfrac{1}{2}y_p,$$
$$[i,-i+p]=\tfrac{1}{2}t_p-\tfrac{1}{2}z_p,$$
$$[i,-i-p]=\tfrac{1}{2}t_p+\tfrac{1}{2}z_p,$$

and thus all the coefficients of R will be calculated up to a given decimal

17 We have already shown that the preceding method does not permit any error to escape We shall better appreciate the advantages of it by observing that the arbitrary angle α may be adopted once for all, and that thus it would be easy to determine beforehand the numerical values of the coefficients of the formulæ (24), (26), so as to have no longer any but linear expressions to calculate, which is always very rapid

It is very important to observe, that in virtue of the excentricities of the orbits the degree of convergence of the disturbing function is different for the different values of the mean longitude l' of the disturbing planet so that there are great advantages in not being obliged to employ in all the cases the same number of values of the function R We should not be able to arrive at this in the method whereby the circumference is divided into equal parts, without being obliged to change incessantly this division, which is impracticable By the preceding calculations, on the contrary, we never employ more than the number of the numerical values strictly necessary for each of the positions of the disturbing planet, without having to change the formulæ (24), (27), (29), , which we shall begin by establishing for the case in which the series is the least convergent There will afterwards be only to suppress one or more of the last terms of these formulæ in proportion as the calculation itself will indicate the possibility of this, and taking care to neglect at first the functions R_0 and R_2, then R_1 and $R_{2(i-1)}$, and so forth

SCIENTIFIC MEMOIRS.—PART XVIII.

In the last Part of the Scientific Memoirs, the remark appended to the Note containing the extracts from Laplace should have been cancelled, it having been written inadvertently. The velocity of the ray in the interior of the crystal, according to the emission system, to which Laplace was referring, is the inverse of the velocity on the undulatory hypothesis; and it was from inadvertence as to this point that the remark was made, and which the Translator had not an opportunity of correcting before publication.

SCIENTIFIC MEMOIRS.

VOL V—PART XIX

Article VIII

*On the Repulsion of the Optic Axes of Crystals by the Poles of a Magnet** By M Plücker, *Professor of Natural Philosophy in the University of Bonn*

[From Poggendorff's *Annalen*, Vol lxxii, No 10, October 1847.]

1. The object of the present memoir is to make known a series of new observations, which form a sequel to the last discoveries of Faraday, from which the idea of making them originated The results of these observations, when arranged in the form of a general expression, lead to the following empirical laws —

When any crystal having a single optic axis is placed between the two poles of a magnet, this axis is repelled by each of the two poles If the crystal has two optic axes, each of these two axes is repelled by each of the two poles with the same force

The force which produces this repulsion is independent of the magnetic or diamagnetic condition of the mass of the crystal, it diminishes less, as the distance from the poles of the magnet increases, than the magnetic or diamagnetic forces emanating from these poles, and acting upon the crystal

2. To facilitate as much as possible the survey and critical investigation of my observations, and the conclusions deduced from them, the best method appears to me to be that of detailing them in exactly the same course as that by which I was conducted to the above results It will however be indispensable previously to relate briefly the method and means used in my experiments.

First, with the view of repeating Faraday's experiments upon

* Translated by Dr. J W Griffith

magnetism and diamagnetism, as also the rotation of the planes of polarization of light by magnetic action. I had a powerful electro-magnet constructed by Etter, the university mechanician, under my own superintendence; and to be sure of obtaining at least the same action, the iron nucleus in it was made of the same dimensions as those specified by Faraday for his large horse-shoe magnets, substituting the *Parisian* for the English foot. The surfaces of the ends of the poles were therefore circles, the diameter of which was 102 millim. ($3\frac{3}{4}$ French inches), and the centres of which were 284 millim. ($9\frac{5}{8}$ French inches) apart. The iron nucleus weighs 84 kilogrammes, and each of its two perpendicular arms is covered with four layers of copper wire, each of which consists of ninety-two coils. This wire is 4.36 millim. in thickness (2 Rhenish lines), whilst the wire of Faraday's magnet was 0.17 of an English inch. On theoretical grounds I chose the greatest thickness which could be conveniently obtained in wire which had been well heated to redness and covered. The wire weighs about 35 kilogrammes. The coil of wire extends to the surfaces of the poles; to fit each of these, an appendage of soft iron is ground; the surface of this is of the same diameter, and 48 millim. in height. The two appendages are perforated in the centre of their height, and in the perforations, which are 20 millim. in diameter, two moveable cylinders of soft iron, which fit and are conically pointed at their extremities, are inserted and fixed by screws. The conical apices, in which the magnetic action is concentrated, can be approximated or separated from each other at will, and are removable either in connexion with, or separately from the appendages. A glass case, containing a Coulomb's torsion balance, is placed upon the leaf of a table, which can be raised or lowered, and is furnished with two round holes, through which the arms of the electro-magnet pass. A strong thread, composed of a large number of separate silkworm threads, winds up and down upon the arm of the balance, to which heavy bodies, weighing as much as half a kilogramme and more, can be immediately suspended, for instance in a light little boat, to a small hook. As regards the suspension of light bodies, from the first, in my magnetic and diamagnetic experiments, I found it requisite to dispense with this boat; since I could not obtain any substance which, when placed between the two poles of my electro-magnet, did not appear either magnetic or diamagnetic. I suspend all such

light bodies immediately in a double loop of a single or double silkworm thread, from 60 to 300 millim. in length, and fastened with a little wax to the end of the thick thread which passes over the arm of the torsion-balance. The suspensions are best made in the glass case itself, because the single silkworm thread is more readily seen there; and after a little practice it is easy to make twenty or thirty different suspensions of such a thread in an hour. Moreover, in this manner we can test as to their magnetism or diamagnetism, bodies which are not greater or heavier than a single stamen of a cherry-tree flower.

To excite the magnetism in the electro-magnet, I used throughout the following experiments merely three or four small Groves's elements, combined to form a battery. The platinum was immersed in commercial nitric acid, the zinc in a mixture of 1 part of concentrated sulphuric acid and 9 of water.

3. During the month of May I made a large number of experiments, which appear to yield the general result, that in each individual plant (and perhaps animal) constant magnetic and diamagnetic counteractions are in play, and are in *connexion* with their physiological development. I shall reserve the details for a future communication, and merely remark, that these experiments led me to examine whether the arrangement of the fibre exerted any influence upon the position assumed by vegetable structures suspended between the two poles of the magnet; and here the question incidentally occurred to me, whether the crystallographic structure of a crystal suspended in the same manner exerted any influence. The very first experiment decided this positively.

4. I took, for instance, a green plate of tourmaline, as prepared by M. Soleil, for the polarization of light. It was 3 millim. in thickness; its largest surfaces were nearly square, being 12 millim. in length and 9 in breadth. The longitudinal direction of the plate corresponded with the direction of its optic axis. On suspending this plate by a silk thread, so *that the direction of the thread coincided with the direction of the optic axis*, it assumed the same position as any other magnetic body of the same form would have done, *i. e.* so that the direction of its breadth coincided with a straight line connecting the two apices of the poles, and the plate maintained this position decidedly, even after the polar apices were removed. This result I had anticipated, because the plate of tourmaline was so strongly *magnetic*, that

when suspended very near one of the polar apices, it was attracted by it

5 The same plate of tourmaline was now suspended in such a manner that the direction of its breadth coincided with that of the silk thread and thus *the optic axis could oscillate freely in a horizontal plane* The apices of the poles were not too near each other and were at last entirely removed As a magnetic body, the plate should have assumed such a position, that its longitudinal and axial direction coincided with the line of the apices of the poles However, it assumed that position which a diamagnetic body of the same form would have done, *i e* with its axial and longitudinal direction *perpendicular to the line of the apices of the poles*

6 The same plate of tourmaline was lastly again suspended so that it consequently its optic axis could oscillate horizontally Again, as on its second suspension, it assumed the same position as a diamagnetic body of the same form would have done, the direction of its breadth being in the line of the apices of the poles and its longitudinal and axial direction *perpendicular to it*

7 By opening and closing the circuit in each of the three positions of suspension, the tourmaline could be turned round and retained in exactly the opposite position

8 If we admit that an equal repulsive force is excited by each of the two poles of the electro magnet upon the axial direction of the plate of tourmaline, and that this repulsion is stronger than the attraction of the same poles excited upon the same axis in consequence of the magnetic distribution in the ferruginous mass of tourmaline, we obtain a comprehensive point of view, under which the phenomena above described may be conceived

As in the position assumed by the plate of tourmaline in experiments 5 and 6, the magnetic attraction must first be overcome by the new force producing the repulsion, it might be anticipated that the phenomena in question would be modified when the magnetic rectilinear force was so increased that the form of the crystal was such that the dimensions of its axial direction were very considerably greater than its other dimensions, and the two apices of the poles were approximated as much as possible Therefore, after having again convinced myself of the accuracy of my former experiments, and having found them confirmed by means of a second plate of tourmaline of the same dimensions,

I selected a dark brown, almost opake crystal of tourmaline, having the form of a six-sided prism, which was about 36 millim. long and 4·5 millim. in thickness, and placed the apices of the poles so near together, that it could only just oscillate freely between them. The magnetic attraction caused the tourmaline to assume such a position, that the axis of the prism, which is also its optic axis, coincided with the line of the apices of the poles. The more the latter were separated from each other, the less intense was the force with which the crystal assumed this position; and when their distance amounted to more than 80 millim., it rotated 90°, as if it had become diamagnetic, so that its axis was now perpendicular to the line of the apices of the poles. On the further separation of the latter, the force which retained it in the position just described increased; and in this it continued distinctly to remain after the apices of the poles had been entirely removed.

The apices were again inserted and pushed forwards until the tourmaline assumed an axial position (in the line of the apices). When it was now raised or lowered, by winding or unwinding the thread by which it was suspended, it turned round, at a certain elevation or depression, 90°, at the same time assuming an equatorial position (perpendicular to the line of the apices of the poles) As far as the limit to which this rotation extended, the axial rectilinear force diminished until it finally vanished; the equatorial rectilinear force then came into play, increasing when the crystal was further raised or lowered, and finally again diminished, being however distinctly perceptible when removed from 200 to 250 millim. from the line of the apices of the poles.

In all these experiments, by opening and closing the circuit, the tourmaline could be rotated 180°, and retained in the opposite position.

9. The passage of the tourmaline from one position to the other still appeared to take place to exactly the same extent when the power of the electro-magnet was either increased or diminished.

10. If we retain the hypothesis of a repulsive action exerted by the poles of the magnet upon the axial direction, in accordance with the experiments which have been detailed in the two last paragraphs, we must necessarily admit that the force which produces the repulsion diminishes *more slowly* with the increase of the distance than the force of the magnetic attraction emanating from the same poles.

11. Before examining other crystals in the same way as the tourmaline, it appeared to me best to investigate as completely as possible the remarkable phenomena and their modifications in this mineral, in which I first discovered them. Two other crystals of tourmaline resembling that which was used in experiment 7, but heavier, one being 47 millim. in length and about 3 millim. in thickness, the other of the same length and about 5 millim. in thickness, yielded in general exactly the same results. All these tourmalines were attracted into the immediate vicinity of one of the poles of a magnet, in their entire mass.

12. A rubellite, 9 millim. long and 6 to 7 millim. thick, when the apices of the poles were 13 millim. apart, became placed axially in the line of the apices, but turned round 90° even when raised about 20 millim. or lowered about 30 millim., at the same time assuming an equatorial position.

A red, transparent tourmaline, 30 millim. in length, did the same.

I then examined four smaller crystals of tourmaline from the isle of Elba, 4 to 9 millim. long, the first half light and half dark green, a second entirely light green, a third light green in the middle but dark at both ends, and the fourth red. All these, as also the two former, were strongly magnetic; the first was more strongly so at the dark than the light green end. They exhibited all the phænomena described in paragraph 7, except that the third and smallest could not be made to assume the axial position, because, when the apices of the poles were approximated with this intention, before it could attain this position, it was drawn away by one of the apices of the poles.

13. Lastly, I must mention a perfectly colourless tourmaline, the long dimensions of which coincided with the direction of the axis, but which in other respects was irregular. The mass of this crystal proved to be *diamagnetic* throughout; whence it appears that both the red and the green colour of this mineral are produced by iron; consequently the diamagnetic force emanating from the poles of the electro-magnet must act upon the axial direction in the same manner as the repulsive force, to produce the equatorial position of the crystal.

14. As regards the tourmaline, after the preceding remarks, I considered myself justified in considering the following law determined, viz. that its axial direction is repelled by the poles of

a magnet; and that this repulsion, in ordinary cases when the mass of the tourmaline is magnetic, at a sufficient distance from the pole itself, and when the largest dimensions of the crystal are in the axial direction, overcomes the produced magnetic attraction of the axis.

Here it naturally occurs to us to inquire, whether the above phænomena may not perhaps stand in close relation with the well-known fact, that the tourmaline, when heated and cooled, exhibits electric polarity. That must however be decidedly answered in the negative.

15. It might certainly have been possible, that by touching the tourmaline whilst suspending it, electric currents had been excited in its substance. But after the tourmaline had remained undisturbed under the glass case of the torsion-balance for twenty-four hours, and the electro-magnet was then set in action, it exhibited exactly the same phænomena; and no difference could be perceived in it even when it was rendered electrical at its extremities by being heated whilst between the poles of the magnets.

16. When the tourmaline was suspended in water, it also assumed a decided position, from which it was not disturbed when the water was heated to near the boiling-point. With the moistening of the surfaces of the crystal which occurs in this experiment, no electric tension can occur; if this tension, however, is the consequence of internal electric fluctuations, it might perhaps be expected that the latter were promoted by the continual conduction of the free electricity appearing on the surface.

17. The most decided proof of the inaccuracy of the assumption of *electric currents within the crystal which have not been primarily excited by the magnet* being the cause of the phænomena in question, exists in the fact, that these currents, whatever direction may be ascribed to them, would produce a *polar* repulsion of the axial direction. But experiment contradicts this most decidedly.

18. Since the repulsion of the axial direction by the poles of the magnet is therefore not of pyro-electric origin, we should expect that it would not be confined to tourmaline, in which I had first accidentally met with it, and to a few other crystals. After a preliminary experiment upon a small piece of rock-crystal had shown it to exist in this also, I commenced examining various crystals, and first those which are *uniaxial*. In doing

this regard had first to be paid to distinguishing whether the substance of the body was *magnetic* or *diamagnetic*. It was specially important to test a crystal of some diamagnetic substance as well as tourmaline which derives its magnetism from the iron contained in it. To obtain an effect which could in nowise be ascribed to diamagnetism, a crystal of such form must be selected, or such a shape must be given to it by artificial means, that its shortest dimensions coincide with the direction of the axis. The form of the colourless diamagnetic tourmaline mentioned in paragraph 13 was not adapted for the purpose of a decisive experiment.

19. I therefore first examined calcareous spar, the substance of which is decidedly diamagnetic. A colourless crystal, bounded by natural faces of cleavage, the length of the angles of which were 60 millim., 50 millim. and 28 millim., was suspended, without using the apices of the poles, in such a manner that its axis could oscillate horizontally between the poles. This axis became placed exactly equatorially, whereby the crystal assumed a position in which neither a magnetic nor a diamagnetic mass of the same form would have rested when acted upon by the magnetic and diamagnetic action of the poles of the electro-magnet.

20. I then took a smaller crystal of the primary form, which, overcoming the diamagnetism, arranged itself between the approximated apices of the poles, so that its axial direction became exactly equatorial.

A second such crystal, but larger, the length of the angles of which was 15 millim., and the obtuse angles of which were ground off perpendicularly to the optic axis to such an extent that the thickness of the crystal in its axial direction was 10 millim. only, was suspended in the same manner as the two former, and assumed the same position, as regards its axis, as those. When, however, the apices of the poles were so far approximated, that on account of its large dimensions it could no longer remain in the line of the apices of the poles, it rotated 90°, assuming the same position as a diamagnetic body, so that its axial direction coincided with the line of the apices of the poles, and the diamagnetic repulsion of the mass determined the position of the crystal.

21. I then examined several other plates cut perpendicularly to the axis, all of which exhibited the same phænomenon. One

of them, which was from 26 to 30 millim. broad and long and 6 millim. thick, when suspended in the same manner and oscillating freely, assumed the same position as a diamagnetic mass; but on further separating the poles, or on shortening or elongating the silkworm thread, it rotated 90°, and remained as if it had become magnetic, the axis being turned perpendicularly to the line of the apices of the poles.

22. The experiments described above point out uniformly that a repulsive force is excited by the poles of the magnet upon the axial direction of the calcareous spar, and that, when by the abbreviation of the dimensions in this direction, an attraction towards the axial direction arises from the diamagnetic repulsion of the substance of the crystal, the poles being sufficiently separated, this attraction is less than the repulsion.

23. Whilst the colourless and transparent calcareous spar is diamagnetic, a white opake crystal of calcareous spar is *magnetic*, and one of them presented the same phænomena as tourmaline.

24. Rock crystal is diamagnetic like calcareous spar, and like it exhibited the repulsion of the axial direction; but this repulsion is *less* intense. When a plate cut perpendicularly to the axis (which exhibits the rotation of the plane of polarization) is about three times as long and broad as thick, on suspending it with the axis horizontal, it assumed the same position as a diamagnetic body, and no longer rotated 90° on separating the poles, which decidedly occurred with plates the dimensions of which were less contracted in the direction of the axis.

25. In a Soleil's apparatus for exhibiting the conjugate hyperbolæ with polarized light, two similar prisms are ground out of rock-crystal; their height amounts to 50 millim., and their total base is an almost regular octagon, the two opposite sides of which are 26 millim. apart, and cut at right angles by the optic axis. The two prisms are cemented together to form a single prism, in such a manner that the axial directions in the two halves are at right angles to each other. When the entire prism is suspended so that its axis (the axis of the prismatic form) coincides with the direction of the silkworm thread, and thus the optic axes can oscillate horizontally, when raised or lowered, so that at one time the lower, at another time the upper half gets between the apices of the poles, it successively assumes two different positions, either of which passes into the other by a rotation of 90°, whereby each time the direction of the optic axis of

that half vibrating in the line of the apices of the poles becomes placed perpendicularly to this line

26 The following crystals were very decidedly proved to be those the substance of which was magnetic, and which, in consequence of their form when suspended in the line of the apices of the poles, under the influence of the magnetic attraction of the poles of the electro magnet, arranged themselves like the tourmaline, in this line but when raised or lowered, after a rotation of 90 assumed an equatorial position —

1 An opake and perfectly crystallized piece of *quartz* from Hagen, the longitudinal and axial dimensions of which were 10 millim

2 A square octahedron of *zircon*, with truncated angles and edges, from Siberia

3 A six sided crystal of *beryl* from Siberia, 11 millim in length and from 11 to 13 millim in thickness

4 Two yellowish green transparent crystals of *emerald*, one of which was 27 millim long and 11 millim thick, the other much larger, and weighing several hundred grammes

5, A black *idocrase* from Siberia, crystallized in perfect square prisms, the edges truncated, the angles acute, and one of the apices truncated

6 A large *corundum*

27 I found *two* crystals, which were strongly magnetic, and which could *not* be removed from the axial position, in which the magnetism fixed them, even by the removal of the poles It is worthy of remark, that both these crystals exhibited *magnetic polarity** It appears to me probable, that by using a more powerful current, which would allow of a greater separation of the poles, these crystals, overcoming the magnetism of the substance, would also have become arranged equatorially, which they would probably have done if I had been able to contract their longitudinal and axial dimensions

These two crystals were the following —

1 An opake brownish crystal of *pinite* from Auvergne, a regular six sided prism, 12 millim long and from 6 to 7 millim thick

2 A small crystal of *sapphire*

28 After the experiments which have now been detailed, it

* One of the plates of tourmaline above mentioned was also polar, but slightly so only and in a direction which did not coincide with the axis

appears to me that the empirical law laid down in paragraph 1, so far as it relates to uniaxial crystals, is sufficiently established, and applies indifferently both to *positive* and *negative* crystals.

29. Hence it might further, with tolerable certainty, be supposed, that an analogous action would occur also in *crystals with two optic axes*, to that found in uniaxial crystals. As such we might expect either a repulsion of the *two optic axes*, or merely a repulsion exerted against their *central line*, *i. e.* against that direction which subdivides the *acute* angle formed by the optic axes. Experiments are in favour of the first more universal assumption, which comprises the latter.

30. I cut a circular disc, about 22 millim. in diameter, from a plate of *mica*, and suspended it by a silkworm thread so that it could oscillate horizontally. As is well known, the two optic axes of mica lie in a plane which is at right angles to the direction of the laminæ in it, inasmuch as it forms with the normal the same angle on both sides, this I estimated at 22½°. When suspended as above, the planes of the two axes could rotate around their mesial line placed vertically. Between the two poles of the magnet, the lamina of mica assumed such a position that its plane coincided with the equatorial plane. In this position the equatorial direction was thus marked upon the lamina of mica, and it was afterwards found that the two optic axes, *i. e* those two directions which, when viewed by polarized light, correspond to the central point of the two systems of rings, lie in the same plane, which is placed at right angles to the plate of mica in the equatorial direction.

Mica exhibits the properties of a magnetic body.

31 With the view of modifying the experiment described in the last paragraph, with regard to the concluding remark, the planes of the two optic axes of a plate of mica were determined, and a hexagon with parallel opposite sides cut out of it, so that its longest dimension, which was 26 millim., was in the plane just determined, whilst the breadth of the lamina was only 18 millim. The lamina was then again suspended as before; and when the poles were approximated as much as possible, it arranged itself with its longitudinal direction, hence with the plane of the two axes, in the line of the poles. When the lamina was elevated or lowered, it rotated 90°, so that the planes of the two optic axes became perpendicular to this line.

32. I then took a transparent and colourless *topaz* from Scot-

land, which was cut in such a manner that it approximated in form to a right rhombic prism, with its two pairs of opposite lateral surfaces perpendicular to the two optic axes. The length of the crystal was 19 millim., its thickness taken in the direction of each of the two axes 10 millim. The middle line between the two optic axes therefore corresponded to the shorter diagonal of the rhombic prism. Upon two of its adjacent sides two *thin* plates of tourmaline were cemented (which did not of themselves exert any perceptible influence), so that polarized light, which passed through both the optic axes of the crystal, and then through the corresponding plate of tourmaline, yielded one of the two systems of rings. The substance of the crystal was diamagnetic.

1. When the crystal was suspended so that the plane of its two optic axes could revolve vertically around its middle line, on approximating the apices of the poles as much as possible, these planes, in consequence of the diamagnetism, became placed axially; but overcoming the diamagnetism, they rotated 90° and became equatorial when the crystal was elevated or lowered.

2. On suspending the crystal, so that the planes of its two optic axes could oscillate horizontally, the middle line, in the form of crystal above described, again assumed an axial position; and when elevated or lowered, overcoming the diamagnetism, it became equatorial.

33. In the experiments described in the last paragraph, the topaz may be replaced by crystallized *sugar*. Thus, those planes in the direction of which a crystal of sugar is most easily cleft, are perpendicular to one of its two optic axes; and when one of these surfaces of transmission is polished (to effect which, mere scraping with a piece of glass is sufficient) and placed between two crossed plates of tourmaline, the plane of the two optic axes is recognised even by the position of the black band seen through the system of rings; and we then know, from the known angle of about 50° which these axes form with each other, the direction of the second axis and the middle line between them.

34. Crystals of Brazilian *topaz, arragonite, nitre, sulphate of soda*, and of many other substances, which exhibited the phenomena of diamagnetism and crystallized in different prismatic forms, when placed between the apices of the poles, as also at every distance from them, arranged themselves with their longitudinal and prismatic axis, which (although not in every instance,

yet on the average) is at the same time the central line between their optic axes, equatorial. These phænomena, owing to the diamagnetic action, of themselves prove nothing; only it was requisite that they should not furnish a different result. But an experiment made with a crystal of *staurolite*, which was magnetic and became axially placed when in the line of the apices of the poles, but equatorially when raised or lowered, was conclusive. This experiment was also performed, even before that on the topaz, with the most decided result; but subsequently, to my astonishment, with somewhat doubtful effect; and at the conclusion of these experimental investigations it was repeated in a new point of view, so that I shall return to it more fully in a subsequent paragraph (40).

A crystal of *lepidolite* was so strongly magnetic, that when raised it could not be made to rotate; just so with a beautifully perfect crystal of *hornblende* (a thick hexagonal prism with acute extremities). The former did not exhibit the least polarity, whilst in the latter it was very decided.

35. It must moreover be remarked, that all biaxial as well as all uniaxial crystals, which in the above experiments assumed an axial or equatorial position of a certain dimension, were rotated 180°, and retained in this position by opening and closing the circuit.

36. The phænomena detailed in paragraphs 30 and 35 are perfectly explained by the assumption, that a repulsive force is exerted upon both the optic axes by the poles of the magnet, and then, in accordance with the second experiment upon the topaz (33), we must add to this supposition, that the repulsion exerted upon both the optic axes is of equal intensity. The two experiments with mica (30, 31), as also the first with topaz (32), do not allow of the assumption of a repulsive force exerted merely upon the central line, instead of upon both axes.

In the second experiment with the plate of mica, the repulsion exerted upon the axes overcame the *magnetic*, and in the case of the experiment with the topaz and sugar the *diamagnetic* force, which tended each time to bring the crystal, in conformity with its form, into a position differing by 90°.

37. Two other experiments, suggested by a theoretical combination, were performed, which are conclusive as regards biaxial crystals, and deserve special attention. However, before describing them, for the purpose of facilitating the survey of the

phenomena concerning biaxial crystals as produced by diamagnetism and magnetism, the position of the two axes, the external form of the crystal and the direction of the suspension of these axes, I shall premise the following general considerations.

In a biaxial crystal three principal positions of suspension may be distinguished in three directions at right angles to each other.

1. In a direction Z which is perpendicular to the plane of the two optic axes.

2. In the direction of the central line X between the two optic axes.

3. In the direction Y which is in the plane of the optic axes at right angles to their central line.

In the first position, the plane XY of the two optic axes oscillates horizontally, and in consequence of the repulsion of the axes the central line X takes an equatorial position.

This action,

a, when the crystal is magnetic and its dimension according to the middle line X is less than that at right angles to it in the direction Y;

b, when it is diamagnetic and its dimension in the direction Y is greater than that in the direction X,

is in the one case increased by the magnetic attraction, in the other by the diamagnetic repulsion of the mass. When, on the other hand,

* In the text we have implied that the inclination of the axes to each other is not great. Strictly speaking [illegible] crystals when suspended at right angles [illegible] produces a double position of equilibrium [illegible] we admit that the single axis [illegible] in the horizontal plane on each side [illegible] becomes the middle line X the unstable position [illegible] but its stability is at first slight [illegible] opened, and when these axes form with each [illegible] lity of the two positions of equilibrium [illegible] suspended in the direction X or Y [illegible] plate of iron the two optic axes of which [illegible] such an external form that the magnetic attraction [illegible] would present two equally stable positions of equilibrium [illegible] with a thin circular (diamagnetic) plate of [illegible] diameter in the plane of which the [illegible] only with difficulty return it in the [illegible] stable position of equilibrium [illegible]. The experiment must be repeated with a larger plate or with the poles more pointed [illegible] at the same time it must be [illegible] consequence of the form the diamagnetic action [illegible] vanished it diminishes the stability of the equilibrium.

c, the crystal is magnetic, and its dimension in the direction Y is greater than that in the direction X;

d, the crystal is diamagnetic, and its dimension in the direction Y is less than that in the direction X;

the magnetic attraction and diamagnetic repulsion of the mass must previously be overcome before the repulsion of the axes can be apparent.

In the second and third normal positions of suspension, the plane X Y of the two optic axes oscillates, so that it each time constantly remains vertical, and each time it is forced by the repulsion of the axes into the equatorial position. In the second position of suspension, the direction X, in the third the middle line Y, oscillate horizontally and become equatorial. The axial action is increased by the magnetic attraction and diamagnetic repulsion of the mass, when

a, the crystal is magnetic, and its dimension X in the second case and Y in the third case are less than Z;

b, the crystal is diamagnetic, and its dimensions X and Y are greater than Z These forces must moreover be overcome before the axial repulsion can be perceived, when

c, the crystal is magnetic, and its dimensions X or Y are greater than Z,

d, the crystal is diamagnetic, and its dimensions X or Y are less than Z

38 Arragonite crystallizes in right rhombic prisms, which by truncation of the sides usually become six-sided. From one of these crystals, which was perfectly transparent, I had a piece ground at right angles to the axis and polished, and then, to ensure perfect certainty, determined the position of the two axes by viewing them by polarized light. These axes, as is well known, form with each other an angle of full 18°; and the middle line, subdividing this angle, coincides with the axis of the prism. The planes of the two optic axes were perpendicular to those two parallel lateral surfaces, the distance of which from each other was extremely minute, and was 10 millim. The height of the prism was 12·5 millim., and the largest diagonal of its terminal facets, which was at right angles to the plane of the two optic axes, was 22 millim. The directions in which these three dimensions were taken, coincide respectively with the directions which we indicated in the last paragraph by X,

Y and Z, that is, still indicating the corresponding dimension by the same sign

$$Z > Y > X$$

The substance of arragonite is strongly diamagnetic. When suspended in the direction Z, the crystal assumed such a position that Y became equatorial, and as Y > X, this action was increased by the diamagnetic action exerted upon the mass of the crystal.

In consequence of the diamagnetic action, when the crystal is suspended so that it can oscillate with its longitudinal direction Z horizontal, it rotates so that this direction becomes placed at right angles to the line of the apices of the poles, and the force with which this is effected is evidently (neglecting magnitudes, which cannot here be taken into account) *of the same intensity in whatever direction of the plane* X Y *the crystal is suspended*. However, in the two ordinary positions of suspension in X and Y, if we admit the existence of a repulsive force, exerted by the two poles upon the two axial directions, the action exerted by it is very different as regards intensity. This can in each case consist of a single rotation only of the two axial directions around the alternate line of suspension, whence each optic axis passing through any one point of the crystal describes a one sheeted hyperboloid of rotation or when it especially cuts the line of suspension, a conical surface. The moment of rotation, however, on suspension in Y, is less than on suspension in X, and, in fact, the more so the more acute the angle which the two optic axes form with each other, so that with this angle (if the crystal be uniaxial) the former moment of rotation completely vanishes. Hence it follows that when, as in the experiments with topaz and sugar (32, 33), the axial action, on removing the apices of the poles, overcomes the diamagnetic action, both of which (because Z > X and Z > Y) act in opposite directions, this must ensue *later* when the suspension is in the direction of the middle line Y than when in that of X.

On approximating the apices of the poles as much as possible, the crystal each time became placed like a diamagnetic body with the longitudinal direction Z equatorial, and on shortening the silk thread, rotated on each occasion 90°. *When the crystal was suspended in the direction of the middle line* Y, *this took place when the crystal was raised about* 40 *millim. above the line*

of the apices of the poles; but when the suspension was in X, *it occurred after an elevation of* 11 *millim. only.*

39. A binaxial crystal of the prismatic form, and the substance of which is magnetic, when suspended freely between the apices of the poles, must necessarily be differently acted upon from a uniaxial crystal of about the same external form. Thus the force with which a uniaxial crystal is moved into the equatorial position, remains exactly the same in whatever way it may be suspended, provided that its longitudinal direction can oscillate in the horizontal plane. This was confirmed by direct experiment, on three times repeating with the same tourmaline the experiment described in paragraph 8, so that, the apices of the poles remaining undisturbed, the tourmaline was suspended in three different directions, lying in a plane at right angles to the axis of the prism. Each time an elevation of exactly 24 millim. was requisite to turn the tourmaline round 90°, so that it should assume an equatorial position.

This *cannot* be the case in a binaxial crystal, whatever may be the position of the two optic axes. We will suppose in such a crystal that the middle line Y coincides with the axis of the prism. Whatever may then be the direction in the plane X Z in which we suspend the crystal, the magnetic action always fixes it with the same force in the axial position, whilst the repulsive force exerted by the poles against the two axes tends to place it equatorially but with different force. Thus when the crystal is suspended in the direction Z, and hence the two optic axes can oscillate horizontally, the forces from each of the poles of the magnet acting upon the two axes tend to produce rotations in the reverse direction. However, when the crystal is suspended in the direction of the axis X, and thus the planes of the two axes constantly remain vertical during the oscillation of the crystal, the forces emanating from each pole of the magnet accumulate, and the resulting force is obviously greater than in the previous method of suspension. Moreover, it is clear that the moment of rotation increases in proportion as the direction of the silk thread is removed from the direction Z and approximates the direction X.

40. The above considerations led me to the experiment with the staurolite mentioned in paragraph 34; and I had no doubt that what had previously embarrassed me as being an inex-

plicable anomaly, would now yield a beautiful confirmation of my theoretical views.

The crystal of staurolite was transparent and strongly magnetic. It formed a prism 18 millim. in length, the transverse section of which was an irregular hexagon, A, B, C, D, E, F. The opposite parallel lateral surfaces were about the same distance apart; this was 6 millim. The apices of the poles were approximated as much as possible, and in all the experiments retained undisturbed in the same position. The crystal was suspended, *at first* perpendicularly to the lateral surfaces A, F and C, D, *secondly* perpendicularly to the lateral surfaces A, B and D, E. In both cases an elevation of exactly 25 millim. above the line of apices of the poles was requisite to cause the crystal to become equatorial. The crystal was then, *thirdly*, suspended at right angles to the lateral surfaces B, C and E, F, and the crystal could now no longer be rotated 90° with the strength of the current used; when raised 100 millim., it continued to maintain at least its axial position. Two other suspensions were then made, the *fourth* in the direction of the line of subdivision of the angle at A, which was at right angles to the line of the third suspension, and the *fifth* in the direction of the line of subdivision of the angle at B. In the fourth suspension, the crystal, when raised 23 millim., was rotated 55 millim.; in the fifth, at an elevation of 50 millim., to the same point. Thus the greatest moment of rotation produced by the repulsion of the optic axes obtained in the fourth, and the least in the fifth position of suspension. We thus draw the conclusion, that the general direction denoted by X subdivides the angle at A, whilst the direction Z is perpendicular to the lateral surfaces B, C and E, F.

The angles of the prism were next measured, and the angles at A and D found to be about 129°. But the primary form, as is well known, is a right rhombic prism, in which the obtuse angles are equal to the one measured. The plane of the optic axes thus passes through the two obtuse angles of the primary form. The two surfaces B, C and D, F, by which the two acute angles are truncated, are parallel to the plane of the two optic axes.

The above supposition, that the axis of the prism is the middle line between the two optic axes, might require confirmation. For this purpose I suspended the prism of staurolite longitudinally; it became placed so that the plane passing through the

two obtuse angles was equatorial. After this experiment, there can be no further doubt as regards my supposition.

41. The method of observation used in the last paragraph may be altered, by placing the prism of staurolite in such a position, that instead of raising it in the different horizontal directions of suspension above the line of the apices of the poles, it is made to oscillate around the position of equilibrium in the line of the poles, and the axial effect estimated by the different duration of the oscillations. To effect this, I completely removed the apices of the poles; the prism then became placed axially in the third position of suspension, and equatorially in the fourth. I then again inserted the two apices, and moved them forward until even in this position of suspension the equatorial position became axial. As the magnetic force, when the apices are at the same distance apart, remains constant in the different suspensions, the position of the prism is in each case determined by a force which is equal to this constant force *minus* the variable force acting upon the axes. The proportion of this latter force in different suspensions may be ascertained in this way. The more the direction of suspension approximates to the direction above denoted by X, the slower the crystal oscillates.

This method of observation, where the object is merely to obtain a general view, is less convenient, because it requires greater care, it possesses however the advantage of a more extensive application; it may be used even when the crystal contains so much iron that the axial action cannot overcome the magnetic attraction, as was found to be the case in a crystal of lepidolite. It is also applicable when the crystal is diamagnetic, and even in consequence of its form assumes an equatorial position. A crystal of topaz affords an appropriate example. *In* this case the diamagnetic repulsion and the action upon the axes *combine* to produce the action observed.

42. The last paragraphs contain the first example of the manner in which the optic axes of a crystal may be determined by means of a magnet; and, what must appear surprising, the crystal may be opake, and every trace of crystalline form have disappeared.

In the same way we can obtain an answer to the question, whether a solid, transparent or opake, uniaxial or biaxial crystalline mass, consists of elementary crystals (if I may be permitted to use this un-mineralogical expression), in which an

axial direction predominates, or where this is not the case, as, for instance, the optical condition of melted sugar, which shortly after solidification, comes under the latter, but after the lapse of some time, under the former category.

43. When a sphere (or even a rotating cylinder with its axis vertical) is suspended between the two apices of the poles, we do not find any rotation occur, whether the mass be magnetic or diamagnetic. The *only* action which can come into play in this case, is the repulsion of the axes. If the sphere is made from a uniaxal crystal, each time it is suspended the axis becomes equatorially arranged. However, before it becomes fixed in this position, it makes oscillations about it, the rapidity of which is greater in proportion to the difference of the direction of suspension from the direction of the axis. A preliminary experiment with a sphere of rock crystal 57 millim. in diameter, has shown that we may in this case obtain accurate admeasurements*.

If in any two different positions of suspension, we mark on the sphere the equatorial plane in each case, the section of the two planes determines the axial direction of the crystal.

In the last determination the external form is of no consequence, provided we have convinced ourselves that in the mass under examination the axial direction has overcome its magnetism or diamagnetism.

Lastly, when the mass is made from a biaxial crystal, the middle line between the two axes in this determination assumes the place of the single axis.

44. Faraday has already observed the modifications which occur when a body is suspended between two *surfaces* of the poles instead of the apices. As regards diamagnetic bodies, the following experiment is characteristic.

I placed upon each pole a parallelopipedal keeper 189 millim. in length, so that the surfaces of the poles, which formed a right angle 67 millim. broad and 27 millim. in height, were directly opposite each other at such a distance that a cylinder of bismuth 34 millim. long and 6 millim. thick could oscillate freely between them. This cylinder of bismuth being thus suspended so that

* I may remark here that some *other* crystalline mass is best used for these determinations; for in rock crystal the axial action was feeble beyond expectation, and the phænomena in several experiments were but slightly apparent; less so than in any of the other experiments.

its centre of gravity was in the horizontal middle line between the two surfaces of the poles, arranged itself *equatorially* as long as the centre of gravity was within the two planes of the lateral surfaces of the keeper. As soon as it passed beyond one of these two planes, *two* stable positions of equilibrium were apparent, one in the direction of the middle line (axial), the other perpendicular to it (equatorial); when the centre of gravity was moved further away, the second only of these two stable positions of equilibrium remained. All these phænomena are perfectly explicable on the assumption of a non-polar repulsion of the mass of bismuth by the poles of the electro-magnet.

45. These phænomena, as also the position which a magnetic mass between the surfaces of the poles assumes, must be borne in mind when we allow a crystal to oscillate between the surfaces of the poles, and we wish to determine *à priori* the phænomena about to result. The prism of tourmaline mentioned in paragraph 8, when suspended in the same manner as the cylinder of bismuth, became placed in the centre *axially*, in the planes of the lateral surfaces of the keeper *equatorially*, and further away again *axially*. The large plate of calcareous spar (paragraph 20), in all positions of suspension, arranged itself with its axis towards the middle line, overcoming the diamagnetism of the mass within the two planes of the lateral surfaces of the keeper, but beyond them supported by the diamagnetism.

46. There could hardly be any necessity for confirming the fact, that the electro-magnet acts in the above experiments exactly in the same manner as a *permanent magnet*, but it was of interest to determine whether the latter possessed power enough to render the repulsive force of the axes visible.

At my request, M. vom Kolke, who with great talent and patience acted the part of my assistant during all the experimental investigations, repeated the experiments described in paragraphs 4 to 6, first with the magnet of Ettinghausen's magneto-electric rotation apparatus, and subsequently with a small horse-shoe magnet, which sustained barely a kilogramme at each pole. To approximate the poles, he placed at each end of the magnet, lying horizontally, a thick iron rod, the extremities of which were situated at a proper distance, so as to allow the plate of tourmaline to oscillate between them in the air. The result was perfectly decisive.

47. The experiments detailed in the present memoir are, in

my opinion sufficient to establish the general law laid down in the first paragraph and the existence of a new force which had not hitherto been indicated by any phænomenon. The relation of the new results obtained by their means to the two discoveries of Faraday, which form epochs in science, is too intimate to allow of my passing it over unnoticed.

It appears to me that philosophers do not yet attribute to one of these discoveries, viz. that *all* bodies without exception are either magnetic or diamagnetic, that importance which it really possesses. Faraday has not only observed and described individual phænomena, like others before him, who could only have been acquainted with half of them, when they designated them as transversal magnetism; but he has announced a *general law*, to which I without hesitation subscribe; and he has pointed out an entirely new action of the magnet in diamagnetism, whereby the nature of magnetic attraction, which is already essentially so enigmatical, is rendered still more so. I have made many but unsuccessful experiments to discover a diamagnetic polarity, or a test for matter in a state of diamagnetic excitation. The simplest hypothesis at present appears to me, especially if we wish to retain the established ideas regarding magnetic distribution, to be that in which diamagnetism is regarded as a general repulsive force of matter.

As regards Faraday's other discovery, I agree with the common view, that in the observed rotation of the plane of polarization there is no direct action of the magnet upon the light, but that this is primarily produced by a magnetic or diamagnetic action upon the ultimate particles of the mass, of which we have permanent instances in many bodies occurring in nature, but among crystals in rock crystal alone, and this only in the direction of the axis.

48. My experiments have shown that a peculiar action is exerted by the poles of a magnet upon every uni- or biaxial crystal, for which we find an explanation when we suppose the resulting action to be a repulsion of the axial directions, which is independent of the magnetic and diamagnetic property of the matter. This repulsion is evidently connected with the form of the ultimate particles of the crystal, and appears to come into play where the magnetism is not in a condition to produce a transient molecular change, the result of which is the rotation of the plane of polarization discovered by Faraday.

Can we admit that the new repulsive force is a modification of diamagnetism, produced by the form of the ultimate particles of matter? It would then be remarkable, that this force is so strong, that, when the crystal is suspended in a certain position, it is capable of overcoming the originally much stronger magnetic or diamagnetic directive force, when the poles of the magnet are further removed apart. The experiments proved that the new force diminishes less in proportion to the increase of distance than this directive force.

At all events the forms of the ultimate particles of matter and the magnetic forces stand in mutual relation, which has brought us to the remarkable result, that we can determine crystalline forms by a magnet. Moreover, this renders the existence of a relation between those forces which are in action during crystallization and the magnetic forces extremely probable. The most important point of view however is evidently this, that the directions, the repulsion of which results from the new exertion of force, are the very ones which stand in peculiar and exclusive relation to light, in which it does not suffer double refraction when transmitted through it. This relation will not remain long thus isolated.

Article IX.

On the Relation of Magnetism to Diamagnetism.[*] *By* M. Plucker, *Professor of Natural Philosophy in the University of Bonn.*

[From Poggendorff's *Annalen*, October 1847.]

1. Faraday has completely disproved the view laid down by other philosophers, viz. *that diamagnetism is merely another manifestation of ordinary magnetism*, by the single fact, that whilst a magnetic body (as instanced in iron) is attracted throughout its mass by each of the two poles of a magnet, a diamagnetic body is repelled by each pole throughout its entire mass.

2. Hence the simplest supposition would be that, *in which the magnetic and diamagnetic forces called into action opposite conditions of matter neutralizing each other*: a supposition which at a glance is seen to be supported by the phenomenon constantly observed by Faraday, viz. that on mixing a magnetic and a diamagnetic substance an intermediate condition is produced, which depends upon the proportions of the mixture: we need then only consider that the magnetic forces in ordinary cases are incomparably stronger than the diamagnetic. Every diamagnetic body, when gradually mixed with a comparatively small quantity of a substance containing iron, appears at first less and less diamagnetic, and soon becomes magnetic. On the other hand, it is only in the case of very feebly magnetic substances that we can succeed in converting the magnetic behaviour of a more considerable mass into the diamagnetic by the admixture of a diamagnetic substance in not excessive quantity. We must here however suppose that by such an admixture the action of the magnetism is diminished, and that in a greater degree than if the substance added acted like an indifferent inactive mass.

3. But what is even more opposed than anything to the supposition in the last paragraph, is the fact, that whilst an iron rod, magnetically excited between the poles of a magnet, exhibits polarity at its extremities: hitherto, notwithstanding all attempts, no trace of polarity has been detected in a substance diamag-

* Translated by Dr. J. W. Griffith.

netically excited between the poles of a magnet. However, every idea of the tenability of the view in question must necessarily be given up in consequence of the experiment which I shall next detail.

4. Even in my earliest experiments on the magnetic or diamagnetic state of different vegetable and animal structures, in which especially very small masses were examined with the nearest possible proximity of the poles, I often found what appeared to me as anomalies, that these bodies, although placed so near one of the poles as to touch it, were repelled by it, yet they arranged themselves between the poles like a magnetic body. The wings of the cockchafer especially, which arranged themselves magnetically between the poles, *i. e.* with their longitudinal direction from one pole to the other, when placed with their broad surfaces next one of the poles, were decidedly repelled by it, like a diamagnetic body. This was an anomaly the explanation of which I reserved for future experiments, because at this time the experiments on the action of the poles of the magnet upon the optic axes had temporarily engaged my whole attention. These experiments, to which the former treatise has been devoted, were originated by the inquiry, as to what was the cause of the magnetism of certain vegetable structures, and whether the direction of the fibres did not perhaps exert some influence upon the position which vegetable structures assumed when suspended by a silkworm thread between the poles of a magnet. On resuming this question, I placed the barks of several trees, all of which were magnetic, so as to oscillate; and on doing so, especially with a piece of the bark of the cherry-tree of a rectangular form, about 15 millim. long and half this breadth, I obtained the unexpected result, that when this was suspended so that whilst its longitudinal direction oscillated horizontally between the two apices of the poles, which were approximated as much as possible, it could still move freely, *it placed itself equatorially like a diamagnetic body*; but when the poles were removed further apart, or when the bark was raised above or lowered below the line of the poles, *it became axial, like a magnetic body*. It is evident that in this experiment, which I repeated with different pieces of cherry-tree bark of different sizes and with the fibres variously arranged, *there were two distinct forces in a constant state of activity, and that one, the magnetic, diminished less in proportion to the increase of the distance than the other, the diamagnetic.*

5. It appeared to me to be hazardous to form general conclusions upon magnetism and diamagnetism from the experiment detailed in the last paragraph, which I had made even before writing my former treatise, inasmuch as it was to be feared that, with the complicated structure and chemical properties of the substance used, some unknown extraneous cause might have produced the phænomenon observed. Further experiments must decide whether *all* substances, which at a certain distance from the pole become placed (with slight force) magnetically, when moved nearer to the pole react diamagnetically; and the next problem was, to find more simple substances which were magnetic in the slightest possible degree. For this purpose, I took tinfoil, which was magnetic (probably from its containing iron), and fused it with some bismuth. With the proper proportions in this alloy (more bismuth than tin), which I poured out upon paper into a thin bar of about 15 millim. in length, I obtained my object. The bar was affected in exactly the same manner as the piece of cherry-tree bark, *i. e. it became axial or equatorial, according as the apices of the poles were more or less distant from each other*. From this I draw the conclusion, that it forms a general law, and not merely an isolated phænomenon. A similar one has already been observed by De la Rive in charcoal*. I may remark here, that whilst Faraday found charcoal magnetic, I found common wood charcoal, as also box-wood charcoal prepared for electrical experiments, diamagnetic, which cannot surprise us, because the smallest quantity of iron mixed with it, which may even arise from the body, must make charcoal magnetic, and thus, among the specimens of charcoal examined by Faraday and myself, there would be some which are affected in the same manner as the above alloy.

6. I therefore adhere to the following hypothesis, which at present has the best foundation, *that the magnetic and dia-*

* The part of the *Bibliothèque Universelle* for June, p. 171, contains my first notice upon the relation of magnetism to the optic axes, as published in the *Comptes Rendus*, from a letter to M. Arago dated June 11th, to which the following note is appended by M. De la Rive:—

Like M. Plücker, I have made a great many experiments upon the action of the magnet upon different bodies. I shall mention one here which has yielded a similar result to those obtained by M. Plücker; it relates to the action of the electro-magnet upon charcoal, a substance which I have sometimes found magnetic, sometimes diamagnetic, according to its molecular state, and sometimes also according to its distance from the poles of the magnet.

It was in consequence of this note that in the present short essay I treat of an object which it was my intention to discuss subsequently in connexion with some others.

magnetic forces coexist simultaneously, and that, because the first of these forces diminishes less in proportion to the increase of distance from the poles of the magnet than the last, the same body may react, according to circumstances, at one time like a magnetic, at another like a diamagnetic body.

Several questions, which are important in a theoretical point of view, are connected with the above law, and lay open new paths to us

First, a conclusion drawn by Faraday from his observations is overturned, and, on the other hand, it is proved that it is impossible by the mixture of substances, the reactions of which are of the opposite kind, to procure one which is *indifferent* as regards magnetism and diamagnetism.

It moreover appears, from the results which have been obtained, necessarily to follow that the same body, *e. g.* of a spherical form, at a greater or less distance from one of the poles of a magnet, throughout its whole mass, may be at one time repelled, at another attracted; moreover, that a smaller and a larger sphere, composed of the same substance, and each time placed near one of the poles of the magnet, may be respectively repelled and attracted.

7. The answer to the following questions would be more unsatisfactory.

Can the reaction of any diamagnetic body, when the power of the magnet is increased, be converted into a magnetic reaction by the augmentation of the distance? If so, *at a certain distance no diamagnetic body in Faraday's sense would exist.* Is it not therefore probable, that if, by a more delicate method of suspension it could be so managed that all bodies assumed a direction in consequence of terrestrial magnetism, as they now do under the influence of a moderately-powerful magnet in one or the other manner, this direction would always be that only of a magnetic body? On the other hand, to what extent are we capable of diminishing the magnetic action or converting it into the diamagnetic, even in strongly magnetic substances, by approximating the central point to the action of the extremities of the poles as much as possible, and using the substance in small fragments?

Is a substance, which at one time reacts like a magnetic, at another like a diamagnetic body, necessarily a mixture of magnetic and diamagnetic substances? Or, what still appears to me probable in accordance with my theoretical view, may not a

simple body react in this manner? I consider that a direct answer to this question by means of experiment is at present impossible, because we could not ensure certainty that those simple bodies which exhibit slight magnetic and diamagnetic properties, according to Faraday, are really chemically pure

8 When we conjoin the observations described in the present treatise with those which I have detailed in the previous memoir, it results, that of the threefold action emanating from the poles of a magnet, viz

1st, the magnetic action in a strict sense,

2nd, the diamagnetic action discovered by Faraday,

3rd, the action exerted upon the optic axes of the crystals (and that producing the rotation of the plane of polarization, which probably corresponds to it)†,

the second diminishes more with the distance than the first, and the first more than the third

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9 As I was looking through the above memoir, it occurred to me (in which case the conclusion made at the end of paragraph 5 would retain its general correctness) that Faraday might have found charcoal magnetic, and I diamagnetic, because I placed it so as to oscillate at a less, and he at a greater distance from the poles Moreover, it appeared to me desirable to confirm the general result in paragraph 6 by new experiments I therefore made the following experiments, in which I again proceeded in the manner described in the former memoir, but using ten feebly excited Grove's elements instead of the former five

10 I at first found my supposition completely confirmed I examined four different pieces of charcoal successively, all of which were acted upon in exactly the same way, and, according as the distance of the extremities of the poles was more or less, arranged themselves magnetically or diamagnetically I shall only detail one experiment One of these pieces of charcoal (ordinary wood charcoal) was cylindrical, about 14 millim long and 6 millim thick When the poles were 17 millim apart, it arranged itself equatorially, but when raised 24 millim above

† The apparently equal illumination of the *entire* field by respectively equal intensities of colour after the rotation of the planes of polarization in Faraday's experiments proves that the action observed here does not diminish very rapidly with the distance

the line of the apices of the poles, the equatorial position was exchanged for the axial, in which it was distinctly retained even at an elevation of 54 millim. Again, when the two apices of the poles were separated 55 millim. from each other, on suspension in the centre between them, it became axial; but when suspended at a third of the distance, it assumed an equatorial position. The latter observation, that the same body, the apices of the poles being the same distance apart, but in different parts of this distance, reacted at one time like a magnetic, at another like a diamagnetic body, might have been anticipated from our view.

11. A piece of dry apple-tree wood and two pieces of deal, cut in different directions, and placed between the apices of the poles approximated to 17 millim., were more strongly diamagnetic than the charcoal; but, when raised, arranged themselves distinctly, but slightly, like a magnetic body

A cylindrical piece of lump-sugar, 19 millim. in length and 8 millim. in thickness, exhibited the transition from the equatorial to the axial position perfectly.

12. A fresh last year's shoot of an almond-tree, 15 millim. in length, the apices of the poles being 16 millim. apart, was diamagnetic, and remained so at all elevations; its entire bark was also diamagnetic, but at an elevation of 24 millim. rotated into the magnetic position.

A last year's shoot of a cypress-tree, 16 millim. in length, was diamagnetic at every elevation; the entire bark was the same. But the brown external bark alone was decidedly magnetic, as long as it could oscillate between the two apices of the poles*.

When the apices of the poles were approximated to 6 or 7 millim., and the piece of bark placed between them, it assumed a strongly diamagnetic position, and *indeed was ejected from the line of the apices of the poles*. When raised 4-5 millim., it again became magnetic.

13. In a hen's egg, magnetism only occurs in the white mem-

* A general result, at which I arrived in the commencement of my experimental investigations, but which I can only allude to here, is, that the outermost bark of all plants is *magnetic*. All those experiments which have been made without a knowledge of the results detailed in the present memoir, although the general conclusions are not deprived of their accuracy, must necessarily be indefinite and inaccurate as regards the details, and require repetition under the new point of view

brane which lines the shell internally. A piece of this membrane was also magnetic or diamagnetic between the poles according to the distance*.

* To obtain the above result with certainty, it is indispensable, especially if the body to be suspended is light, to avoid giving it the requisite form with instruments of iron (I always use glass), or taking it up with the fingers, if they have previously been in contact with iron and have not been subsequently washed. A piece of dry wood charcoal, after having been rasped with an iron file, was magnetic under all circumstances.

Article X.

Investigations on Radiant Heat. (Second Memoir.) By H. Knoblauch*.

[From Poggendorff's *Annalen*, vol. lxxi. part 1, April 29, 1847.]

V. *Comparison of the amount of Heat diffusely reflected by different bodies.*

It is well known that reflexion *occurring in all directions* (called diffuse) must be distinguished from that which takes place from reflecting surfaces *at a certain angle* only.

The latter, in reference to heat, has long formed the subject of numerous investigations, which have shown that the intensity of the reflected heat is dependent upon the nature of the reflecting bodies †, the condition of their surface ‡, as also the inclination of the rays incident upon these surfaces §, but that heat from different sources (in all bodies) undergoes this reflexion in the same manner || ¶.

* Translated by Dr. J. W. Griffith. The first Memoir will be found at p. 188 of the present volume.

† P. v. Musschenbroek, *Introd. ad Philos. Natur.*, 1762, vol. ii. (De Igne), p. 653. Leslie, An Experimental Inquiry into the Nature and Propagation of Heat, 1804, p. 98.

‡ P. v. Musschenbroek, *Introd. ad Philos. Natur.*, vol. ii. p. 654. Leslie, An Experimental Inquiry, &c., p. 99.

§ Forbes, Proceedings of the Royal Soc. of Edinb., March 18, 1839.

|| Leslie, *l. c.* Maycock, Nicholson's Journal, vol. xxvi. p. 75. H. Davy, Elem. of Chem. Phil., vol. i.

¶ Of course we only allude to *true reflexion*, which in the ordinary method of proceeding frequently cannot be directly observed. If, with different sources of heat, imperfect diathermanous substances be used for this purpose, *e. g.* a glass mirror, we should perceive differences on that side in the direction of which the reflexion occurs, accordingly as the incident rays emanate from one or other source of heat. But these differences would not arise from these rays being reflected by the reflecting surface with unequal intensity, but depend solely upon the unequal absorption which they experience on transmission through the glass, before they arrive at the posterior surface, the reflexion of which is in this case observed at the same time.

Experiments with diathermanous bodies can only lead to accurate results with respect to this point when, as Melloni and Biot have ingeniously done, in the effects observed, the influence of the absorption occurring on transmission is taken into account. They otherwise yield either indefinite results, as the experiments of Forbes, Edinb. Trans., vol. xiii., p. 302, and of Buff, Wöhler and Liebig's *Annal.*, vol. xxvii. p. 170, or merely confirm the phenomena of transmission, as *e. g.* certain observations of Leslie (An Experimental Inquiry, &c., p. 102–107), in which the amount of heat reflected by a coated metallic mirror

With this is connected the fact, that a number of dissimilar rays of heat are not altered by it in their properties, *e g* in their capability of passing through certain diathermanous bodies, as Melloni has shown in his experiments, he used well polished metallic mirrors for this purpose

Observations on the diffuse reflexion occurring upon rough surfaces were first made by Herschel and Leslie, they could not however lead to accurate results, because in them the heat emitted by these surfaces themselves was not separated from the reflected heat The diffusion was first accurately proved by Melloni, who protected the thermoscope from the rays of heat emitted by the reflecting body itself by a glass screen, whilst *e g* the heat of a flame diffusely reflected by a white plate, and which passed through the glass, exerted a perceptible action upon the instrument

As on reflexion at a definite angle, so also on diffuse reflexion, the intensity of the reflected heat of course varies according to the properties of the reflecting body and the structure of its surface, a result which is evident from the phænomena of absorption which have been already detailed, to which the phænomena of reflexion are complementary The magnitude of the angle of incidence of the rays which reach the diffusely reflecting surface, in this case exerts but very slight influence upon the intensity of the reflected rays An important distinction from simple reflexion consists in various kinds of heat being reflected in a different manner by one and the same body Melloni, to whom science is indebted for the great advances made in all these departments, discovered this phænomenon also He observed that a white surface reflected the heat of a Locatelli's lamp, according to whether it was used with or without a glass chimney, as also that of red hot platinum and copper heated to 752° F, with different degrees of intensity

Metallic plates only, the surfaces of which are rough, reflect heat from all sources in an equal degree, whilst lamp-black exhibits a scarcely perceptible amount of diffusion in any

It has not hitherto been determined *whether heat, on diffuse reflexion, experiences changes in its properties which distinguish it from that which is not reflected*

I therefore instituted a series of experiments on this point,

was found to diminish in proportion to the thickness of the layer applied, a phænomenon which simply depends upon the fact, that heat is absorbed to a greater extent by a diathermanous substance of greater thickness than by one of less

the chief results of which M. Magnus did me the honour of laying before the meeting of the Royal Academy of Berlin on the 29th of May 1845, a notice of which was taken from the Monthly Report of the Berlin Academy, and inserted in Poggendorff's *Annalen*, vol. lxv. p. 581–592. In the following memoir I shall give the details of this investigation.

It has been already mentioned, that of the two means which we possess of detecting differences in heat, that of radiation through diathermanous bodies is preferable to absorption, and at p. 227–230 instances are given of the great delicacy of this test-method.

I therefore adopted it in the present instance also, and examined "whether radiant heat permeates the same diathermanous media in dissimilar proportions, according as it is unreflected or diffusely reflected by different bodies."

Great differences were in fact found. Thus, when the heat of an Argand lamp radiated upon the pile, so that the needle of the multiplier was deflected 25°, it receded to 15°·19 when a plate of calcareous spar 3·7 millim. in thickness was inserted between the source of heat and the thermoscope. The 15°·19, as is known, arose from the heat transmitted by the calcareous spar. But when the heat of the lamp, diffusely reflected by a carmine-surface, had produced the deflection of 25°, the needle receded to 22°·31, when the same plate of calcareous spar was inserted at exactly the same spot between the reflecting surface and the thermal pile. The heat reflected by the carmine was therefore transmitted by the calcareous spar comparatively better than that unreflected. The same was the case with other diathermanous media.

The rays of heat diffusely reflected by a large number of bodies were compared, in the manner above described, both with that unreflected and with each other (as regards their transmission through diathermanous substances). However, before proceeding to the results of these experiments, I must premise some remarks upon the method of proceeding which was adopted.

To ensure the action of reflected heat alone upon the thermoscope, care must of course be taken to avoid heating the reflecting bodies. This was effected by employing them in the form of the lateral surfaces of metallic cubes which contained water of the temperature of surrounding bodies. Thus, those which were to be compared with each other were spread upon different cubes,

so that each of them was exposed to the rays of heat for as short a time as possible The following observations will show that the object was attained by this means —

1 When a surface has been heated (for the purpose of examining the heat reflected from it by three diathermanous bodies) by exposure to the rays of heat for four minutes, it exerts no perceptible action upon the thermoscope, for the needle returns almost immediately to its original position as soon as the source of heat is removed, the position of the surface itself being unaltered as regards the pile

The deflection produced by the direct radiation, and which was controlled before each new insertion of a diathermanous substance is therefore not perceptibly increased even in this space of time, which should occur if the heat of the reflecting body itself were added in a constantly increasing quantity to the reflected heat

Within the 1 5 to 2 hours which comprised a series of observations and during which time a cube was exposed to the rays of heat at the most four times the temperature of the water was not raised more than 0 5 R in any cube by the radiation However, as the heat acquired by the surfaces inclined towards the thermal pile merely produces a deflection of about 1 in the multiplier, the errors arising from this cause in the observations made after the insertion of the diathermanous bodies cannot exceed half degrees, within which the differences of the numbers subsequently given may be considered as accurate

2 On repeating the experiment several times, after the insertion of the diathermanous substances, the same deviation of the needle is constantly observed, hence the quantity of heat which passes through it is always the same If the deflection which ensues on direct radiation were produced on the repetition of the experiment partly by the heat of the reflecting surfaces themselves, the needle, on the insertion of the diathermanous media, should exhibit less deviation than before, because the heat radiated by these bodies is comparatively less perfectly transmitted by these substances than the reflected heat of an Argand lamp (which was used in these experiments)

Thus in five experiments a recess of 6° 25–6° 5 of the needle was found when red glass was used, 4 5 with blue glass, and 3° 5–3° 75 with alum, when the heat of the lamp was reflected by black velvet, and when the deflection produced before the

insertion was 13°. The repetitions of every three of these observations were made at intervals of five minutes only.

3. The radiation yields the same values when (as shown in subsequent experiments) one and the same reflecting substance is used in different degrees of roughness, although in this case unequal amounts of heat are absorbed and different amounts would be radiated, if the water contained in the cubes did not prevent this*.

I therefore believe, that by the above process *the absorption of heat by the reflecting surfaces is sufficiently diminished* to allow of the assumption, that it has not perceptibly interfered with the effects of the reflected heat.

The constant fundamental deflection, which must be produced by the heat reflected by the different bodies before the insertion of the diathermanous media between the reflecting surface and the thermoscope, might have been effected by approximating or removing the latter or the source of heat. But in both cases it was impossible to protect the thermal pile from all external influences, and especially from the immediate action of the source of heat during the reflexion. I therefore preferred producing this deflection by a measured withdrawal and inclination of the reflecting surface in regard to the instrument; but, even then, to be enabled really to judge of the changes which it has under-

* These relations do not exist when the cube is exposed to the rays of heat *without water*. The heat acquired by them then deflects the multiplier several degrees, and causes a diminution of the variations after *the insertion of the diathermanous bodies* in proportion to the extent of the share they have had in the constant direct deflection. This is evident from the following examples:—

Reflecting surfaces exposed to the rays of an Argand lamp.	On a metallic cube.	Deflection by the direct radiation of the reflected heat.	Deflection after the insertion of Red glass 1·8 millim.	Blue glass, 1·4 millim.	Alum, 1·4 millim.
Diesbach blue.	With water	13°	7·50	5·50	3·25
The same	Without water	.	5·25	3·75	1·75
Red velvet	With water.	13°	6·75	4·75	4·50
The same	Without water		4·75	3·50	3·25
White paper	With water	13°	18·50	15·50	14·00
The same .	Without water		16·50	14·00	12·00
Carmine .	With water .	13°	8·50	5·50	5·00
The same . ..	Without water		7·50	5·00	4·00
Black lac	With water	13°	10·75	8·25	6·50
The same . .	Without water	..	9·75	7·75	5·50
Metal . ..	With water..	26°	18·25	14·25	10·00
The same .. .	Without water	......	16·25	13·00	10·25

gone by the reflexion of various bodies, I was obliged previously to satisfy myself that even the altered position of the reflecting surfaces, as regards each other, did not produce a change in the transmission of the heat by the diathermanous substances used as tests.

Experiment showed that e. g. on inserting the red glass a constant recess of 15° in the needle occurred whether the direct deflection of 21° was produced by the diffuse reflexion of the

TABLE XXII.

Surface reflecting the rays of an Argand lamp.	Distance of the surface from the thermal pile in Rhenish inches.	Inclination of its normal to the longitudinal axis of the pile.	Deflection produced by the direct radiation of the reflected heat.	Deflection after transmission through Red glass 1.5 millim.	Deflection after transmission through Blue glass 1 millim.	Surface reflecting the rays of an Argand lamp.	Distance of the surface from the thermal pile in Rhenish inches.	Inclination of its normal to the longitudinal axis of the pile.	Deflection produced by the direct radiation of the reflected heat.	Deflection after transmission through Red glass 1.5 millim.	Deflection after transmission through Blue glass 1 millim.
White lead	8.00	1	21	15	11	Red wool	8.5	[illegible]	[illegible]	15.[illegible]	10.75
	9.00	5[illegible]		15	11		9.0	5		15.[illegible]	10.70
	9.75	58		1[illegible]	11		9.5	0		15.25	10.[illegible]0
	10.00	20		15	11		10.0	30		15.00	10.75
	10.50	30	[illegible]1	15	11		10.5	[illegible]5	[illegible]	1[illegible].25	10.75

It is hence evident that the transmission of the diffusely reflected heat by the diathermanous plates, within the limits of these observations, is perfectly *independent of the distance, inclination and size of the reflecting surfaces*, provided that before the insertion the same action is excited upon the instrument.

The position of the diathermanous substances as regards the thermal pile was the same as in the other experiments made with them, in which it has already been shown (pp. 203 and 204) that free radiant heat was the real agent concerned.

1. In the first series of experiments which I instituted to determine the heat diffusely reflected by different bodies, I used the *Argand lamp* which has been so frequently mentioned: this was kept at a constant level, had a double current of air and a cylindrical wick, and was used without the glass chimney. To allow of the reflecting bodies being accurately compared in different respects, they were divided into certain groups, which were never so far extended as to cause any fear of altering the conditions during the course of the experiments.

I first examined a number of *colouring matters*, and very

heat of an Argand lamp, by a surface of white lead, the centre of which was 8 inches distant from the thermoscope, and the normal of which was at an inclination of 4° to its longitudinal axis, or at a distance of 9·75 inches and an inclination of 58°. The same constant was found at every other distance and inclination, as also whatever was the size of the reflecting surface.

The experiments which were instituted on this point are contained in the following table:—

TABLE XXII.

Surface reflecting the rays of an Argand lamp	Distance of its centre from the thermal pile in Rhenish inches	Inclination of its normal to the longitudinal axis of the pile	Size of the reflecting surface	Deflection by the direct rays of the reflecting [illegible]	Deflection after the insertion of	
					Red glass [illegible]	Blue glass [illegible]
Tin, ground dull	8·5	5° or 55°	8 centim. square	21	13·00	10·50
	16·5	20°	8 centim. square		13·00	10·50
	11·0	15°	4 centim. square	18	12·00	9·00
	11·0	0°	5 centim. square		12·25	8·75
	10·0	20°	15 centim. 7 millim. sq.	18	13·00	9·00

marked differences were discovered in them. Thus, when the direct radiation of the Argand lamp (through a diaphragm) upon the thermoscope had deflected the needle of the thermo-multiplier 13°, on inserting the red glass it receded to 6°·59; but when a similar deflection of 13° was produced by the heat of the lamp diffusely reflected by vermilion, on inserting the same glass it placed itself at 7°·01; and when the heat reflected by carmine had produced this deflection, at 8°·33. Thus the reflected heat, of the same direct intensity, passed through red glass better than the unreflected, and that reflected by carmine in a greater degree than that reflected by vermilion. The same occurred with calcareous spar. Thus the unreflected rays of the Argand lamp which passed through the plate of calcareous spar to the pile, deflected the galvanometer-needle 15°·19, those reflected by vermilion 17°·81, and those reflected by carmine 22°·31, when the deviation of the needle before the insertion of the calcareous spar amounted in each of the three cases to 25°.

The following table exhibits the differences which occurred with other reflecting pigments and other diathermanous bodies:—

TABLE XXIII

I	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]				[illegible]	[illegible]
1 5	Red glass	13	6 [illegible] *	$\overline{7\ 83}$	8 33 *
1 1	Blue glass		5 17	5 67	5 5[illegible]
1 4	Alum		3 11	4 08	4 06 *
1 1	Rock salt	25	21 91 *	23 13	23 38 *
3 7	Calcareous spar		15 19 *	19 01	22 31 *
1 1	Gypsum		12 2 *	$\underline{15\ 60}$	18 2 *

(The numbers marked with an deserve notice on account of their difference

The above, and all subsequently mentioned reflecting surfaces, were 8 centim square in size The numbers, which refer to a direct deflection of 13 in these as in all other cases in which it is not expressly otherwise stated, are each arithmetic means of six observations, those relating to a deflection of 25 , of every four observations [*] The former were obtained by means of a multiplier, which M Schellbach was good enough to lend me the latter by my own, which has been described at pp 189, 190, the delicacy of which allowed me to produce greater deflections by the direct radiation of the reflected heat, without any fear of disturbance from the absorption of heat by the reflecting surface

I repeated each of the experiments six times merely because they were the first which I instituted with regard to this point and thus I convinced myself, with perfect certainty, of the correctness of my results and of the limits of their accuracy Subsequently, when I experimented after greater practice, and in summer under more favourable conditions of temperature, four repetitions appeared to me more than sufficient

The relative superiority of the transmission by radiation of the reflected heat in comparison with that unreflected, which was observed in all (Table XXIII) the instances detailed, completely lays aside any doubt which might remain regarding the origin

[*] Thus each of the former is the result of twelve separate readings from the multiplier since the direct deflection serving for comparison was controlled be f re each n w insertion of a diathermanous body and each of the latter is the re ult of eight

Table XXIII.

after the insertion when the heat of the Argand lamp is reflected by

Madder red	Red cinnabar	Paris green	Green cinnabar	Chrome yellow	Diesbach blue	Ultramarine
7 83	7 04 *	= 7 75	= 7 75	7 79	7 03	7 58
5 51	5 50	5 07	5 07	5 03	5 50	5 50
1 75	3 38	3 75	3 42	4 17	3 25	3 67
* 23 31	22 94	23 13	23 25	23 06	* 23 00	23 13
21 88	17 81 *	19 94	19 88	19 94	20 00	19 25
18 25	11 13 *	15 75 =	15 69 =	15 81	15 75	15 31

those series marked with an =, from their similarity.)

of the differences observed from the addition of the heat of the reflecting surfaces themselves to the rays from the original source of heat, for, as we know (see Table XV. pp. 225 and 226), the former are less perfectly transmitted by all the diathermanous media used than those of the Argand lamp. Thus, to give a single instance, the rays of heat emitted by a body below 234° F., which have produced a direct deflection of 25°, after the insertion of the calcareous spar, cause the needle to deviate 5°·69; those from an Argand lamp, exerting the same direct action, after the insertion of the same plate, deflect the needle 15°·19. Hence, if the former were added to the latter, so as when united with them to produce a deflection of 25°, a less deviation than 15° 19 should occur after the insertion of the calcareous spar. An increase to 22°·31, however, as we found *e. g.* with the carmine (Table XXIII.), would be utterly impossible.

It appeared to me interesting, with regard to the present question, to compare *the same substances, but of different colours*, with each other.

On so doing, it appeared that, *e. g.* the heat reflected by white and black satin, as also that by white and black taffeta, could not be distinguished by radiation through the above bodies; for the rays of heat emitted by all these surfaces, which passed through the red glass, deflected the needle of the multiplier 7° 54–7°·58, when their direct action produced an indication of 13°, and those passing through the calcareous spar caused a deviation of 17°·12–17°·50 in the needle when the deflection before

the insertion amounted to 25 The heat reflected by black and white velvet therefore radiates through the above substances in very unequal proportions for when the heat was reflected by the former, the needle, on the insertion of the red glass, moved from 13° to 8° 16, but when the reflexion was produced by black velvet, it receded from 13 to 6° 5 On inserting calca

TABLE XXIV

II [illegible]	[illegible]	D[illegible]	D[illegible] Argand lamp	D[illegible] White [illegible]	Black [illegible]	White [illegible]	Red [illegible]
1 5	Red glass	13	7 00 *	7 54	7 51	7 58	7 50
1 4	Blue glass		5 19	5 28	5 21	5 28	5 21
1 4	Alum		3 31 *	3 [illegible]	3 00	1 13	1 38
1 4	Rock salt	25	21 63	22 1	22 12	22 1	22 31
3 7	Calcareous spar		13 62 *	17 25	17 12	17 38	17 12
1 4	Gypsum		10 50 *	14 10	14 12	14 12	14 12

Corresponding results were obtained on the comparison of other substances of different colours Thus, the heat reflected by white paper, which had directly deflected the needle 13°, on the insertion of the red glass produced a deviation of 8° 29, that reflected by black paper a deflection of 6° 12, and on the insertion of the calcareous spar, the former caused the needle to deviate to 19° 81, the latter to 13° 38, when the direct deflection amounted to 25° If these values be compared with those representing the portion of the unreflected rays which passes through the red glass and the calcareous spar 7° 16 and 14° 56, it is evident that the transmission of the heat through these bodies is

reous spar, it receded in the first case from 25° to 19°·62, and in the second from 25° to 15°·5.

Similar differences were observed in other coloured surfaces composed of smooth silk and velvet, as shown by the annexed table, which also exhibits the relation of the transmission of the reflected heat to that of the unreflected[*]:—

TABLE XXIV

insertion when the heat of the Argand lamp is reflected by

Green taffeta	Black taffeta	White velvet	Dark red velvet	Light red velvet	Green velvet	Blue velvet	Black velvet
7·71	= 7 58	8 16 *	6 71	6 92	6 67	6 60	* 6 50
5 12	5 33	5 67	4 75	4 71	4·03	4 58	4 58
4 08	3 96	5 86 *	4 13	4 67 *	4 13	3 50	3 50 *
22 38	22 81	21 60	20 88	20 01	21 00	20 88	20 01
18 38	17 50	10 62 *	15 62	10 62	10 62	15 50	15 50 *
15 25	14 06 =	16 75 *	12 75	11 31	11 25	12 38	12 31

comparatively facilitated after reflexion by white paper, but impeded by that from black paper.

The same was observed with the other diathermanous media. The following table contains these differences, as also those which occurred on the reflexion of the heat by other substances of the same kind, of different colours:—

* That the numbers which give the transmission of the unreflected heat through the diathermanous bodies in the different tables are not the same, depends upon their having been observed on different days, and thus under altered conditions. It would have been useless to have reduced them, because the comparison of the reflecting surfaces is never extended beyond a single group, within the limits of which it is of full value.

TABLE XXV

III Thickness	Substance inserted	[illegible]	Deflection [illegible]	White paper	Blue paper	Black paper	Deflection [illegible]
1 5	Red glass	13	7 16 *	8 20 *	8 20	6 12 *	7 75 *
1 4	Blue glass		,01	[illegible] 25	4 91	4 38	4 92
1 4	Alum		3 71 *	4 02 *	4 81	3 17 *	3 07 *
4 4	Rock salt	25	22 10	22 75	22 68	21 06	22 18
3 7	Calcareous spar		11 ,6 *	10 81 *	10 81	13 18 *	11 18 *
1 4	Gypsum		12 00 *	16 62 *	16 62	10 88 *	11 81 *

Differences were also perceptible on reflexion by *bodies of the same colour*.

On comparing together a number of white surfaces, I found* *e g* that the heat diffusely reflected by ivory, the direct deflection of which produced a deviation of 13° in the needle after the insertion of the red glass, produced a deflection of 7°37 in the thermo multiplier, that reflected by white velvet 9°01 On inserting the calcareous spar, in the first instance, a recess of

TABLE XXVI

IV Thickness in millimetres	Substance inserted	[illegible]	Deflection [illegible]	Gypsum	Chalk	White lead	[illegible]
1 5	Red glass	13	7 63 *	9 00	9 00	9 00	9 21
1 4	Blue glass		5 79 *	6 58 *	6 50	6 54	6 16
1 4	Alum		4 38 *	5 71	75	5 71	5 71
4 4	Rock salt	25	22 25	23 06	23 06	23 13	23 00
3 7	Calcareous spar		14 91	20 19 *	20 25	20 25	18 18 *
1 4	Gypsum		11 75	16 87 *	16 85	16 75	15 12 *

I also made the same experiments with black surfaces as with the white It was then found, that *e g* the heat reflected by

Table XXV.

Deflection after the insertion when the heat of the Argand lamp is reflected by							Deflection after the insertion by the unreflected rays of the Argand lamp	Deflection after the insertion when the heat of the Argand lamp is reflected by		
Red woollen tapestry	Green woollen tapestry	Blue woollen tapestry	White wool	Red wool	White cloth	Black cloth		Yellow leather	Brown Spanish leather	Black Spanish leather
8 67 *	8 71	8 71	8 35	8 42	7 11	7 33 *	7 83	9 00	8 83	8 92
5 92	5 88	5 88	6 00	6 00	5 16	5 00	5 50	5 75	5 58	5 50
5 83 *	5 37	5 16	5 56	5 62	4 66	4 54	4 08	5 25	4 92	4 75
23 50	23·00	22 91	23 00	23 00	22 75	22 75	22 19	22 31	22 11	22 37
21 31 *	20 75	20 75	20 56	20 62	19 62	19 50	11 75	18 25	18 41 *	17 25 *
18 75 *	17 25	17 25	17 88	17 81	16 56	16 63 *	11 56	14 88	11 88 *	13 88 *

the needle from 25° to 17°·44 occurred; in the second, from 25° to 21° 31.

The following table contains the observations which were obtained on the transmission of the heat reflected by different white bodies through the diathermanous media used for testing them. On comparing it with the unreflected, it is evident that all except that which is reflected by silver having a "dead" polish can be distinguished by means of transmission from the direct:—

Table XXVI

after the insertion when the heat of the Argand lamp is reflected by										
White oil paint	Porcelain	White satin or taffeta	White velvet	White linen	White paper	White cotton	White wool	Mother of pearl	Ivory	Silver
9·01	9 00	9 01	9 01 *	9 21	9 20	9 21	9 00	8 33 *	7·87 *	7·58 *
6 21	6 58	6 50	6 13	6 46	6 46	6·50	6 40	6 13	5 96	5·83
5 54	5·67	5 46	6 67 *	5 71	5 75	5 83	6 58 *	6 13 *	5 83	4 38 *
23 00	22·94	23 13	23·00	22 56	22 50	23·13	23 13	22 56	21 91	22 25
20 19	20 31	19 50	21 31 *	20 31	20 25	20 41	20 81	19 06	17 14 *	14 80 *
16 88	16 81	16 81	18 69 *	16·88	16 88	16 91	18 31 *	17 56	15 81 *	11·59 *

black paper, which caused a direct deflection of 13° in the needle, on introducing the red glass, produced a deflection of 8°·03;

and that reflected by black lac, under the same circumstances, a deflection of 10°.64. That portion of the former which passed through the plate of calcareous spar returned the needle of the galvanometer at 14.75; the portion of the latter permeating it, at 20.38, when the direct radiation of the reflected heat had

TABLE XXVII.

V.							
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			
				[illegible]	[illegible]	[illegible]	[illegible]
1.5	Red glass	13	9.30	9.98	10.14	10.01 *	9.01
1.4	Blue glass		7.66	7.86	7.89	8.18	8.07 *
1.4	Alum		85	6.29	6.01	6.57	6.71
1.4	Rock salt	25	22.25	23.12	22.61	2 .88	22.01
3.7	Calcareous spar		16.75	20.06	20.14 *	20.38	19.81
1.4	Gypsum		14.00	16.00	16.88	16.12	16.10

Excepting black wood charcoal and brown coal, none of the surfaces of the varieties of coal used reflected sufficient heat to allow of their being experimented upon by transmission.

The diffusion is most feeble in lamp black and animal charcoal. This renders the changes which the heat suffers on reflexion from the surfaces first mentioned the more remarkable. Thus the rays reflected by brown coal pass through red glass, alum, calcareous spar and gypsum to a greater, those reflected by black vegetable charcoal through the same bodies to a less extent than the unreflected rays of the lamp. The following numbers correspond to the quantities of heat which were diffusely reflected upon the thermal pile at the same and the most favourable position with regard to the Argand lamp, by Indian ink, lamp black, animal charcoal, fossil coal, coke and graphite:—

TABLE XXVIII.

VI.							
[illegible]	[illegible]	[illegible]					
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8 centim. square	3°	6.00	2.80 *	1.90 *	7.80	6.10	7.05

† Black Spanish leather e / produced under the same circumstances (1

placed it at 25°. Only heat diffusely reflected by sheet iron passed through the diathermanous substances in the same manner as the unreflected heat, as may be seen from the subjoined table, which contains the details of this investigation:—

TABLE XXVII.

heat of the Argand lamp is reflected by						Deflection after the insertion by the unreflected rays of the Argand lamp	Deflection after the insertion when the heat of the Argand lamp is reflected by	
Black velvet	Black paper	Black cloth	Black Spanish leather	Black glass	Iron plate		Black wood charcoal	Brown coal
8·70	8 03	9 32	9 50	8 25	9 20	9 08	8 25	10 25
*	*				+	*	*	*
7 25	6 06	7·75	7 43	6 70	7 11	7 88	7 50	7·50
	*							
6 11	5 36	7 07	6 71	5 94	5·57	5 75	5·31	6 06
	*	*			*			
21 50	21 12	22 12	22 75	20 88	22 12	22 12	21·31	22 12
18 62	14 75	20 00	20 06	15 12	16 81	16 50	11 69	19 50
*	*				*	*	*	*
15 31	12 75	17 25	16 69	12 62	13 69	14 11	13 12	16 56
*	*	*			*	*	*	*

Thus black bodies, as regards luminous rays, observe the same relations as those which are coloured to heat

The rays of heat reflected by *certain homogeneous bodies* passed through the diathermanous media in an unaltered proportion. Thus, on inserting the red glass after a direct deflection of 13° had been produced by birch-wood, cork or mahogany, the needle became placed at 8°·08 to 8°·17, and on the insertion of the calcareous spar, at 18° 50 to 18°·62, by whichever of these three surfaces the heat was reflected, to deflect the needle 25°.

The same occurred with metals and alloys, in which the peculiarity was observed, that the heat diffusely reflected by their surface after it had received a "dead" polish, is undistinguishable on transmission, a result which is in conformity with the proposition laid down by Melloni, that rough metallic surfaces act towards heat in the same manner as white bodies upon light, *e. g.* on insertion of the red glass, the needle of the multiplier receded from 13° to 7°·91–7°·75, and on inserting the calcareous spar, from 25° to 15°·33–15°·08, when the rays of heat either directly reached the pile, or were diffusely reflected by gold,

platinum, mercury copper, lead brass or any other metal or alloy

The following table shows how great this agreement of the

TABLE XXIV

VII Thi k mill t	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	D fl
				Bl [illegible]	C [illegible]	M [illegible]		C [illegible]
1 5	Red glass	13	7 67	8 17	8 08	8 17	7 87	7 91
1 4	Blue glass		5 12	6 33	6 33	6 12	6 25	6 20
1 4	Alum		4 33	5 50	5 50	5 50	4 96	4 91
4 4	Rock salt	25	22 81	22 81	2 75	22 88	22 7	22 07
3 7	Calcareous spar		15 11	18 12	18 50	18 50	15 33	15 17
1 4	Gypsum		12 75	16 00	16 00	16 00	12 33	12 08
							19 direct	
1 5	Red glass	17	10 06	11 75	11 58	11 75	21 00	21 00
1 4	Blue glass		9 12	9 25	9 17	9 17	16 25	16 50
1 4	Alum		6 83	8 42	8 33	8 33	11 25	11 75
							40 direct	
4 4	Rock salt	30	27 00	27 00	27 2	26 90	30 0	30 7
3 7	Calcareous spar		18 00	21 25	21 00	21 25	25 75	25 50
1 4	Gypsum		11 25	18 00	18 00	18 25	19 75	19 75

The comparison of the heat reflected by some *perfectly heterogeneous surfaces* led to the same results as the experiments just detailed

Thus, the rays of heat reflected by green oil cloth, which passed through the red glass, produced a deflection of 7° 0, those reflected by white calico, which were transmitted through the same plate, a deflection of 8 5, when the needle had deviated 13° before the insertion and those passing through calcareous spar produced in the first instance a deflection of 18°, in the second of 20 81, when the thermoscope by the direct action of the reflected heat indicated 25°

The heat reflected by yellow marble and birch wood, however,

TABLE XXV

VIII Thickness millim	Substances [illegible]	[illegible]	[illegible]	[illegible]	
				B [illegible]	White [illegible]
1 5	Red glass	13	7 75	8 08	8 50 *
1 4	Blue glass		4 92	5 79	6 25 *
1 4	Alum		3 67 *	5 91	5 25
4 4	Rock salt	25	22 38	23 00	23 25
3 7	Calcareous spar		14 38	20 50	20 81 *

deflections was, in various surfaces of woods and metals, for all the diathermanous media used, and at greater deflections than those above mentioned —

TABLE XXIX

tion after the insertion when the heat of the Argand lamp is reflected by

Silver	Platinum	Mercury	Iron	Tin	Zinc	Copper	Lead	Alloy of lead and tin	Brass	German silver
7 87	7 79	7 83	7 79	7 79	7 75	7 83	7 79	7 79	7 79	7 87
6 33	6 21	6 25	6 29	6 25	6 21	6 29	6 21	6 21	6 25	6 29
4 91	4 91	4 91	4 91	4 96	4 88	4 96	4 88	4 88	4 88	4 96
22 75	22 67	22 58	22 58	22 58	22 58	22 67	22 75	22 58	22 58	22 67
15 25	15 08	15 25	15 25	15 08	15 08	15 17	15 08	15 17	15 25	15 17
12 17	12 25	12 17	12 25	12 25	12 08	12 08	12 08	12 17	12 33	12 17
21 00	21 00	21 00	21 25	21 00	20 75	21 00	21 00	21 00	20 75	21 00
16 50	16 25	16 25	16 50	16 25	16 25	16 25	16 25	16 25	16 50	16 25
11 75	11 25	11 25	11 25	11 25	11 00	11 25	11 00	11 25	11 50	11 25
36 25	36 50	36 25	36 75	36 50	36 50	36 50	36 25	36 50	36 75	36 50
25 50	25 50	25 50	25 50	25 50	25 25	25 25	25 25	25 50	25 75	25 25
19 25	19 50	19 50	19 50	19 75	19 50	19 50	19 25	19 50	19 75	19 75

radiated through the diathermanous media in exactly the same proportion Thus, on inserting the red glass, each deflected the needle of the galvanometer 8° 14 to 8° 17, the direct deflection being 13°, and on inserting the calcareous spar, 17° 62, when the direct deflection amounted to 25° They are however well distinguished by the heat diffusely reflected by a metallic surface, which, like that unreflected, on inserting the glass, caused the needle to recede from 13° to 7° 67, and on introducing the calcareous spar, from 25° to 14° 41

The subjoined table contains the details of the observations on these and some other reflecting surfaces, as found with the different diathermanous bodies —

TABLE XXX

when the heat of the Argand reflected by			Deflection after the insertion by the unreflected rays of the Argand lamp	Deflection after the insertion when the heat of the Argand lamp is reflected by		
Gray calico	Green oil cloth	Black velvet		Birch wood	Yellow marble	Metal
8 33	7 00*	6 67*	7 67	8 17	8 01	7 67
6 25	5 00	4 83*	4 92	5 33	5 33	4 96
5 50*	3 75*	3 06	3 33	4 50	4 50	3 17
23 25	22 25	22 19	22 81	22 81	22 81	22 88
20 75	18 00*	17 50	14 41	17 62	17 62	14 41
17 50	14 69	14 41	14 50	14 09	14 50	14 75

By these results it is therefore placed beyond all doubt, *that heat, on diffuse reflexion, is very differently modified, by some bodies to a great extent, by others it is unchanged*

It is evident, from the following observations, that these changes, when unpolished bodies are used are *independent of their degree of roughness*, for, *e g* a deflection of 7° 63 to 7 75 is constantly found on the insertion of the red glass, whether the heat of the Argand lamp is reflected by a more or less rough surface of wood, to cause a deviation of 13° in

TABLE XXXI

IX Thickness in millimetres	Substances through which the heat passed	Deflection by direct rays [illegible]	Deflection after [illegible]			
			B[illegible]			
			S[illegible]	S[illegible]	R[illegible] gl[illegible]	Still gl[illegible]
1 5	Red glass	13	7 71	7 75	7 63	7 75
1 4	Blue glass		6 75	5 70	5 83	5 70
1 1	Alum		5 21	5 33	5 25	5 38
4 4	Rock salt	25	22 81	22 88	22 75	2[illegible] 75
3 7	Calcareous spar		18 [illegible]0	18 50	18 02	18 [illegible]0
1 4	Gypsum		15 75	15 6[illegible]	15 50	15 75

As was to be expected, the alteration of the transmission of the heat after reflexion in one diathermanous substance, bears no relation to its passage through any other

Rays of the same direct intensity reflected by carmine (see p 389) white velvet (Table XXIV), and many other surfaces, are transmitted by all the six diathermanous bodies with which we are acquainted in a greater degree than the unreflected, and those reflected by black paper (see p 394, Table XXV) and wood charcoal (p 397) are transmitted by all in a less degree than the un

TABLE XXXII

X Thickness in millimetres	Substances [illegible]	Deflection by [illegible]	Deflection [illegible] Argand lamp	Deflection after [illegible]					
				Bl[illegible]	C[illegible]	[illegible]	Whit[illegible]	Whit[illegible]	Car[illegible]
			b	*c*	*f*		*l* *f* *k*	*i* *i* *i*	*l*
1 5	Red glass	13	7 00	6 50	6 67	6 71	9 00	9 01	10 01
1 4	Blue glass		5 10	4 58	4 63	4 7	6 58	6 13	6 03
1 4	Alum		3 31	3 60	4 13	4 13	6 71	6 07	6 17
4 4	Rock salt	2	21 69	20 91	21 00	20 88	23 01	23 00	21 00
3 7	Calcareous spar		13 69	15 50	16 62	15 02	20 10	21 31	21 31
1 4	Gypsum		10 0	12 31	11 25	12 7[illegible]	10 87	18 00	17 00

the needle. The same is the case with the other diathermanous substances.

In the case of metallic surfaces, the diffuse reflexion from which does not exert the least influence upon the transmission of the heat by radiation through those diathermanous plates which have been hitherto experimented upon (see Table XXIX.), it is, in fact, a matter of indifference whether they are used in a reflecting or any other condition of the surface. In this case also the subjoined table contains the values observed:—

Table XXXI.

the insertion when the heat of the Argand lamp is reflected by

Sheet tin				Copper		Lead	
Reflecting.	Scratched longitudinally	Scratched in both directions	Scratched in a cloudy manner	Smooth	Deeply engraved in both direct	Smooth	Scratched in both directions
7·27	7·27	7·27	7·27	7·25	7·00	7·25	7·25
5·38	5·38	5·38	5·38	5·25	5·25	5·50	5·25
4·58	4·58	4·58	4·58	4·50	4·25	4·50	4·50
22·67	22·58	22·75	22·58	22·50	22·75	22·75	22·75
15·44	15·25	15·17	15·50	15·50	15·25	15·25	15·50
12·50	12·33	12·17	12·41	12·25	12·25	12·50	12·25

reflected rays. But the heat reflected *e. g.* by green velvet (see p. 393) radiates through alum, calcareous spar and gypsum better than, red glass as well as, and rock-salt less perfectly than the direct rays of the flame. The same occurs with other surfaces.

In the following table I have grouped some examples which are characteristic in this point of view, in which those columns especially which are marked with the same letter when compared exhibit peculiar relations:—

Table XXXII.

when the heat of the Argand lamp is reflected by

White wool.	Red tapestry	Peroxide of tin	Ivory	Mother of pearl	Red taffeta	Green taffeta.	Brown Spanish leather	Black Spanish leather	Black lac	Black cloth
a, k	*k*	*l, m, f*	*l g*	*m*	*n, i*	*n*	*o*	*o*	*p*	*p*
9·00	9·25	9·21	7·87	8·33	9·00	9·21	8·83	8·92	10·64	9·32
6·16	6·38	6·16	5·96	6·13	6·16	6·07	5·58	5·50	8·18	7·75
6·58	6·70	5·71	5·83	6·13	5·58	5·28	4·02	4·75	6·57	7·07
23·18	23·62	23·00	21·94	22·50	23·12	23·19	22·11	22·37	22·88	22·12
20·81	21·50	18·38	17·44	19·06	18·56	19·86	18·11	17·25	20·38	20·00
18·31	19·12	15·12	15·81	17·50	15·56	16·69	14·88	13·88	16·12	17·25*

* Those numbers separated by the dark lines are not comparable with each other.

Thus the rays of heat reflected by white velvet pass through alum and gypsum in a greater, through calcareous spar in an equal, and through red and blue glass and rock salt in a less proportion than those reflected by carmine. The heat reflected by peroxide of tin passes through red glass more freely, blue glass, alum and rock salt in the same proportion, and calcareous spar and gypsum in a less degree than that reflected by mother-of-pearl. The values obtained on the transmission of the heat reflected by black lac and black cloth through the above bodies exhibit a similar variation. The rays reflected by red and green velvet permeate red and blue glass, alum and rock salt in the same, calcareous spar and gypsum in a different manner.

It would be tedious to consider in detail the other instances, which lead to similar results, and completely confirm the position already advanced (p. 202) viz. that the transmission of heat through diathermanous media depends solely upon the nature

TABLE XXXIII.

Th k i milli m t	S b t t l	D fl t ly l t di tl	D fl tl ft tl tl ly: A gn l l mp	R l l ot l l tl	I l f l l l	M t lll yl l	Defl c t ly ll t ll tl	D fl ll ftc: A g l l l	R l l t l l tl
1 5	Red glass	18	10 50	7 00	5 50	3 25	25	18 50	11 50
1 4	Blue glass		8 50	6 00	1 00	3 00		12 25	11 00
1 4	Alum		6 00	4 75	2 50	2 00		9 50	7 50
4 4	Rock salt	20	18 25	17 00	16 00	14 00	25	22 00	21 00
3 7	Calcareous spar		12 50	9 50	8 50	6 50		16 75	10 75
1 4	Gypsum		10 50	8 50	6 75	7 00		12 50	9 25

I. I first produced reflexion from those surfaces which in the previous observations had exhibited the *greatest differences*. The results to which the experiments with the *Argand lamp* led,

TABLE XXXIV.

Th kn n m ll m t	S b t i e t l	D fl ti n by ll t di tion	D fl tl fte: M t l ot refl t l	Gyp m	C r l
1 5	Red glass	13	8 50	8 75	10 00 *
1 4	Blue glass		6 50	6 75	7 00
1 4	Alum		5 00	6 25	7 25 *
4 4	Rock salt	20	18 00	18 00	18 37
3 7	Calcareous spar		12 12	15 50	16 75 *
1 4	Gypsum		9 87	13 00	13 87 *

of these bodies, by virtue of which they transmit some rays more easily than others.

2 The next question was, how the modifications of heat produced by diffuse reflexion, which ensue on their transmission through diathermanous substances, would be affected *by different sources of the heat.*

For this investigation, in addition to an Argand lamp, I made use of platinum at a red heat (p. 163), the flame of alcohol (p. 193), and a metallic cylinder, which was heated by being placed over the flame of the Argand lamp (p. 198)

We know that the rays of heat emitted by them are of different kinds, by the unequal proportions in which they pass through the same diathermanous bodies, and as may also be seen in the following table, in which they are given for different direct deflections :—

TABLE XXXIII.

the insertion by		Deflection by direct radiation	Deflection after the insertion by				Deflection by direct radiation	Deflection after the insertion by			
Flame of alcohol	Metallic cylinder		Argand lamp	Red hot platinum	Flame of alcohol	Metallic cylinder		Argand lamp	Red hot platinum	Flame of alcohol	Metallic cylinder
11 00	7 00	30°	20 25	18 25	11 87	8 00	50°	36 00	30 00	20 25	17 50
10 50	6 75		16 75	16 25	11 12	7 87		32 50	28·50	28 25	16 50
6 25	6 25		11 75	8 50	7 87	7 62		21 50	16 25	15 50	15 50
19 50	17 50	30°	26 50	25 37	23 50	21 50	50°	46 00	45 50	43 25	12 50
10 00	7 25		17 50	11 87	11 25	7 62		35 00	25 50	21 00	12 50
7·50	7 50		13 50	9 87	8 52	8 00		27 50	20 00	16 50	15·00

are contained in the following table, and require a new arrangement for this method of grouping them :—

TABLE XXXIV.

the insertion when the heat of the *Argand lamp* is reflected by							
Oxide of copper	Red taffeta	White velvet	Black velvet	Black paper	White wool.	Wood	Green oil cloth.
8 50	9 00	9 00 *	8 25 *	7 00 *	9 00	8 50	8 25 *
6 50	6 50	6 50	6 00	5 75	6 75	6 25	6 00
5 50	6 50	7 75 *	6 25 *	5·00 *	7 50	6 00	6·00 *
18 00	18 00	18 00	17 87	16 62	18·00	17 87	17·50
15 37	15 12	16 50 *	13 37 *	10 50 *	16 87	15 12	13 62 *
12 00	12 25	14 62 *	11 37 *	8 75 *	14 14	12·50	11 87 *

It is hence evident that e. g. the needle of the galvanometer, which had been deflected 13° by the reflexion of the heat from the above source, receded to 10° on inserting the red glass when the heat was reflected by carmine, and to 7° when by black paper; or when it had deviated 20° on the insertion of

TABLE XXXV.

Thickness in millimetres.	Substances interposed.	Direct deflection by the radiation.	Deflection after. Metal reflector.	Gypsum.	Carmine.
1.5	Red glass	13	7.01	7.71	8.79*
1.4	Blue glass		6.00	6.10	7.00
1.4	Alum		3.60	3.70	[illegible]*
4.4	Rock salt	20	18.13	18.00	18.50
3.7	Calcareous spar		10.63	12.01	11.81*
1.4	Gypsum		9.25	11.13	[illegible]*

It was found that the heat reflected in this instance by carmine, when it again produced a direct deflection of 13°, on inserting the red glass produced a deflection in the thermo-multiplier of 8° 79, and that reflected by black paper under the same circumstances an indication of 6° 42; or on the insertion of the calcareous spar, the former a deflection of 11° 81, the latter of 9° 69, when the direct action of the reflected heat had caused the needle as before to deviate to 20°.

Thus the differences in the heat reflected by carmine and

TABLE XXXVI.

Thickness in millimetres.	Substances interposed.	Direct deflection by the radiation.	Deflection after. Metal reflector.	Gypsum.	Carmine.
1.5	Red glass	13	4.87	5.00	5.75*
1.4	Blue glass		4.00	4.25	4.62
1.4	Alum		2.50	2.50	2.62*
4.4	Rock salt	20	15.50	16.00	16.50
3.7	Calcareous spar		8.68	9.50	10.75*
1.4	Gypsum		7.00	8.25	8.87*

Thus, when the heat of this flame was reflected by carmine, so as to deflect the needle of the thermoscope to 13°, on inserting

the calcareous spar, in the first case to 16° 75, in the second to 10° 5

The repetition of the same experiment with the rays of *red hot platinum* yielded the results detailed in the following table —

Table XXXV

the insertion when the heat of the *red hot platinum* is reflected by

Peroxide of copper	Red taffeta	White velvet	Black velvet	Black paper	White wool	Wood	Green oil cloth
8 33	7 92	7 25 *	7 00 *	6 42 *	7 25	7 21	7 33 *
6 67	6 63	6 25	5 79	5 12	6 25	6 01	6 08
3 75	4 08	4 50 *	3 71 *	3 17 *	4 25	3 92	3 71 *
18 00	18 00	18 00	17 50	16 63	18 00	18 00	17 38
12 88	12 50	13 63 *	12 25 *	9 69 *	13 14	12 38	12 25 *
10 56	10 69	11 56 *	10 38 *	8 56 *	11 13	10 63	10 31 *

black paper, when transmitted through red glass and calcareous spar, are less with the rays of red-hot platinum than with those of the Argand lamp The same applies to the differences which occur in the other diathermanous bodies, and in consequence of reflexion from all other surfaces

The reflexion of the heat of *a flame of alcohol*, when examined in the same manner by means of transmission, led to the values expressed in the subjoined table —

Table XXXVI

the insertion when the heat from *the flame of alcohol* is reflected by

Peroxide of copper	Red taffeta	White velvet	Black velvet	Black paper	White wool	Wood	Green oil cloth
4 87	5 50	4 75 *	4 62 *	4 25 *	4 75	4 37	4 50 *
4 25	4 87	4 25	4 12	3 62	4 12	4 37	4 25
2 12	2 62	2 25 *	2 50 *	2 37 *	2 25	2 62	2 50 *
16 00	16 00	15 57	15 50	14 75	16 00	16 00	15 50
9 87	10 00	9 75 *	9 25 *	8 00 *	9 87	9 87	9 25 *
7 87	8 25	8 00 *	7 87 *	6 87 *	8 00	8 18	8 00 *

the red glass it receded to 5° 75, and to 4° 25 when the direct deviation of 13° was produced by reflexion from black paper,

or the needle receded the first time from 20° to 10°.75, the second time from 20° to 8°, when the plate of calcareous spar was introduced between the reflecting surface and the thermal pile.

On comparing these results with the former, it is evident that the differences which were found on the passage of the heat of the flame of alcohol after reflexion by carmine and black paper through red glass and calcareous spar, were less than those which

TABLE XXXVII.

Thickness in millimetres.	Substances interposed.	Deflection by direct heat.	Deflection after reflexion.		
			Metallic surface.	Gypsum.	Carmine.
1.5	Red glass	13	3.25	3.25	3.25
1.4	Blue glass		2.87	2.75	2.87
1.4	Alum		2.00	2.00	2.00
4.4	Rock salt	20	10.00	10.25	10.00
3.7	Calcareous spar		5.25	5.25	5.25
1.4	Gypsum		0.00	0.00	0.00

Hence, in this instance, the heat reflected by carmine and black paper was transmitted through red glass and calcareous spar in exactly the same manner; for when their direct action produced a deviation of 13°, they both deflected the needle of the galvanometer to 3°.25 when the red glass was inserted, or, the direct deflection being 20° to 5°.25–5°.0 when the plate of calcareous spar was introduced between the reflecting surface and the thermoscope. Nor were the rays, when reflected by other surfaces, more distinguishable by any one of the diathermanous substances, either from each other or from those unreflected.

Thus with these sources of heat no differences could be detected after diffuse reflexion.

TABLE XXXVIII.

Thickness in millimetres.	Substances interposed.	Deflection by direct heat.	Deflection after reflexion.		
			Metallic surface.	Gypsum.	Carmine.
1.5	Red glass	16	10.12	11.83	12.78
1.4	Blue glass		8.50	8.92	9.33
1.4	Alum		7.08	7.07	8.12
4.4	Rock salt	30	27.00	27.00	28.00
3.7	Calcareous spar		18.00	22.25	21.00
1.4	Gypsum		11.25	19.00	20.00

red-hot platinum exhibited after reflexion by the same bodies. The same was the case in the differences noticed with other diathermanous bodies, as also in those which were observed in the rays reflected by the other surfaces on transmission through red glass, blue glass, alum, rock salt, calcareous spar and gypsum.

An examination of the heat of an *iron cylinder heated* to about 212° F., reflected by the same bodies, yielded the results contained in the following table:—

TABLE XXXVII.

the insertion when the heat of the *hot metallic cylinder* is reflected by

Peroxide of copper	Red taffeta	White velvet	Black velvet	Black paper	White wool	Wood	Green oil cloth
3·25	3·25	3·50	3·25	3·25	3·25	3·25	3·25
2·87	2·87	2·75	2·87	2·87	2·87	2·75	2·62
2·00	2·00	2·25	2·25	2·12	2·00	2·25	2·00
16·00	16·00	16·00	16·25	16·00	16·25	15·75	16·00
5·25	5·25	5·25	5·25	5·50	5·50	5·50	5·25
6·25	6·00	6·25	6·00	6·25	6·25	6·25	6·00

Hence these experiments prove that the modifications which heat experiences on reflexion are very considerable in the case of the heat emanating from an Argand lamp, that with the heat of the red-hot platinum they diminish; with the heat of the flame of alcohol they are still less; and in the case of the heat emitted by a heated iron cylinder, of whatever temperature it may be, between 79° and about 234° F.*, they absolutely vanish.

To render this still more distinct, I repeated the experiments detailed also at greater deflections than those already given. The numbers which were found are arranged in the following table:—

TABLE XXXVIII.

the insertion when the heat of the *Argand lamp* is reflected by

Peroxide of copper	Red taffeta	White velvet.	Black velvet	Black paper	White wool	Wood	Green oil cloth
11·75	11·83	11·67	10·00	8·92	11·67	11·33	10·50
8·92	8·58	8·12	8·00	7·33	8·12	8·12	8·58
6·75	7·75	8·50	7·00	5·83	8·50	7·25	6·92
26·75	27·00	26·75	26·25	21·75	27·00	27·00	26·25
21·75	21·50	23·50	19·50	16·50	22·75	21·25	19·50
17·00	18·00	21·50	15·75	18·50	20·50	18·00	15·75

* The limit, 320° F., given in the Monthly Report of the Berlin Academy for May 1845 is too high. When the temperature of the cylinder is above 234° F. (thus at a temperature which is far too low to produce a visible red heat), differences do occur in the transmission of the heat reflected by different bodies.

TABLE XXXVIII (*continued*)

Thickness in millimetres	Substance interposed	[illegible]	Diffuse reflexion [illegible]: M [illegible]	Gypsum	C [illegible]
1 5	Red glass	20	10 12	11 25	1 83
1 4	Blue glass		9 17	9 75	10 08
1 4	Alum		8 08	8 17	8 75
4 4	Rock salt	20	18 13	18 00	18 50
3 7	Calcareous spar		10 63	12 91	11 81
1 4	Gypsum		9 25	11 13	11 68
			Diffusion after [illegible]		
1 5	Red glass	25	9 87	9 87	11 62
1 4	Blue glass		9 00	9 00	9 87
1 4	Alum		5 50	5 75	6 00
4 4	Rock salt	2	20 37	20 50	20 87
3 7	Calcareous spar		11 75	12 25	11 13
1 4	Gypsum		9 50	11 14	11 63
			Diffusion after [illegible]		
1 5	Red glass	25	8 87	9 00	9 00
1 4	Blue glass		8 12	8 12	8 00
1 4	Alum		6 50	6 32	6 50
4 4	Rock salt	30	22 50	22 50	22 25
3 7	Calcareous spar		9 87	9 63	9 63
1 4	Gypsum		10 50	10 25	10 38

It is thus again shown, *that the changes undergone by heat on diffuse reflexion are occasioned both by the nature of the sources of heat and the properties of the reflecting body*

This is connected with the fact, that the rays of heat reflected by different substances, change, in a certain respect, their relations to one another

Thus the heat of the Argand lamp, when reflected by carmine, is transmitted through gypsum less perfectly in comparison with that reflected by white velvet, the rays of red hot platinum reflected by these surfaces, however, permeate this plate in the same manner and the heat of the flame of alcohol passes through it after reflexion by carmine comparatively better than when reflected by white velvet The same occurs, under similar circumstances, with white lead and white wool The rays of heat of the Argand lamp, when reflected by red taffeta and peroxide of copper, are transmitted by a plate of alum in a different proportion, whilst those of platinum at a red heat, when reflected by the same surfaces, are so in the same proportion

These examples are sufficient to illustrate the process in ques

TABLE XXXVIII (*continued*)

the insertion when the heat of *red hot platinum* is reflected by

Peroxide of copper	Red taffeta	White velvet	Black velvet	Black paper	White wool	Wood	Green oil cloth
11 50	11 42	10 75	10 50	9 58	10 83	10 75	10 75
9 83	9 83	9 50	9 00	8 33	9 33	9 17	9 08
8 08	8 33	8 58	8 17	7 33	8 58	8 33	8 17
18 00	18 06	18 00	17 50	16 83	18 00	18 00	17 38
12 88	12 50	13 63	12 25	9 69	13 11	12 38	12 25
10 56	10 69	11 56	10 38	8 56	11 13	10 03	10 31 *

when the heat of an *alcoholic flame* in Berzelius's lamp is reflected by the above bodies:

11 37	11 00	9 50	9 87	8 75	9 50	9 75	10 12
9 87	9 62	8 75	8 37	8 25	8 75	8 37	9 00
5 62	6 25	5 50	5 62	5 12	5 50	6 00	6 12
20 37	20 50	20 37	20 12	19 63	20 50	20 50	20 25
13 75	13 00	12 75	12 25	11 13	12 87	13 00	12 25
11 12	11 00	10 50	10 00	9 25	10 50	10 88	9 87

when the heat of the *hot metallic cylinder* is reflected by the same bodies:

8 87	8 87	8 87	8 87	8 75	8 87	8 87	8 87
8 12	8 00	8 12	8 00	8 00	8 12	8 00	8 00
6 50	6 50	6 62	6 62	6 50	6 62	6 50	6 62
22 50	22 37	22 37	22 50	22 62	22 37	22 62	22 37
9 63	9 63	9 50	9 50	9 50	9 50	9 50	9 63
10 50	10 38	10 38	10 50	10 38	10 38	10 50	10 50

tion, which was observed in the same manner in other reflecting surfaces and diathermanous bodies

II It still remained to be determined whether those surfaces which exert a *similar influence* upon the rays of the Argand lamp, *i e* which they reflect in such a manner that the heat reflected by the one is transmitted by the diathermanous media used for testing in the same proportion as that reflected by the others, would also reflect the heat from the other sources, so that the rays reflected by them would pass through these substances in the same manner

To ascertain this, I repeated the experiments performed with the Argand lamp with the red-hot platinum, the flame of alcohol and the cylinder at a dark red heat, using for reflexion those surfaces the similar action of which upon the rays of heat of the former appeared to me especially remarkable

The following table contains the numbers which were found on the transmission of the heat of the *Argand lamp* reflected by them through red glass, blue glass, alum, rock salt, calcareous spar and gypsum —

* When exposed to the rays of the red hot platinum, none of the surfaces by reflexion were capable of producing a greater deflection than 20°

TABLE XXXIX

Thickness in millimetres	Substances inserted	Deflection by direct radiation	Deflection after the insertion by the unreflected rays of the Argand lamp	Deflection after the				
				Silver	Sheet iron	White satin	Black satin	White taffeta
1 5	Red glass	13	8 13	8 13	8 30	8 70	8 79	8 81
1 4	Blue glass		6 92	7 00	7 03	6 83	6 79	6 81
1 4	Alum		5 66	5 61	5 57	5 29	5 20	5 16
4 4	Rock salt	25	22 10	22 19	22 12	22 81	22 81	22 81
3 7	Calcareous spar		15 64	15 56	15 50	19 75	19 62	19 88
1 1	Gypsum		12 16	12 00	11 88	16 69	16 62	16 62

It is thus seen that *e g* the rays of heat reflected by silver and sheet iron, which directly deflected the needle 13°, produce a deviation of 8° 39 to 8° 13 on transmission through the red glass, or of 15° 50 to 15° 56 when transmitted by the plate of calcareous spar, the direct deflection amounting to 25° The heat reflected by white oil paint, as also that by black lac, pass through red glass or calcareous spar in the same proportion, for in the former instance a recess of the needle from 13° to 8° 37–8° 62 occurs with both, in the second, from 25° to 20° 94–20° 88

TABLE XL

Thickness in millimetres	Substances inserted	Deflection by direct radiation	Deflection after the insertion by the unreflected rays of red hot platinum	Deflection after the				
				Silver	Sheet iron	White satin	Black satin	White taffeta
1 5	Red glass	13°	7 00	7 00	7 00	8 50	8 50	8 50
1 4	Blue glass		6 00	6 00	5 87	7 00	7 00	7 00
1 4	Alum		3 66	3 60	3 66	4 75	4 75	5 00
4 4	Rock salt	20	18 13	18 13	18 30	17 87	17 87	17 87
3 7	Calcareous spar		10 63	10 63	10 50	12 50	12 50	12 50
1 1	Gypsum		9 25	9 00	8 88	10 50	10 50	10 37

Neither do we here find any difference in the transmission of the heat reflected by the bodies which are compared, for the needle recedes from 13° to 7° on the insertion of the red glass; and on introducing the calcareous spar, from 20° to 10° 63–10° 5, whether the heat of the red hot platinum is unreflected or diffusely reflected by silver or sheet iron The rays reflected by white oil paint and black lac, which produced a direct deflection of 13°, caused the needle to deviate 8° 5 on transmission through red glass, and when the direct deflection was 20°, on transmission through calcareous spar, produced a deviation of 12° 25 to

Table XXXIX.

Insertion when the rays of the *Argand lamp* are reflected by

Black taffeta	White cloth	Black cloth	White oil paint	Black lac	Yellow leather	Brown Spanish leather	White wool	Red wool	Red cinnabar	Peroxide of copper
8 83	8 58	8 50	8 37	8 62	9 00	8 83	8 87	8 91	8 75	8 50
6 88	6 79	6 63	6 62	6 62	6 75	6 58	6 91	6 91	6 50	6 50
5 29	6 04	5 92	6 25	6 12	7 25	6 92	7 06	7 12	5 50	5 50
23 00	22 00	22 00	22 88	22 88	22 31	22 14	23 00	23 00	22 50	22 50
20 00	19 12	19 06	20 91	20 88	19·25	19 11	20 56	20 62	17 50	17 75
16 50	16 81	16 88	17 12	16 88	15 88	15 88	17 88	17 81	11 00	14 00 *

Also on inserting the other diathermanous bodies, no differences were observed Neither can the rays of the Argand lamp, reflected by white and black silk, light and black cloth, yellow leather and brown Spanish leather, white and red wool, and cinnabar and peroxide of copper, be distinguished from each other by means of any one of the above substances.

The values observed with the heat of the *red-hot platinum*, on the repetition of these experiments, are arranged in the following table —

Table XL.

Insertion when the heat of the *red-hot platinum* is reflected by

Black taffeta	White cloth	Black cloth	White oil paint	Black lac	Yellow leather	Brown Spanish leather	White wool	Red wool	Red cinnabar	Peroxide of copper
8 50	8 19	8 25	8 50	8 50	8 56	8 56	7 50	7 58	8 25	8 33
7 00	6 75	6 81	6 81	6 87	7 38	7 14	6 50	6 50	6 51	6 07
5 00	5 69	5 63	4·81	4 75	6 00	6 00	4 50	4 58	3 75	3 75
17 87	17 37	17·50	17 62	17 75	17 75	17 87	16 87	16 75	17 87	17 75
12 50	11 50	11 50	12 37	12 25	12 50	12 25	11 50	11 50	12 25	12·25
10 50	9 63	9 75	10 25	10 37	10 50	10 62	9 87	9 75	10·25	10 25

12° 37. The same occurs with blue glass, alum, rock salt and gypsum. The other surfaces arranged in the table, in the case of each and the same pair, also reflect the heat of the red-hot platinum in such a manner that it permeates the diathermanous bodies in the same proportion.

Under the influence of the rays of the *flame of alcohol*, as shown by the following observations, this agreement also obtains :—

* The numbers separated by the thicker lines are not comparable with each other.

Table XLI

Thickness in millimetres	Substances tested	Direct deflection [illegible]	Deflection [illegible]	Deflection [illegible]				
				Silver	[illegible]	White [illegible]	Black [illegible]	White [illegible]
1 5	Red glass	13	4 87	4 87	4 87	7 87	7 87	7 87
1 4	Blue glass		4 00	4 08	4 20	7 25	7 25	7 25
1 4	Alum		2 50	2 50	2 50	5 50	5 0	5 50
1 4	Rock salt	20	15 50	15 50	15 50	15 25	15 00	15 00
3 7	Calcareous spar		8 63	8 38	8 13	9 63	9 50	9 50
1 4	Gypsum		7 00	7 25	7 37	7 37	7 50	7 50

On this occasion the needle of the thermo multiplier became placed at 4 87 on the insertion of the red glass when the direct deflection of 13 was produced either by the unreflected heat of the flame or by that reflected by silver or sheet iron, and on inserting the calcareous spar at 8 63 to 8 13 the heat of the flame of alcohol being either unreflected or diffusely reflected by the two metals just mentioned to produce a deflection of 20 After the reflexion from white oil paint and black lac, in each case a recess of the needle of the galvanometer from 13° to 7° 42 was observed when the red glass, and from 20° to 9 5 when the plate of calcareous spar was introduced between the reflecting

Table XLII

Thickness in millimetres	Substances tested	Direct deflection [illegible]	Deflection [illegible]	Deflection [illegible]				
				Silver	Sheet iron	White [illegible]	Black [illegible]	White [illegible]
1 5	Red glass	13	3 25	3 25	3 18	3 25	3 25	3 25
1 4	Blue glass		3 38	3 25	3 25	3 00	3 25	3 25
1 4	Alum		1 50	1 50	1 62	1 50	1 50	1 50
4 4	Rock salt	30	22 25	22 50	22 50	22 50	22 50	22 50
3 7	Calcareous spar		10 00	9 75	9 75	10 00	10 00	10 00
1 4	Gypsum		10 50	10 50	10 50	10 50	10 50	10 50

A deflection of 3 25 to 3 5 is now constantly obtained as often as the red glass is inserted, and of 9 5 to 10 as often as the calcareous spar is introduced the direct deflection of 13 , and subsequently of 30° being previously produced by either the unreflected heat of the cylinder, or by reflexion from any one of the surfaces under consideration The same was found to be the case with the other diathermanous bodies The bodies subjected to investigation, which exhibited the same de

Table XLI.

Insertion when the heat of *the flame of alcohol* is reflected by

Black taffeta.	White cloth	Black cloth	White oil paint.	Black lac	Yellow leather	Brown Spanish leather	White wool	Red wool	Red cinnabar	Peroxide of copper
7 87	6 50	6 50	7 42	7 42	7 50	7 50	7 25	7 33	4 87	4 87
7 25	5 67	5 67	6 66	6 58	6 50	6 50	6 12	6 50	4 25	4 25
5 50	4 50	4 58	1 83	1 75	5 50	5 37	5 83	5 83	2 25	2 12
15 25	14 75	14 75	16 00	16 00	15 00	14 75	15 00	15 25	16 12	16 00
9 38	9 00	9 00	9 50	9 50	8 50	8·62	9 37	9 50	9 87	9 87
7 50	7 12	7 25	8 37	8 37	7 50	7 50	7 50	7 50	7 87	7 87

surface and the instrument. The same uniformity was observed in the thermoscopic indications with these surfaces, when the other diathermanous bodies were used. How perfect it is in the case of the other pairs of reflecting substances also, is evident from the above table.

It was not to be expected that any differences would occur in the present instances, when exposed to the influence of the *iron cylinder* at a dark red heat, with which even the greatest differences perceived in other sources of heat vanish (see p. 403–409). Observation has confirmed this :—

Table XLII.

Insertion when the heat of the *hot metallic cylinder* is reflected by

Black taffeta	White cloth	Black cloth	White oil paint	Black lac	Yellow leather	Brown Spanish leather	White wool	Red wool	Red cinnabar	Peroxide of copper
3 25	3 50	3 50	3·50	3 50	3 50	3 50	3 50	3 50	3 25	3 25
3 25	3 00	3 00	3 25	3 25	3 00	3 25	3 50	3 25	3 25	3 25
1 50	1 50	1 50	1 50	1 50	1·50	1 75	1 75	1 50	1 62	1 50
22 75	22 25	22 50	22 25	22 50	22 50	22 50	22 25	22 25	22 75	22 50
10 00	10 00	10 00	10 00	10 00	9 75	9 50	9 50	9 75	10 00	10 00
10 25	10 00	10 00	10 50	10 25	10 25	10 50	10 50	10 25	10 50	10 50

portment when exposed to the rays of the Argand lamp, also reflected the rays of the other sources of heat in the same manner; for the rays of heat reflected by silver and sheet iron, white and black silk, light and black cloth, white oil-paint and black lac, yellow leather and brown Spanish leather, white and red wool, cinnabar and peroxide of copper, on transmission through all those bodies which have as yet been used, are always found, in the cases mentioned, to act the same.

Although the tables given at p 103–109, which exhibit the variations in the heat reflected when different sources of heat are used are arithmetical means of four repetitions of the experiments, the latter (p 110–113) are derived from two series of observations only, which appeared to be sufficient, because the object was merely to make an accurate comparison of each pair of surfaces, and, moreover, there is less cause for mistrust in cases of similarity than where differences are concerned

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In the following table, those bodies which diffusely reflect the rays of heat in such a manner that when transmitted through red glass, blue glass, alum, rock salt, calcareous spar and gypsum, they are undistinguishable from one another, are arranged in vertical series In those under 1 only are no differences perceptible after reflexion in this manner on comparison with the unreflected heat —

TABLE XLIII

1	2	3	4	5
Gold	Gypsum	Birch wood	White satin	Blue velvet
Silver	Chalk	Cork	Black satin	Black velvet
Platinum	White lead	Mahogany	White taffeta	
[illegible] silver	White oil paint	Yellow marble	Black taffeta	
Iron	Porcelain			
Tin	Linen			
Zinc	White paper			
Copper	Blue paper			
Lead	White cotton wool			
Alloy of lead and tin	Grey cotton			
Brass	Paris green			
German silver	Green cinnabar			
Sheet iron	Chrome yellow			
	Black lac			

6	7	8	9	10
Yellow leather	Light cloth	Blue woollen tapestry	White wool	Cinnabar
Brown Spanish leather	Black cloth	Green woollen tapestry	Red wool	Deutoxide of copper

Those of the following substances, which are arranged in one and the same column, exhibit very similar, although not identical deportment in this respect —

11	12	13	14.
Carmine Madder-red Red woollen tapestry	White velvet White wool. Green woollen tapestry.	White lead Diesbach blue	Black velvet Green oil cloth

15	16.	17
Black paper. Black glass	Fossil coal Coke. Graphite	Lamp black Animal charcoal

The following bodies cannot be referred to either of these groups as regards the reflexion of heat:—

TABLE XLIV.

Ultramarine	Peroxide of tin	Tannate of iron	Indian ink
Pale red velvet	Green velvet	Black Spanish leather	Brown velveteen.

Red taffeta	Green taffeta.	Dark red velvet.
Mother of-pearl	Ivory	Black wood-charcoal. Brown coal

3. It was an important question, *How the alterations in heat by reflexion, proved in the above manner to occur, could be explained.*

In regard to this point, two cases might occur. They either consisted in a change of the rays of heat, which rendered them more capable of permeating one or the other diathermanous substance, or they were the consequence of a selective absorption of the reflecting surfaces for certain rays of heat transmitted to them, as appeared the most probable view from the experiments of Baden Powell and Melloni.

In the first case, the differences in the reflected heat should not occur until it was transmitted through the diathermanous media; in the second, it must be recognizable in it, even before its entrance, from the intensity with which the different rays of heat would be reflected by the different surfaces, because the intensity of the reflected heat is the reciprocal expression of the absorption of heat (see p. 384).

Experiment decided this point as follows:—We have learned that *e. g.* the heat reflected by carmine is transmitted compara-

tively better through red glass and calcareous spar than that by cinnabar (p. 390 and 391). Therefore if this arose from the carmine absorbing a larger portion of the rays, which do not pass freely through these bodies than cinnabar, it must, on comparison with the latter, reflect the heat of any source less freely the more it emitted to it these rays which are badly transmitted by red glass and calcareous spar. It is moreover known that the heat emitted by the cylinder at a dark red heat permeates red glass and calcareous spar less freely than that of an Argand lamp (p. 402 and 403). Hence carmine, in comparison with cinnabar, should reflect the heat of the cylinder proportionately less freely than that of the Argand lamp, if this alteration in the heat after reflexion were really produced by selective absorption. Experiment has confirmed this, for when the heat of the Argand lamp was reflected by cinnabar, the surface being in a certain position, a deviation of 29°·75 in the needle of the multiplier was obtained, whilst the reflexion by carmine, when the reflecting surface was of the same size and in the same position as regards the thermal pile and the source of heat, produced a deflection of 18°. However, when the heat of the metallic cylinder reflected by cinnabar had produced a similar deflection of 29°·75, when reflected by carmine under the same circumstances, it deflected the needle only 14°·37. Thus the intensity of the heat reflected by carmine was really diminished in the manner expected.

The same was found in the other cases. We know that the heat reflected by white paper is transmitted in a much greater degree by red glass and calcareous spar than that reflected by black paper (see p. 392–394). If this were a consequence of selective absorption, white paper also, in accordance with the above consideration, should reflect the heat of the cylinder at a dark red heat, which transmits principally those rays which are but slightly susceptible of transmission, through red glass and calcareous spar comparatively less freely in comparison with black paper than that of the Argand lamp.

However with black paper the contrary ought to occur. In comparison with white paper it should reflect the heat of the Argand lamp less perfectly than that of the cylinder. This was found to be the case. The heat of the Argand lamp, reflected by white paper, caused the needle of the galvanometer to deviate to 21°·25, whilst that reflected by black paper, under the same circumstances, deflected it to 18°; but the heat of the metallic cylinder, which, when reflected by white paper, caused a devia-

tion of 24° 25, when reflected by black paper produced a deflection of 34° 5 Hence the proportion was really inverse

When the rays of heat reflected by two surfaces could not be distinguished from each other, as *e g* those reflected by white and black satin, which passed through the diathermanous media used in the same manner (see p 391 to 392, 110 to 113), if this depended upon both absorbing the rays of heat sent to them in the same proportion, the relation of the intensities with which they reflect the heat should remain unchanged under the influence of radiation from any source

This was also shown by experiment to occur most distinctly Thus the heat of the Argand lamp, when reflected by white satin, produced a deflection of 31°, after reflexion from black satin, of 27° 5, and that of the cylinder at a dark red heat, in the former case a deviation of 27° 25, in the latter of 23° 5 Hence white satin reflected each kind of radiant heat better in the same degree than black satin

Yellow leather and brown Spanish leather, which also reflected heat in such a manner that it was transmitted by diathermanous bodies in the same proportion (see p 395, 411 to 413), reflected both the heat of the Argand lamp and that of the heated cylinder with the same intensity Under the influence of the rays of the former, a deflection of 28° 37 to 28° 5 was obtained in the case of each of these surfaces, and a deviation of 18° 75 with the rays of the metallic cylinder

To ensure still greater certainty in these experiments, in addition to the heat of the Argand lamp and of the metallic cylinder, I caused also that of red hot platinum and the flame of alcohol to be reflected from all those of the surfaces which have been previously mentioned, which I had of the same size (8 centim square)

The experiment consisted simply in exposing these surfaces with their normal at an inclination of 32° to that of the longitudinal axis of the thermoscope, and their centre at a distance of 7 inches from the latter and 1 5 from the source of heat, *seriatim* to the above four sources of heat, and observing the deflections which the heat reflected by them produced in the thermo multiplier The numbers obtained in this manner (each the arithmetical mean of two observations)* are contained in the following tables —

* Accurate to within 1° 0

TABLE XLV

I C [illegible] p 90 [illegible] 1891	S[illegible] f[illegible] [illegible]t	D[illegible] [illegible]ly		
		W[illegible] l[illegible]	([illegible]	M[illegible] l[illegible]
Argand lamp		[illegible] 17	18 50 *	10 00
Red hot platinum		11 00	10 1[illegible]	10 37
Flame of alcohol		20 00	11 3	11 [illegible]0
Metallic cylinder at a dark red heat		21 87	11 37 *	1[illegible] 00

TABLE XLVI

II C [illegible] p 92 [illegible] 1893	S[illegible] f[illegible] [illegible]t	D[illegible] [illegible]ly			
		W[illegible]	B[illegible]	W[illegible]	R[illegible]
Argand lamp		31 00	27 [illegible]0	27 50	27 00
Red hot platinum		11 [illegible]5	1[illegible] [illegible]	11 00	11 37
Flame of alcohol		10 87	17 37	1[illegible] [illegible]	1[illegible] 25
Metallic cylinder at a dark red heat		27 [illegible]	[illegible] 50	22 00	1[illegible] 75

TABLE XLVII

III C [illegible] p 394 [illegible] 1895	S[illegible] f[illegible] [illegible]t	D[illegible] [illegible]ly				
		W[illegible]	B[illegible]	B[illegible]	R[illegible]	G[illegible]
Argand lamp		21 25 *	19 13	18 00 *	20 50	19 [illegible]0
Red hot platinum		11 75 *	11 13	12 87 *	8 12	7 87
Flame of alcohol		11 50	13 [illegible]3	18 12 *	10 [illegible]5	10 02
Metallic cylinder at a dark red heat		21 25 *	22 25	31 50 *	11 25	14 12

TABLE XLVIII

IV C [illegible] p 394 [illegible] 1395	S[illegible] f[illegible] [illegible]t	Deflection by			
		Gyp[illegible]u[illegible]	W[illegible]	I[illegible]	W[illegible]
Argand lamp		25 50	20 50	21 25 *	21 50
Red hot platinum		12 12	1[illegible] 00	12 87 *	11 25
Flame of alcohol		11 50	15 75	17 50 *	11 00
Metallic cylinder at a dark red heat		29 25	30 75	30 50 *	27 87

Table XLV.

direct radiation when the heat is reflected by

Red cinnabar	Paris green	Green cinnabar	Chrome yellow	Diesbach blue	Ultramarine.
29 75 *	20 50	21 75	22 50	10·50	22 50
17 50	11 50	13 12	13 12	9 62	13 12
26 75	16 62	18 25	19 12	15·37	19 12
29 75 *	16 87	20 12	18 75	15 37	18 75

Table XLVI.

direct radiation when the heat is reflected by

Green taffeta	Black taffeta	White velvet	Dark red velvet	Light red velvet	Green velvet	Blue velvet	Black velvet.
25 50	25 00	19 87 *	19 75	19·50	19 25	18 25	21 50 *
11 50	11 25	8 50	9·50	9 50	9 50	8 50	10·50
15 37	14 37	10 50	12 12	11 37	11 75	11 12	14·50
19 75	19 00	15 87 *	17 87	17 00	17 87	16 25	21 50 *

Table XLVII.

direct radiation when the heat is reflected by

Blue woollen tapestry	White wool	Red wool	Light cloth	Black cloth	Yellow leather.	Brown Spanish leather	Black Spanish leather
19 50	23 62	22 75	21 75	23 75	28·37	28 50	30 25
7 75	10 00	9 00	10 71	10 37	10 87	10 87	13 25
10 37	12 75	12 00	13 37	12·87	13·75	13 75	15 87
12 00	13 75	12 75	16 37	15 12	18 75	18 75	22 37

Table XLVIII.

direct radiation when the heat is reflected by

White satin	White taffeta	White velvet	White paper	White cotton	White wool	Ivory	Silver.
28 00	21 75	17 12	21 00 *	21 50	19 00	18 25 *	63 25
12 50	11 50	7 87	10 75 *	10 37	8 37	9 75 *	50 75
15 12	13 75	9 25	13 62 *	13 25	11 37	13 50 *	50 00
20 25	27 75	18 75	26 50 *	22 75	21 00	28 50 *	72 50

Table XLIX

V / C mp 1 3J0 1397 / VI / C p 1 396	S fl t	Deflec t n ly					
		P ll t ll	1 t f i	A ll lt l	Bl k u	Bl l t ff t	Bl k l t
Argand lamp		r r0	19 75	21 ?r	2 00	22 75	21 00
Red l ot platinum		13 1	9 37	11 75	12 37	10 25	10 2
Flame of alcol ol		15 ?5	11 r0	11 J7	10 r0	13 00	1 1 87
Metallic cylinder at a dark red heat		27 50	18 00	21 75	26 00	?1 75	21 75

Table L

VII / C p c p 308 1390 / VIII / 1 398 1399	S fl t	Deflection by			
		Bl l ool	Co k	M logany	G ll
Argand lamp		31 00	J1 (2	28 0	61 00
R l l ot platinum		12 62	11 ?5	12 12	52 00
Flame of alcohol		1r 62	18 62	13 r2	51 00
Metall c cyl i der at a dark red heat		21 12	?5 62	19 50	72 75

Table LI

C mp p 40 d 03 / P 410 d 411	S u f l eat	D flc tion by				
		Gyp i	Car l o	I c ll c ll c	R l taff t	Wl lt l t
Arga d lamp		2? 50	20 00 *	22 50	23 75	16 50
Red hot platinum		16 50	14 50 *	16 r0	11 37	10 75
Flame of alcohol		12 37	11 2r *	11 87	13 75	10 00
Metallic cyl n ler at dark redness		27 25	17 6? *	27 50	21 50	15 7r

† Thus among all the bodies subjected to examination *lamp black* and *animal la coal* (Tabl XLIX) refl ct every kind of radiant heat in the least degree an l excej ting th se oke graphite and coal only exhibit a comparatively

Table XLIX.

direct radiation when the heat is reflected by

Black paper	Black cloth.	Black Spanish leather.	Black glass	Sheet iron.	Black wood-charcoal	Brown coal	Indian ink	Lamp black	Animal charcoal	Coal	Coke	Graphite
21 00 *	22 00 *	26 00	18·75	30 00	10 00 *	19 50 *	6 00	2 80 *	1·00 *	7 80	5 10	7 05
13 37 *	10 37 *	14·00	11 75	18 75	10 00 *	9 50 *	4 00	1 50 *	1·75 *	2 50	2·75	2 25
18 12 *	12 87 *	16 75	11 25	27 50	12 50 *	10 50 *	4 50	2 00 *	1·75 *	4·25	3 00	1 00
33 50 *	21 75 *	28 25	20 25	17 75	16 50 *	13 00 *	10 50	3 50 *	3 50 *	7 50	6 00	7·00

Table L.

direct radiation when the heat is reflected by

Silver.	Lead	German silver	Brown velveteen	White cotton	Green oil cloth	Black velvet.
63 25	42 50	55 00	21 00	25 75	21 00	22 87
50 75	32 00	13 25	8 50	11 12	11 00	10 12
50 00	31 50	42 50	11 25	15 00	11 00	11 50
72 50	59 00	67 25	13 12	15 00	10 00	17 00

Table LI.

direct radiation when the heat is reflected by

Black velvet	Black paper	White wool	Wood	Green oil-cloth.	Silver	Sheet iron.	White oil paint	Black lac.	Red cinnabar.	Peroxide of copper
19·00	18 50 *	18 12	25 12	16·75	61 75	30 50	21 75	18 75	20 50	22·50
12 87	16 50 *	11 87	15 50	13 50	51 00	23 50	12 25	11 25	21·37	16 50
13 37	13 87 *	10 62	15 25	11·75	50·50	23 50	16·00	14 37	22·12	14 37
22 50	30 87 *	17 62	27 02	21 50	72·50	40 00	20·80	21 50	37·00	27 50

feeble diffusion, which is also apparently independent of the nature of the source of heat; consequently these only can be considered "black bodies" as regards luminiferous and calorific rays

When these results are compared with those which were obtained on the transmission of the heat diffusely reflected by the above bodies through diathermanous substances, it appears—

1. That a surface which reflects heat in such a manner that it is transmitted by red glass, blue glass, alum, rock salt, calcareous spar and gypsum in a greater degree than that reflected by any other, in comparison with the latter, reflects the heat of the Argand lamp best, that of red hot platinum next, that of the flame of alcohol in a less degree, and that of the heated cylinder least of all, which also involves the reverse proposition, that a reflecting surface, which in comparison with any other diminishes the transmission of the heat by the above bodies, reflects the rays of the Argand lamp in comparatively the least degree, that of red hot platinum better, that of the flame of alcohol with still greater intensity, and that of the dark cylinder comparatively best.*

(Compare white and black velvet, p. 393 and 419, black wood charcoal and brown coal, p. 397 and 421, carmine and black paper, p. 401 and 405, and p. 420 and 421.)

2. That a substance by which heat is so reflected that it is transmitted by some diathermanous media better or in the same manner, by others less freely than that reflected by any other surface, in comparison with the latter, sometimes reflects the rays of the one, and sometimes of the other source of heat comparatively the best.

(Compare red and green taffeta, pp. 392, 418 and 419, asphalt-lac and black taffeta, p. 396 and 420, gypsum and peroxide of copper, pp. 402, 403 and 420.)

3. But that two surfaces which reflect the heat so that it permeates the diathermanous plates in the same manner, also constantly reflect the rays of the different sources of heat with the same relation to the intensities as has been found in either of them.

(Compare white and black satin, p. 392 and 418, yellow leather and brown Spanish leather, p. 395 and 419, silver and sheet iron, p. 410 and 421.)

Now when we recollect that through red glass, blue glass, alum, rock salt, calcareous spar and gypsum the heat of the Argand lamp passes best, that of red hot platinum less freely, that

* The rays of the above four sources of heat reflected by one and the same surface could not be directly compared, because it was impossible to give the same direct intensity to the rays emitted by them in the position above described.

of the flame of alcohol in a still less degree, and that of the dark cylinder worst, we find, with regard to the above considerations (p. 416 and 417), that all these phænomena confirm the position, "*That the changes experienced by heat on diffuse reflexion are merely the result of a selective absorption of the reflecting surfaces for certain rays of heat transmitted to them*."

The great uniformity existing in the results which have been detailed will certainly contribute to establish the view which I endeavoured to found at the commencement, viz. that these results really depend upon diffusely reflected heat, uninterfered with in any perceptible manner by foreign influences. (Compare p. 385-387.)

In the last investigation, the complementary selective absorption was determined to exist from the unequal intensity of the reflexion of the heat. That it might have been equally well proved by the surfaces exposed to the sources of heat becoming heated, is evident from the observations on carmine and black paper, the former of which, as shown by a previous experiment (p. 206), becomes less heated when exposed to the rays of the Argand lamp than those of the dark cylinder, and the latter less by the heat of the cylinder than the rays of the flame; whilst the later experiments (p. 420 and 421) show that carmine reflected the heat of the flame with greater intensity than that of the cylinder, whilst black paper reflected the rays of the latter in a greater degree than those of the lamp.

The reason why in this case the experiments by means of reflexion were preferred to those by absorption of heat, was, that they were not only more quickly performed, but, as seen on comparison of the indications, p. 206, and pp. 420 421, afforded a more delicate test-method than the latter.

It might be concluded, even from the phænomena of absorption (p. 206 and 207), that *the diffusion of heat is independent of the temperature of its source*. The direct investigation of the latter has confirmed this, and thus removed all doubt on the point.

* From what has been stated, it is clear that we can judge of the degree in which a body absorbs certain rays of heat, from the deportment of the heat diffusely reflected by it under certain circumstances. It is well known (Fusinieri, *Annali delle Scienze del regno Lombardo-Veneto*, 1838, *Gen. et Febr.*, p. 38, and Melloni, *Comptes Rendus*, t. vi. p. 801) that *snow* exposed to the rays of the sun melts more rapidly on trees and bushes than on a uniform surface.

If any philosopher should compare those rays of the sun reflected by snow with the direct rays, he would find the former transmitted comparatively better by red glass, blue glass, alum, calcareous spar and gypsum than those unreflected.

It has moreover shown most convincingly, *that, excepting charcoal and metals, it cannot be said that any body reflects heat better or worse than any other*, because this relation varies with each kind of radiation.

The differences in diffusely reflected heat which have been alluded to are perfectly analogous to those which are observed in the diffusely reflected *luminiferous rays*, but Herschel and Melloni have already pointed out *that the reflexion of these luminiferous rays is not analogous to that of the calorific rays*.

The investigations which have been detailed have shown this still more distinctly, by proving that certain bodies, which appear of the same colour to the eye, reflect different kinds of heat; and such as are apparently of different colours, so far as experiment has yet shown, reflect similar rays of heat. (See especially Table XLIII.)

It scarcely requires to be mentioned, that this result is not decisive of the identity of luminiferous and calorific rays; for since it has been determined that every *luminiferous* source of heat emits a large number of *invisible rays which are susceptible of reflexion and affect the thermal pile**, the most rigid analogy, i. e. the assumption that every ray which, after penetrating the optic media, excites the retina of our eye to produce vision, in proportion to its strength acts upon a thermoscope coated with lamp-black, would not lead us to expect any agreement in the diffuse reflexion of the luminiferous and calorific rays; for this would be to suppose that we were experimenting with a source which emitted only one kind of luminiferous and one kind of calorific rays.

I shall allude to one more point only in relation to this question.

The different colours under which diffusely reflecting surfaces appear to our eyes, have usually been explained by the assumption†, that certain colours only are reflected by them, the others being absorbed. Since the differences which heat exhibits after reflexion by any body have been shown experimentally to be the consequence of a similar selective absorption (see p. 415–422), the above assumption acquires the highest degree of probability, considering the great analogy which the luminiferous and calorific

* Compare p. 232, where it has been shown that certain rays of heat from the Argand lamp pass through *black glass* and *black lac*, which are therefore emitted *invisibly* from the flame.

† The correctness of which, from there being no accurate photometer, cannot be proved with certainty.

rays present in so many respects, and especially in their deportment after diffuse reflexion itself.

Melloni* has expressed the view, that yellow rays appear the most intense to us because the retina of the eye is yellow. Now if this yellowness, supposing that it does occur in a living, healthy eye, which is denied by most German physiologists, does not arise from a peculiar excitation of the retina, by means of which it emits yellow rays, but, in correspondence with the previous consideration, merely in consequence of its reflecting them, should it even then be assumed that our retina receives a special energetic impression from yellow rays? Certainly not; for the experiments on the absorption of heat have taught us that a body is least affected by those rays which its surface reflects. (See particularly p. 421.)

VI. *On the Sources of Heat.*

In the previous section it was shown that rays of heat undergoing diffuse reflexion by different bodies do not experience any peculiar change, but merely selective absorption, by means of which certain rays are checked, others reflected unchanged. (See p. 415–422.)

Hence it follows, that when *e g.* the rays of an *Argand lamp* reflected by carmine exhibit a different deportment (on transmission through diathermanous substances) to those reflected by black paper (see p. 402–404, and 406, 407), this can only arise from these sources of heat containing different rays, some of which are reflectible by carmine, others by black paper. The greater the differences are which occur after reflexion by various bodies, so much the more heterogeneous must the number of rays be which are emitted by the original source of heat.

Now if we find that the differences which the heat of the *red-hot platinum* exhibits after reflexion from a certain number of different bodies (on transmission through diathermanous media) are all less than the heat of the Argand lamp reflected by the same surfaces examined in the same way evinces (see p. 403–405), we must conclude that the red-hot platinum emits rays of heat which are less heterogeneous than the latter.

Moreover, if we remark that the differences in the rays of heat of the *flame of alcohol*, when reflected by various bodies (as they appear on using the same diathermanous bodies), are all

* E. Seebeck also adopts this view.

less than those observed with red hot platinum (see p. 104 and 105), it is evident that a still less number of different rays of heat emanate from the flame of alcohol than from heated platinum. Lastly, when we see how the heat of the *cylinder heated* to 212° F. fails to exhibit the slightest differences from whatever surface it may be reflected (see p. 106–109), which in the instances previously considered produced very considerable alterations, we must admit that the metallic cylinder at the above temperature emits a single kind of radiant heat only.

Thus if the sources of heat used in the previous investigation be compared in respect to this point, *the variety of the rays of heat emitted is greatest with the Argand lamp, less with red hot platinum, still less when the flame of alcohol is used, and has entirely disappeared with the cylinder heated to* 212° F.

There is a means of testing this in a different manner.

Thus if a number of heterogeneous rays of heat, as *e. g.* are emitted by an Argand or a Locatelli's lamp, be made incident upon different diathermanous bodies, different kinds of rays pass through them according to the nature of the substances. Therefore these, according as they appear in the one or the other of them, permeate a second diathermanous plate in a different manner.

The differences thus found will evidently be so much greater the more varied the original source of heat is. But if only one kind of rays of heat were allowed to enter these substances, so that only one could pass through them, on a second transmission they would not yield any differences from whatever diathermanous bodies the heat issued.

Now if it is found that the heat radiating from a cylinder heated below 231° F. constantly permeates red glass, blue glass, alum, rock salt, calcareous spar and gypsum in the same manner whether it issues immediately from the source of heat, or whether it has previously passed through ivory, post paper, a thin layer of carmine, the black glass, white glass, or any other diathermanous substance (see Table XVIII.), this is a new proof that one kind of rays of heat only is emitted by the cylinder*.

* As in the previous experiments the heat diffusely reflected by various bodies exhibited differences when the temperature exceeded 231° F. (see the note p. 407) in this instance also on surpassing this limit differences were apparent.

For whilst the heat emitted below 231° F., on inserting the red glass, con

If the result previously obtained (p. 233 and 234), which showed that the heat radiated from the most different solid bodies between 88° and 234° F. is *homogeneous*, be simultaneously borne in mind, it is evident *that within these temperatures the rays of heat emitted by them all*—to make use of an expression which reminds us of Melloni's terminology—*are of one and the same "colour."*

We have thus advanced to a certain limit, at which every variation of the rays of heat vanishes, a limit which is not attained until long after the differences in the luminous rays have become invisible.

It appeared to me of interest to ascertain, *How the heterogeneity in the rays of heat emitted by one and the same body is affected by its temperature.*

In investigating this, I had the two means just described at my command, *i. e.* the heat of the heated body, in those stages in which I wished to ascertain its compound nature, might either be reflected diffusely by different surfaces, or be transmitted through different bodies, before passing through certain diathermanous substances. In both cases, as we have seen, differences occur, which appear to be greater and more varied the more different the kinds of rays are which emanate from the source of heat.

I preferred the first process, because it was possible to prevent, by means of cold water, the disturbing influence resulting from the reflecting surfaces themselves becoming heated (see p. 385–387); whilst in the second case the body first radiated through could not be prevented from becoming heated, and its influence upon the experiment could only be eliminated at the expense of the intensity of the effects.

As in the former experiments (p. 201 and 202), to decide the present question, I also heated a spiral of platinum over the chimney of a Berzelius's lamp, first at a temperature below 234° F., then at a red, yellow and white heat.

The heat of *platinum below* 234° F., in correspondence with

stantly caused a recess of the needle from 35° to 10°·1, when immediately transmitted to the thermal pile from the metallic cylinder, or after having permeated post paper or white glass (Table XVIII.), at a certain higher temperature, after the same direct deflection of 35°, on inserting the red glass it produced a deflection of 10°·75 when emanating immediately from the cylinder, of 11°·75 after having permeated the paper, and of 11°·75 when it had passed through the white glass.

The same occurred with the other diathermanous bodies. It might be imagined that this variation in the heat emitted at higher temperatures was dependent upon the alteration of the capacity for heat with the increase of temperature.

the experiments on the heated cylinder, always passes through the diathermanous substances used for testing it, whether unreflected or diffusely reflected by the most dissimilar bodies. Thus in all these cases, on inserting the red glass a deflection of 8°.08 to 8°.25 was obtained; on inserting the calcareous spar, of

TABLE LII.

Thickness in millimetres.	Substance tested.	Deflection by unreflected heat.	Deflection	
			Metal reflected.	Gypsum.
1.5	Red glass	20	8.08	8.08
1.4	Blue glass		7.58	7.75
1.1	Alum		7.08	7.00
1.1	Rock salt	20	11.08	11.08
3.7	Calcareous spar		5.2	5.12
1.4	Gypsum		7.25	7.08

When the heat of *red hot platinum* is reflected by the same bodies, as we know, very distinct differences occur on transmission. Thus *e. g.* the unreflected heat on transmission through red glass produces a deflection of 10°.42, that reflected by black paper of 9°.58, that reflected by carmine of 12°.33, and that portion of the unreflected heat which is transmitted by calcareous

TABLE LIII.

Thickness in millimetres.	Substance tested.	Deflection by unreflected heat.	Deflection.	
			Metal reflected.	Gypsum.
1.5	Red glass	20	10.42	11.25
1.4	Blue glass		9.17	9.75
1.4	Alum		8.08	8.17
1.4	Rock salt	20	16.92	16.83
3.7	Calcareous spar		8.67	10.25
1.4	Gypsum		7.58	8.75

After these results there can be no doubt that the heat emitted by red hot platinum is more heterogeneous than that evolved by this metal at a dark heat.

When the heat of *platinum at a yellow heat* is diffusely reflected by the above surfaces, these differences become still greater. Thus, whilst the unreflected heat which permeates the red glass deflects the needle 8°.83, that reflected by black paper causes it to deviate 7° 42', that emitted by carmine, to 10° 58', and the unreflected heat, when transmitted by calcareous spar,

5° 17 to 5° 12, when the direct radiation upon the pile had deflected the needle 20°

The following table contains the values, each the arithmetic mean of three experiments, observed in the diathermanous bodies mentioned, as also in others —

TABLE LII

after the insertion when the heat of *platinum below* 291 I is reflected by

Carmine	Peroxide of copper	Red taffeta	Black velvet	Black paper	White wool	Wood	Green oil cloth
8 08	8 25	8 08	8 17	8 17	8 17	8 08	8 08
7 58	7 42	7 58	7 58	7 58	7 67	7 67	7 58
7 00	7 08	7 00	7 17	7 08	7 17	7 17	7 17
14 00	13 92	14 08	14 08	14 17	14 08	13 92	13 92
5 17	5 33	5 25	5 25	5 33	5 12	5 42	5 12
7 08	7 42	7 25	7 25	7 25	7 33	7 33	7 33

spar causes the needle to deviate 8° 67, that portion reflected by black paper, 7° 83, and that reflected by carmine, 11° 12, the deflection by direct radiation as before amounting to 20°

The following table contains the observations which were instituted on this point, in addition to those already mentioned (each the arithmetic mean of three experiments) —

TABLE LIII

after the insertion when the heat of *red hot platinum* is reflected by

Carmine	Peroxide of copper	Red taffeta	Black velvet	Black paper	White wool	Wood	Green oil cloth
12 33	11 50	11 42	10 50	9 58	10 83	10 75	10 75
10 08	9 83	9 83	9 00	8 33	9 33	9 17	9 08
8 75	8 08	8 33	8 17	7 33	8 58	8 93	8 17
17 25	16 67	16 83	16 00	15 75	16 75	16 92	16 00
11 42	10 25	9 92	9 33	7 83	10 12	10 08	9 33
9 33	8 33	8 33	7 83	6 92	8 75	8 58	7 83

*

produces a deflection of 6° 08, that reflected by black paper, of 5° 17, and that by carmine, of 9° 75, the direct radiation upon the thermal pile causing a deviation of 20° in the needle of the galvanometer

* The heat reflected by red hot platinum has been previously (p 104 and 105) examined, however it appeared to me requisite to repeat the experiments on this occasion, so as to be enabled to annex them with greater certainty to the others which belong here The relation of the numbers to each other found was of course the same as before, although their absolute values were different from the former, which cannot surprise us, as they were observed almost a whole year subsequently

Moreover, the rays of heat reflected by certain surfaces, as *e g* by gypsum and peroxide of copper, which were previously undistinguishable, now appeared heterogeneous

TABLE LIV

Thickness in millimetres	Substances interposed	Deflection by yellow-hot platinum	Deflection	
			Metallic reflector	Gypsum
1.5	Red glass	20	8.83	[illegible].75
1.4	Blue glass		7.7	7.0
1.4	Alum		15.02	[illegible].67
4.1	Rock salt	20	5.02	10.50
3.7	Calcareous spar		0.08	0.17
1.4	Gypsum		1.58	[illegible].33

Since it results from these experiments that the differences apparent under such circumstances were not observed in single, and but slightly decisive instances, but are almost always greater and more varied than those in the case of the red hot platinum (compare p 128 and 129), the conclusion appears justified, that the heat emitted by platinum at a yellow heat is more heterogeneous than that evolved by red hot platinum

Although, as has been frequently mentioned, we cannot always conclude from a great difference of two deflections when observed between lower degrees than those with which they are compared that there is a greater difference in the effects of the heat, still in the cases just alluded to (*i e* on comparing the observations Table LIII and Table LIV) this was allowed, because this dissimilarity of the thermoscopic indications does not occur until a certain point but the deflections of the needle of the multiplier, within the limits to which the observations extend might be considered as proportional to the thermal influences

When the heat of *platinum*, part of which is *at a white heat*,

TABLE LV

Thickness in millimetres	Substances interposed	Deflection by white-hot platinum	Deflection	
			Metallic reflector	Gypsum
1.5	Red glass	20	11.25	13.12
1.4	Blue glass		10.83	11.26
1.4	Alum		8.58	9.17
4.1	Rock salt	20	17.08	17.67
3.7	Calcareous spar		9.12	12.17
1.4	Gypsum		8.08	9.02

The subjoined table contains the numbers which were found in the individual instances (arithmetical means of three observations):—

Table LIV.

after the insertion when the heat of *platinum at a yellow heat* is reflected by

Carmine	Peroxide of copper	Red taffeta	Black velvet	Black paper	White wool	Wood	Green oil cloth
10 58	9 75	9 50	8 33	7 12	8 83	9 33	8 58
8 67	8 17	8 00	7 08	6 33	7 75	7 33	7 67
7 12	6 50	6 83	5 92	5 50	7 17	6 58	6 25
16 50	16 00	15 92	14 12	11 00	15 25	15 67	11 12
9 75	8 50	8 83	6 58	5 17	9 08	8 58	6 75
7 42	5 75	6 17	5 17	4 00	6 67	5 75	5 17

is reflected by the same bodies as that at a dark red and yellow heat, on transmission through the diathermanous substances it presents still greater differences than the heat of platinum at a yellow heat. Thus, in the previous instances, the needle receded from 20° to 11°·25 on the insertion of the red glass, when these rays were unreflected, from 20° to 10°·25 when they were reflected by black paper, and to 14°·17 when diffusely reflected by carmine, and on inserting the calcareous spar, the needle of the galvanometer deviates from 20° to 9°42 when the unreflected rays act upon the thermal pile, from 20° to 7°·5 when they are reflected by black paper, and to 13°·67 when by carmine.

The rays of heat reflected by black velvet and green oil-cloth, which exhibited as little difference when emanating from platinum at a red as from that at a yellow heat, were now readily distinguishable from each other. In the following table the details of these observations (again the arithmetical means of every three experiments) are contained:—

Table LV.

after the insertion when the heat of *platinum* partly *at a white heat* is reflected by

Carmine	Peroxide of copper	Red taffeta	Black velvet	Black paper	White wool	Wood	Green oil cloth
14 17	12 67	13 00	10 07	10 25	12 50	12 25	11 58
12 17	11 33	10 83	9 42	9 33	10 33	10 58	10 00
9 83	9 42	9 33	8 92	7·83	9 83	9 17	8 92
18 25	17·83	17 67	15 75	15 50	16 33	17 33	16 08
13 67	11 17	12 08	9 67	7 50	11 83	12 00	10 25
10 50	9 08	9 50	8 58	6 12	10 33	9 07	8 33

Hence it is evident that the differences which the rays of hea evolved by the platinum partly at a white heat exhibit after dif fuse reflexion on transmission through diathermanous media, ar all greater than those which are found under similar circum stances with platinum at a yellow heat It must consequentl be admitted that a still larger number of heterogeneous rays o heat emanates from the former than from the latter

Thus the result of this entire investigation is, *That the hea emitted by red hot platinum is more heterogeneous than that ema nating from this metal at a dark heat that from it at a yellow heat more so than that when red hot and that from white ho platinum more so than that which is emitted under any othe circumstances*

Consequently the complexity of the heat emitted by any body as might be expected, appears greater at higher than at lowe degrees of temperature

But it neither increases in one and the same body constantl with the temperature, as is evident, *e g* by its remaining un changed until its temperature exceeds 934 1 , *nor with differen sources of heat*, when numerous other circumstances cooperate *is it always greatest with that which possesses the highest tem perature*

Thus red hot platinum *e g* emits more heterogeneous ray than the flame of alcohol, nevertheless it must be admitted tha the temperature of the former is lower than that of the latter which is capable of raising the platinum wire to either a yellow or white heat The experiments which have hitherto been mad show that, *the differences in the nature of a source of heat have no the slightest possible relation to its radiating power* However, th series which the sources when arranged according to the compoun nature of their rays of heat form (see p 426, and pp 430, 431) is exactly the same as that which they would form if arranged ac cording to the varied nature of the *luminous rays* which they emit for we must *e g* consider the luminous rays also of an Argand lam as more heterogeneous than those of red hot platinum, becaus all bodies which reflect diffusely, when exposed to their influenc appear to the eye of more varied colours, and the luminous ray of red hot platinum as more heterogeneous than those of th flame of alcohol, because, when reflected by differently coloure bodies, they appear to the eye as far more varied than the latter

The same might be said of the luminous rays of platinum at a white, yellow and red heat.

In the previous details I have purposely avoided all theoretical remarks on the nature of the phænomena of heat, so as not to view the facts, which are the only permanent parts of science, from the perishable basis of a hypothesis. I shall not even now enter upon speculations of this kind, which can only lead to the desired object when combined with a fundamental mathematical treatment. Perhaps the observations contained in these essays may contribute to establish greater unity of principle in the theory of heat, in which greater discrepancy of theoretical views has prevailed than in any other branch of physics.

I shall therefore conclude this memoir by briefly summing up the principal results which have been obtained from the experiments detailed.

1. There are two new means of deciding with certainty whether any body transmits rays of heat or not. (See pp. 232, 236, 237.)

2. The transmission of radiant heat by diathermanous bodies has no direct relation to the temperature of its source, but depends solely upon the properties of the diathermanous substances, which are permeated by certain rays of heat in a greater degree than by others, whether these are of a low or high temperature. (See p. 203.)

3. The absorption of radiant heat by a body, when the rays permeating it are of the same uniform intensity, is perfectly independent of the temperature of its source, and is alone occasioned by the nature of the absorbing body, which is more susceptible of some rays than of others. (P. 206 and 207.)

4. A body becomes heated, within certain limits, in proportion to its thickness, and in a degree which is greater the less it is diathermanous to the rays transmitted to it. (P. 209–211.)

5. Absorption and emission of heat correspond to each other so far only as they are functions of one and the same body; and the nature of the rays of heat has no influence on it. (P. 216 and 217.)

6. The position advanced by Melloni is confirmed, viz. that scratching the surface of a body influences its power of radiating heat merely so far as it modifies its density and hardness, and

diminishes or increases it according as it loosens or condenses the parts concerned (P 215)

7 The radiating power of a body is independent of the nature of the rays of heat by the absorption of which it becomes heated (P 221)

8 The heat radiated by the most heterogeneous bodies, of unequal thickness and the surfaces of which are of the most dissimilar nature, has been shown, by the means at present at our command, to be homogeneous and simple, in whatever manner it may be excited in them within the limits of the experiments hitherto made, i e between 88° and 234° F (Pp 233, 234, 427)

9 The diffusion which heat experiences on rough surfaces has no connexion with the temperature of its source (P 121)

10 Radiant heat is altered in very different ways by diffuse reflexion, by some bodies to a great extent, by others it is unaffected In one and the same substance these modifications are independent of the condition of its surface (P 400)

11 The changes produced in heat by diffuse reflexion are occasioned both by the nature of the sources of heat and the properties of the reflecting bodies (P 408)

12 They are merely the consequence of a selective absorption of the reflecting bodies for certain rays of heat transmitted to them

13 Diffuse reflexion of the calorific rays is not analogous to the reflexion of luminous rays (P 121)

14 The heterogeneity of the rays of heat emitted by one and the same body is greater at higher than at lower temperatures, but does not constantly increase with the temperature, and has no perceptible relation to the radiating power (P 432)

15 The series which certain sources form, when arranged according to the amount of difference in their rays of heat, is the same as that which they exhibit when they are arranged according to the heterogeneous nature of the luminous rays which they emit (See p 432)

Article XI.

On the Spectra of Fraunhofer *formed by Gratings, and on the Analysis of their Light* By O. F. Mossotti, *Professor of Mathematics in Pisa.*

[From a separate Memoir, entitled *Sulle Proprietà degli Spettri di Fraunhofer formati dai Reticoli ed Analisi della Luce che somministrano*, Pisa, 1845]

THIS memoir consists of two parts. The first, which may be regarded as the introduction, contains the notice of the mathematical analysis of the solar spectrum, as read in the Physico-mathematical Section of the fifth assembly of Italian natural philosophers held at Lucca. The second part developes the calculus instituted in continuation for the purpose of more accurately deducing from Fraunhofer's experiments those results which were merely announced at the commencement of the investigation.

Part I —Introduction.

1. Those philosophers who have examined the solar spectrum with the view of ascertaining the extent of the colours it contains, the intensity of the light at various parts, and the length of the corresponding fits or undulations, have generally made use of the spectrum formed by refraction. But the figure of the spectrum obtained by refraction is deformed The more refractive parts are elongated, the less refractive shortened; and it is difficult to ascertain the properties of the component parts of a natural ray of light in this manner.

Newton, who first endeavoured to express the length of the portions belonging to the seven distinguishable colours of the spectrum, observed an analogy between the lengths of these portions and the differences in the numbers given by the values of the tones in an octave of the minor mode. This analogy however is purely accidental; the respective lengths of the different coloured parts of the spectrum formed by refraction vary according to the nature of the body which is made use of. From the very supposition that the spectra formed by different substances are similar to each other, Newton drew the erroneous

conclusion, that achromatism in dioptric telescopes was impossible, which has been long since disproved by experience.

Following this analogy, Newton prepared a chromatic circle, which was intended to represent the image of the spectrum, independent of the elongation or contraction which the refraction produces in the different parts of the prismatic spectrum. This, by means of the colours produced by the admixture or superposition of the several component colours, yields very nearly accurate results, but is constructed upon a hypothetical foundation.

Lastly, Newton made use of this same analogy for the formation of a law concerning the places which the different colours occupy in the prismatic spectrum and the length of the corresponding fits. This law leads to a remarkable relation which was first discovered by Blanc*, viz. that the length of the fit of any coloured ray is proportional to that power of $\frac{1}{3}$ the exponents of which are obtained when $\frac{1}{2}$d of the arc at the extremity of which the same colour should be placed in Newton's chromatic circle, is divided by the entire circumference of the circle. But the values obtained for the length of the fits or undulations of the different parts of the spectrum according to this proportion, are found towards its extremities to differ considerably from the truth†.

2. A better method of ascertaining the composition of natural light and the relation which exists *in vacuo* or in the air between the length of the undulations of the rays of which it is composed and the positions of these rays in the spectrum, consists in the use of spectra obtained by means of a grating, and which were first observed by Fraunhofer. In these spectra the only element contributing to their formation is the length of the waves of the different rays composing the natural light; the phænomenon appears in them in its greatest simplicity, without the alterations which the transmission of the rays through a refractive medium produces. Hence in the reticular spectrum (or that formed by gratings) we have a normal spectrum, to which the variable spectra produced in other ways may be referred.

* See Biot, *Précis Elément de Phys.*, 3 edit., Vol. ii. p. 434.

† Notwithstanding this critical remark, it is astonishing that Newton, in the first analysis of the spectrum, knew how to combine the different elements contributing to its formation by simple and elegant, although only approximative laws. See the note at the conclusion.

Following out this idea, I have deduced from Fraunhofer's extremely accurate observations the length of the different parts of the reticular spectrum corresponding to the intervals of the seven principal dark lines pointed out by Fraunhofer. These lines yield so many definite points, to which the different parts of the spectrum may be referred; they are therefore denoted by the letters B, C, D, E, F, G, H, and denominated the principal lines. Fig. 1, Plate II. represents a spectrum of this kind. If this be compared with fig. 2, representing another spectrum which Fraunhofer obtained by refraction by means of his flint-glass prism No. 13*, it will be seen how great the difference is in the extent of the different parts, and how very considerably the refractive spectrum is deformed. The intervals between the principal lines in the reticular spectrum are respectively expressed by the numbers—

BC	CD	DE	EF	FG.	GH.
31	66	61	41	54	35

and in the spectrum produced by refraction—

13	35	46	40	79	71

3. The reticular spectrum is characterized by a peculiar property. In the spectrum formed by refraction, which being larger and brighter allows of more easy observation, Fraunhofer has determined the intensity of the light in those parts which are nearest to the principal lines. The ordinates of the curve, fig 2, Plate II., represents the intensity of light of the subjacent points of the spectrum The dotted line μ between D and E is drawn so as to divide the spectrum into two parts, the quantities of light in the different parts in which form two equal sums, or so that it halves the whole light of the spectrum. If in the reticular spectrum a line μ be drawn between D and E so as to indicate the place which corresponds to the ray μ, it divides the total length of the spectrum into two equal parts. This simplicity of the distribution of the quantity of light in the reticular spectrum is a distinctive character of a normal spectrum.

In the prismatic spectrum the maximum of the intensity of the light, which corresponds to the maximum-ordinate of the curve, falls at m, at about $\frac{7}{21}$ths of the interval DE reckoned from D to E, and is therefore situated beyond the line μ towards the less refractive end of the spectrum. If we consider that

* *Denkschriften d'Acad der Wissenschaften zu Munchen f* 1823

towards this side the portions of the prismatic spectrum constantly contract more and more, it is not difficult to comprehend that the maximum of light which in the normal spectrum occurs at the line μ, in the prismatic spectrum is moved towards the side D, whenever the ordinates of the curves of intensity follow a law of diminution more slowly than that according to which the refraction condenses the luminous rays on that side. In fact, it is found that the intensity of the light in the normal spectrum is at its maximum in the centre, and diminishes symmetrically on both sides, so that the law of its alteration is represented by the curve over fig. 1, which is symmetrical around the line μ, and has its axis in this line.

4. The very important problem treated of by Newton, viz. to establish a relation between the length of the fits or undulations and the corresponding colours, is at once solved by the formation of the reticular spectrum. In fact, in whatever manner this spectrum is produced, the different parts of the reticular spectrum increase nearly in proportion to the lengths of the waves in the corresponding rays. If we imagine the length of the reticular spectrum to be subdivided, like the circumference of a circle, into 360 parts, and denote them by 2π, we find from the data furnished by observation that the length λ_ϕ of the waves of the ray which corresponds to the extremity of the arc ϕ reckoned from the centre of the spectrum, is given by

$$\lambda_\phi = 553.5 + 184.5\,\frac{\phi}{\pi} \qquad (1.)$$

In this formula the arc or distance must be considered as positive towards the red, and negative towards the violet end of the spectrum, and the unit of length in the measure of the lengths of the waves is the millionth part of a millimetre.

The formula resulting from the relation discovered by Blanc, based upon Newton's hypothesis, is

$$\lambda_\phi = 511.6\left(\frac{1}{2}\right)^{-\frac{\phi}{3\pi}}$$

However, towards the extremities of the spectrum it gives values which differ considerably from the length of the waves.

If in the formula (1.), ϕ be first made $= -\pi$, and then $\phi = \pi$, we have

$$\lambda_{-\pi} = 369, \quad \lambda_\pi = 738$$

These values correspond to the violet and red extremities of the spectrum, and as the second value is twice as great as the first, it is evident that the length of the waves of the extreme red ray amounts to twice that of the extreme violet, when these extremes are observed (as was done by Fraunhofer) by means of a telescope, and if we stop at that point where the colours are still perfectly distinguishable.

If, in the same formula (1.), we make $\phi = 0$, we have

$$\lambda_\mu = 553.5,$$

i. e. in the centre of the spectrum the length of the waves amounts to 553.5 millionths of a millimetre. Now we have remarked that the centre corresponds to the maximum of the intensity of the light; supposing then that in every part of the spectrum there exists an equal number of rays, we should say that those, the waves of which have a length of 553.5 millionths of a millimetre, are most active in exciting in us the perception of light, and that this capability of producing the physiological effects of vision, both when the length of the waves increases as well as diminishes, becomes lessened, and finally almost vanishes, when the waves have increased or diminished by one third of the length corresponding to the maximum effect.

5. From the simplicity of these results, we conclude therefore that, to ascertain the distribution and nature of the rays composing solar light, it is of importance to make use of a spectrum formed by means of a grating, as this alone is normal. In this spectrum the light is symmetrically distributed from its centre, and the relation between the length of the waves of the rays and the distances from the centre in which their corresponding colours appear in the spectrum, is by a simple law directly given by experiment.

The properties of the reticular spectra above detailed, and the conclusion which I have deduced from them,—that they yield new numerical data for optical questions,—appeared to me sufficiently important to be communicated to this honourable and learned assembly.

Part II.—Analysis.

The second part contains the mathematical proofs of the deduction announced in the first part.

§ I. *Value of the Refractive Index of* Fraunhofer's *Prism No.* 13* *in Function of the Length of the Waves.*

1. When Fraunhofer observed the solar spectrum formed by a flint glass prism, the angle of refraction of which was 26° 21′ 30″, through the telescope of a theodolite, the prism being in the position of the minimum deviation of the spectrum, he found that the principal line D was refracted to the angle of 17° 27′ 8″; and when he measured the angles between the line D and the other principal lines B, C, E, F, G, H (fig. 2, Plate II.), he obtained

BD	CD	DE	DF	DG	DH
12′ 20″.2	−9′ 1″.2	11′ 50″.0,	22′ 23″.9,	42′ 17″.8,	61′ 5″.8.

From a series of observations upon the solar spectrum formed by a grating and merely observed with the aid of the telescope of a theodolite, Fraunhofer also deduced the following mean values for the length of the waves of the rays contiguous to these principal lines, expressed in millionths of a millimetre:—

B	C	D	E	F	G	H
688	656	589	526	484	429	393. †

Judging from these values, a grating in which the sum of a dark and light interval would amount to about 0.088 millim. (which was the mean of those used by Fraunhofer), would present a spectrum in which the angular distances between the line D and the others B, C, E, F, G, H, measured at the focus of the telescope of the theodolite, would be expressed by

BD	DC	DE	DF	DG	DH
−4′ 15″	−2′ 5″	2′ 13″	4′ 21″,	7′ 3″,	8′ 36″.

In this spectrum, fig. 1, Plate II., which we shall call the *normal*, the intervals between the principal lines vary in proportion to the respective lengths of the waves of the contiguous rays; and if it be compared with the foregoing prismatic spectrum, it is seen that in the latter the intervals BD, DC, &c., compared with those of the former, diminish in extent, whilst the intervals DE, DF, &c. towards the violet end, comparatively increase in extent. This difference in extent depends upon the corresponding rays, the waves of which are shorter, being refracted in an inverse ratio which is greater than the simple ratio, in which the lengths of the waves diminish.

* Gilbert's *Annal. der Physik*, 1817; *Münchener Denkschriften für* 1814–1815.

† *Denkschriften der Münchener Acad. für* 1823.

2. In the communication which I made to the third Scientific Association which was held at Florence, I have given the formula which expresses the refractive index in function of the lengths of the waves. If this formula be extended, by carrying out the approximation to the fourth power of the lengths of the waves, it may be represented by the following function.—

$$\frac{1}{V} = i + h\left(\frac{\lambda_0}{\lambda}\right)^2 + k\left(\frac{\lambda_0}{\lambda}\right)^4 \quad \dots\dots\dots \quad (1.)$$

In this formula V signifies the velocity of the propagation of the light in the refracting medium, taken as unity *in vacuo* or in the air; $\frac{1}{V}$ therefore corresponds to the index of refraction; λ_0 indicates in the air the length of the waves of a single ray of a given colour, and λ the same for a ray of any colour; i, h, k are three constant coefficients, dependent upon the nature of the medium, and which may be experimentally determined for every refracting substance.

3. To carry out this determination in the present instance, we shall have recourse to a known formula, which Fraunhofer also made use of. If $\phi = 26° 24' 30''$ denote the refracting angle of the prism, $\psi = 17° 21' 8''$ the angle of refraction of the ray of the colour situated at the line D, and x the angular distance in the spectrum between the line D and the line which runs through the colour corresponding to the length λ, we have

$$\frac{1}{V} = \frac{\sin \frac{1}{2}(\phi + \psi + x)}{\sin \frac{1}{2}\phi}.$$

If this value of the index of refraction be made equal to the one above, we obtain the equation

$$\frac{\sin \frac{1}{2}(\phi + \psi + x)}{\sin \frac{1}{2}\phi} = i + h\left(\frac{\lambda_0}{\lambda}\right)^2 + k\left(\frac{\lambda_0}{\lambda}\right)^4 \quad \dots \quad (2.)$$

The value of the first member may be calculated for each of the principal bands by means of the magnitudes previously given, thus if, in the second member, we substitute for λ_0 and λ the corresponding values of the length of the waves which have been already given, we obtain an equal number of equations, from which the values of the constants i, h, k may be deduced, applying, if thought necessary, the method of least squares.

To effect this determination more conveniently, the above

equation need only be subjected to a simple transformation First, since when in it we make $\lambda = \lambda_0$ $\imath$ must be $= 0$, we have

$$\frac{\sin \frac{1}{2}(\phi + \psi)}{\sin \frac{1}{2}\phi} = \imath + h + k \tag{3}$$

If now $\imath$ be eliminated by means of this expression, and twice the product of the cosine of half the sum into the sine of half the difference be substituted for the difference of the sines, we obtain

$$2\frac{\cos \frac{1}{2}(\phi + \psi + \frac{1}{2}\imath)}{\left[\left(\frac{\lambda_0}{\lambda}\right)^2 - 1\right]\sin \frac{1}{2}\phi} \sin \tfrac{1}{2}\imath = h + \left[\left(\frac{\lambda_0}{\lambda}\right)^2 + 1\right]k$$

In using this formula for determining the two constants h and k we must substitute the respective values of $\imath$, for the rays situated upon the six lines B, C D, E, F, G

DB	DC	DE	DF	DG	DH
$-12'\,20''\,2$	$-9'\,1''\,2$	$11'\,50''\,0$	$22'\,23''\,9$	$42'\,17''\,8$	$61'\,5''\,8$,

and correspondingly,

$\lambda = 688$	656	526	484	428	394,

λ_0 being $= 589$

On carrying out the calculation, we get the six equations,

$$\begin{aligned}
0\,027991 &= h + 1\,7399\,k \\
0\,027650 &= h + 1\,8061\,k \\
0\,027519 &= h + 2\,2539\,k \\
0\,027491 &= h + 2\,4809\,k \\
0\,028527 &= h + 2\,8850\,k \\
0\,028903 &= h + 3\,2163\,k,
\end{aligned}$$

whence by the method of least squares,

$$h = 0\,025555 \qquad k = 0\,000975,$$

and we then obtain from the equation (3)

$$\imath = 1\,608506$$

With these numerical values the refractive index of flint glass, of which the prism used by Fraunhofer in his experiments was composed, is expressed in function of the length of waves *in vacuo* for the different coloured rays, by

$$\frac{1}{V} = 1\,608506 + 0\,025555\left(\frac{\lambda_0}{\lambda}\right)^2 + 0\,000975\left(\frac{\lambda_0}{\lambda}\right)^4$$

To determine to what degree of accuracy this formula would represent the observations I calculated by means of the equa

tion (1.) the values of x in the six spaces between the principal lines of the spectrum, and obtained,

$$\begin{array}{cccccc} BD & DC & DE & DF & DG & DH \\ -12' \, 19''{\cdot}6; & -8' \, 57''{\cdot}2, & 11' \, 56''{\cdot}0; & 22' \, 48''{\cdot}8, & 42' \, 33''{\cdot}6, & 60' \, 43''{\cdot}1. \end{array}$$

The comparison of these values shows a sufficient agreement with those mentioned above and given by observation

§ II. *On the respective Intensities of Light in different parts of the Prismatic and Reticular Spectrum.*

4. As the prismatic spectrum is larger and of brighter and more distinct colours, Fraunhofer was enabled to measure the intensity of its light near the principal bands, when approximated to and compared with the light of a lamp placed at various distances. The results of his observations are contained in the following table:—

Number of the observations	Intensity of light at							
	B	C	D	Between D & E	E	F	G	H
I	0·010	0·018	0·61	1·00	0·44	0·081	0·010	0·0011
II	0·014	0·096	0·59	1·00	0·38	0·140	0·029	0·0072
III	0·053	0·150	0·72	1·00	0·61	0·250	0·053	0·0090
IV	0·020	0·084	0·62	1·00	0·49	0·190	0·032	0·0050
Mean	0·032	0·094	0·64	1·00	0·48	0·168	0·031	0·0056

The maximum of light assumed as unity falls between D and E. From the nature of the maximum itself, it was difficult to determine accurately the spot where it falls. Fraunhofer places it between one fourth and one-third of the interval DE from D to E.

The ordinates of the curve above the figure of the spectrum, fig. 2, Plate II, represent the mean observed intensities of light at the points of the spectrum situated beneath, corresponding to the same abscissæ. From the inspection of this curve, we see that the intensities of the light become comparatively further extended towards the red than towards the violet end, which may depend upon the index of refraction of the shorter undulations varying more rapidly than in the inverse proportion of their length, and the rays thus respectively being more condensed at the red end and more diffused at the violet end. The proportion in which the density of the rays in the various parts of the prismatic spectrum alters, compared with that in which they are distributed in

the corresponding parts of the spectrum, is proportional to the differential coefficients $\frac{dx}{d\lambda}$, so that when G signifies the intensity of the light at the point i in the prismatic spectrum, I must correspond to the point λ in the reticular spectrum

$$I = n\frac{di}{d\lambda}G, \tag{1}$$

in which n is a constant coefficient

The value of the differential coefficient $\frac{di}{d\lambda}$ is found from the equation (1) which when differentiated yields

$$\frac{di}{d\lambda} = -\frac{4}{\lambda_0}\left(\frac{\lambda_0}{\lambda}\right)^3\left[h + 2k\left(\frac{\lambda_0}{\lambda}\right)^2\right]\frac{\sin\frac{1}{2}\phi}{\sin\frac{1}{2}(\phi+\psi+i)},$$

whence

$$I = -\frac{4n}{\lambda_0}\left(\frac{\lambda_0}{\lambda}\right)^3\left[h + 2k\left(\frac{\lambda_0}{\lambda}\right)^2\right]\frac{\sin\frac{1}{2}\phi}{\sin\frac{1}{2}(\phi+\psi+i)}G$$

If the above mean values be substituted for G, and the data in the preceding paragraph for λ_0 λ, i, ϕ, ψ, we obtain the following values of $\frac{1}{n}I$ for the positions of the principal lines —

B	C	D	E	F	G	H
9054	30851,	291375,	31[illegible]787,	145931	39316,	9171

These numbers give the ratios of the intensities of the light of the reticular spectrum at the points mentioned

§ III On the Curve formed by the Intensity of the Light in the various parts of the Reticular Spectrum

5 Since the intensities of the light of different parts of the spectrum are recognised by means of the eye, they must depend both upon the amount of rays accumulated at one part, and upon the susceptibility of the retina for the peculiar species of those rays The law of the variability of this intensity, being dependent upon both physical and physiological elements, is too complicated to allow of its being deduced *a priori* in the present state of our knowledge However, as we have already determined the proportions of the intensities of the light in the various parts of the reticular spectrum, we may seek *a posteriori* for a formula which connects them by a law of continuity with each other, and thus renders their properties more intelligible

In constructing a formula to represent the observed intensities with a small number of constants, it is important to proceed to the investigation by direct experiments, which yield the given values for interpolation. The inspection of the values of $\frac{1}{n}\Gamma$ previously given, shows that they diminish from the centre towards the extremities in a manner which tends to point to the existence of a similar law of decrease on both sides. In order therefore to represent the intensities of the light in the reticular spectrum, I shall take the ordinates of a symmetrical curve, and select as the axis of the curve the line which passes through that point where the length of the waves λ_μ is $= 553{\cdot}5$. I have adopted the following formula:—

$$z^4 = \tfrac{1}{3}\chi\left\{1 - \frac{3\chi(1-\chi)}{1+4\chi^2 e^{-\frac{3}{x}}}\right\} \quad . \quad . \quad . \quad . \quad (6.)$$

in which, to render the members homogeneous, I have made

$$z = 3\pi\frac{\lambda-\lambda_\mu}{\lambda_\mu} \quad . \quad . \quad . \quad (7.), \qquad \chi = \frac{1-\Gamma}{\Gamma} \quad . \quad . \quad (8.)$$

and have assumed the maximum value of Γ, *i. e.* that which corresponds to the axis of the curve, to be taken as unity.

That this formula may represent the intensity of the reticular spectrum, it must satisfy the two following conditions:—

First. If by means of it the maximum intensity of light in the prismatic spectrum be calculated, this must fall at the interval DE, about one-fourth or one third of it from D towards E.

Secondly. The calculated intensities of light corresponding to the places at the lines B, C, D, E, F, G and H in the spectrum, must agree very closely with those observed, the values of which we have given in No. 4.

6. To ascertain whether the formula (6.) possesses the above property, I may previously remark, that the values of the intensity G must generally be deduced from those of Γ by means of the equation given above —

$$\Gamma = n\frac{dx}{d\lambda}\mathrm{G}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.)$$

To satisfy the first condition, I differentiate this equation, and in the differential equation make $\frac{d\,\mathrm{G}}{d\lambda} = 0$, so that the value of

λ, which it verifies may belong to the maximum of G. I thus obtain

$$\frac{d\Gamma}{d\lambda} = n\,\frac{d^2 i}{d\lambda^2}\,G$$

and from this eliminating $n\,G$ by means of the previous equation,

$$\frac{d\Gamma}{d\lambda} = -\frac{\dfrac{d^2 i}{d\lambda^2}}{\dfrac{d a}{d\lambda}}\,\Gamma$$

and considering that the equation (8) gives

$$\Gamma = \frac{1}{1+\chi},\quad \frac{d\Gamma}{d\lambda} = -\frac{d\chi}{d\lambda}\,\frac{1}{(1+\chi)^2},$$

on substituting χ instead of Γ we have

$$\frac{d\chi}{d\lambda} = \frac{\dfrac{d^2 i}{d\lambda^2}}{\dfrac{d x}{d\lambda}}\,(1+\chi)$$

The values of $\frac{d\chi}{d\lambda}$, and $\frac{d^2 i}{d\lambda^2}$, $\frac{d i}{d\lambda}$ to be substituted in this equation, must be obtained by differentiation from the equations (6) and (5), which gives

$$z^3 = -\frac{\lambda\mu}{4\cdot 3^2\pi}\left\{1-\frac{3\chi(2-3\chi)}{1+4\chi\,e^{-\frac{3}{x}}}+\frac{12\chi^2(3-2\chi+\chi^2)e^{-\frac{3}{x}}}{(1+4\chi^2)+4e^{-\frac{3}{x}}}\right\}\frac{d\chi}{d\lambda}$$

$$\frac{d^2 a}{d\lambda^2} = -\frac{1}{\lambda_0}\left\{\frac{3h+10k\left(\frac{\lambda_0}{\lambda}\right)^2}{h+3k\left(\frac{\lambda_0}{\lambda}\right)^2}\,\frac{\lambda_0}{\lambda}-\frac{\lambda_0}{2}\,\frac{da}{d\lambda}\,\mathrm{tang}\,\tfrac{1}{2}(\psi+\phi+i)\right\}\frac{dx}{d\lambda}$$

and with these values the previous equation takes the following form —

$$z^3 = \frac{1}{4\cdot 3^2\pi}\,\frac{\lambda\mu}{\lambda_0}\,\Pi(1+\chi)\left\{1-\frac{3\chi(2-3\chi)}{1+4\chi^2 e^{-\frac{3}{x}}}+\frac{12\chi^2(3-\chi+2\chi^2)e^{-\frac{3}{x}}}{\left(1+4\chi^2 e^{-\frac{3}{x}}\right)^2}\right\}$$

in which for brevity we have substituted Π for the quantity

which in the second part of the expression of $\frac{d^2 x}{d \lambda^2}$ is contained between the parentheses.

If we had eliminated χ from the latter equation and (6.), and instead of z and x substituted its values (7.) and (2.) in function of λ, the resulting equation would not involve any unknown quantity but λ, and would be capable of giving the value of this magnitude for that place at which the intensity of the light of the prismatic spectrum must be at its maximum. We shall denote this value by λ_μ

The elimination and solution here spoken of would be impracticable if it were required to be carried out to its fullest extent. We may however remark, that the maximum of G must lie very near to that of I, and that the values of I when near the maximum in general vary but little, and still less in our peculiar instance, on account of the form of the equation adopted. As the value of χ in the formula (6.) must be rather small, and the exponential $e^{-\frac{3}{x}}$ becomes a very small magnitude which may be neglected, the equations (6.) and (10.) may be reduced to the form

$$z^4{}_m = \tfrac{1}{5}\chi - \chi^2 + \chi^3$$

$$z^4{}_m = \tfrac{1}{15}\,\Pi\,\frac{\lambda_m - \lambda_\mu}{\lambda_0}\left\{1 - 5\chi + 3\chi^2 + 9\chi^3\right\}.$$

For the purpose of solving these two equations, I have calculated a table of five terms, which gives the values of $\frac{1}{15}\,\Pi\,\frac{\lambda_m - \lambda_\mu}{\lambda_0}$ by means of the assumed values of λ, and those of λ_m which are very near those of λ. Then, assuming a value for χ which is very near the truth, I calculated from the first of the two equations, that of z^4, then that of z, from which I next deduced

$$\lambda = \lambda_\mu + \frac{\lambda_0}{3\pi}\,z.$$

With this value of λ, by entering in the table mentioned above I have deduced that of

$$\tfrac{1}{15}\,\Pi\,\frac{\lambda_m - \lambda_\mu}{\lambda_0}$$

and by means of the second equation obtained a second value of z^4. If this value of z^4 coincided with that already obtained from the first equation, I concluded that the assumed value of χ was

the true one. By this method I obtained for the maximum of the intensity of the light,

$$\chi = 0.02255, \quad \log z^4 = 7.84958, \quad \lambda - \lambda_\mu = 16.96,$$

whence, λ_μ being $= 553.5$, we have

$$\lambda = 570.5.$$

By the formula (1.) $v = 3' 4'' = 184''$ corresponds to this value of λ; so that as the interval $DL = 11' 50'' = 710''$, and consequently $\frac{1}{4} DL = 177''.5$, $\frac{1}{3} DL = 236''.7$, we see that the place found for the maximum of the intensity of the light in the prismatic spectrum falls at one fourth or one third of the interval DL, as required by experiment.

7. χ having the value obtained by the formula (8.), we have

$$\Gamma = 0.978.$$

If in the equation (4.) we make $G = 1$, it should be verified by this value of Γ; whence

$$n = \frac{\Gamma}{\frac{dz}{d\lambda}},$$

and if the calculation be carried out, we find

$$\log n = 1.28391.$$

This value of n is necessary, in order to pass from the value of Γ in the case of the reticular spectrum to that of G, corresponding to the prismatic spectrum, if we indicate as unity the maximum of the intensity of the light in each spectrum*.

8. To ascertain whether the assumed formula (6.) also fulfills the second condition, i.e. represents the intensity of the light at different points of the prismatic spectrum near the principal lines, we have first to deduce from the same formula the values of Γ which correspond to the values of λ belonging to these lines, and then from these values, by means of the formula (4.), those of G.

Fig. 1, Plate II. represents the curve given by equation (6.), presupposing that in this equation instead of χ its expression (8.) was substituted, and Γ indicates the ordinates and z the abscissæ,

* If it were required to fulfill the condition that both spectra should contain the same amount of light, n must be determined by means of the formula $n = \frac{\int \Gamma\, d\lambda}{\int G\, d\lambda}$, and therefore the value of the intensity G obtained from our formula must be divided by this value of n; but in this case the maximum intensity G would no longer be expressed by unity.

counted from the axis μ and measured in parts of the semi-circumference. I first took on this curve, which was drawn in correct proportions, the nearest values of I, corresponding to the values belonging to the lines B, C, D, E, F, G, H, and then corrected these values, so that they accurately satisfied the equation (6), thus I found

Values of	B	C	D	Between D & E	E	F	G	H
I	2 290 0 020[illegible]	1 715 0 0607	0 601 0 [illegible]61[illegible]	0 000 1 000	0 108 0 6931	1 [illegible] 0 [illegible]	2 1[illegible] 0 0[illegible]	[illegible] 0 01[illegible]

From these values of I, from that of n, and from the values of $\frac{d\,i}{d\,\lambda}$ already calculated, I then deduced by means of formula (1)

Values of	B	C	D	Between D & E	E	F	G	H
C	0 038	0 096	0 635	1 000	0 [illegible]1[illegible]	0 108	0 011	0 003[illegible]

These values of the intensities of the light of the prismatic spectrum, arising from the laws expressed by the formulæ (1) and (6), all lie between those given by observation which are detailed in No 1, and they therefore show that the formula assumed is capable of representing the phenomena. In fact, the limits between which the data of the observations detailed range, show how difficult is the determination of these data, and consequently what uncertainty still remains regarding their values. Thus the necessity of philosophers discovering photometric means which are susceptible of greater accuracy, becomes more and more striking. For want of more accurate data, we consider it superfluous to ascertain whether, by an alteration in the formulæ, or rather of their coefficients, a greater approximation of the calculated to the observed results could not be attained.

§ IV *Remarks*

9 The values of z and I in the formulæ (7) and (8) are so expressed that the intensity of the light in the centre of the normal spectrum is at a maximum, when I is equal to the radius or unity, and the abscissæ increase proportionally in parts of the semi-circumference π. If λ be taken as $= \lambda_\mu \pm \frac{1}{2}\lambda_\mu$ we have from formula (7)

$$z_{-1} = -\pi, \qquad z_{1} = \pi,$$

whence as $\lambda_\mu = 553.5$, the abscissæ which on either side of the maximum ordinate are equivalent to the semi-circumference, the lengths of the waves correspond to 553.5 ± 184.5, *i. e.*

$$\lambda_{-1} = 369 \qquad \lambda_1 = 738$$

These two values approximate with sufficient accuracy to those lengths of waves at which the light ceases to be visible. The intensities of light corresponding to these lengths of waves in the points of the normal spectrum would scarcely amount to 0.006 of the maximum intensity, and these points would scarcely differ by $\frac{1}{60}$th of the whole length of the spectrum from the indefinite limits which are given in Fraunhofer's diagram. If we bear in mind that the observations of this skilful optician were made with great care to assist the eye to discern the faintest traces of light, it may be said that ordinarily the distinct perceptibility of light is produced by waves the length of which extends from 369 to 738 millionths of a millimetre, or rather by waves the length of which varies from 1 to 2, and that those are most capable of producing the most lively perception, the length of which amounts to 553.5 millionths of a millimetre, or once and a half the length of the smallest wave.

10. In conclusion, I shall follow the example of Newton in arranging the values of the lengths of the waves corresponding to the principal lines with those of the tones of the diatonic scale:—

do	*re♭*	*re*	*mi*	*fa*	*fa♯*	*sol*	*la*	*si*	*do*
1	[illegible]	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	[illegible]	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
$\frac{1}{738}$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
A	*B*	*C*	*D*	μ	*E*	*F*	*G*	*H*	*I*
738	688	656	589	553.5	[illegible]	484	429	[illegible]	369

The two first lines of numbers express the relative values of the tones, the lower tone (*c*) being expressed by unity or the fraction $\frac{1}{738}$, so that the denominator of the second line represents the lengths of the strings which produce the respective tones. The third line of numbers contains the values of the lengths of the waves corresponding to the principal lines placed above them, expressed in millionths of a millimetre. On comparison, it appears that the lengths of the waves at the lines C, D, H correspond with the lengths of the strings of the tones *re*, *mi*, *si*, whilst in the others we only get an approximation. These coincidences of the dark principal lines, when the proportion is expressed by the direct denominators 4 and 8, and the numerators

are indirect, appear favourable to the supposition that the dark lines are produced by interference; hence it is worthy of remark, that the line F corresponds only approximatively to the *Sol*, the length of the waves of which is $\frac{1}{80}$th less than the length of the strings of the corresponding tone. However, these purely speculative remarks are merely given until we possess more numerous and accurate experimental data.

APPENDIX.—*On Newton's Theory of the Spectrum.*

If the length of Newton's prismatic spectrum, fig 3, Plate II., be taken as unity, and the commencement of the coordinates be placed in the outermost point O, at the distance of one of the red boundaries, the abscissæ X of the boundaries at which the different colours cease are given by the following numbers:—

$$\begin{array}{cccccccc} Xo & Xr & Xa & Xg & Xv & Xl & Xi & Xu. \\ 1- & \frac{9}{8} & \frac{6}{5} & \frac{4}{3} & \frac{3}{2} & \frac{5}{3} & \frac{16}{9} & 2. \end{array}$$

The letters r, a, g, &c. express respectively the colours red, orange, yellow, &c

Inversely we have

$$\begin{array}{cccccccc} \frac{1}{Xo} & \frac{1}{Xr} & \frac{1}{Xa} & \frac{1}{Xg} & \frac{1}{Xv} & \frac{1}{Xl} & \frac{1}{Xi} & \frac{1}{Xu} \\ 1 & \frac{8}{9} & \frac{5}{6} & \frac{3}{4} & \frac{2}{3} & \frac{3}{5} & \frac{9}{16} & \frac{1}{2}. \end{array}$$

The lengths λ of the fits of the colours corresponding to these limits follow, according to Newton, the numerical values

$$\begin{array}{cccccccc} \lambda o & \lambda r & \lambda a & \lambda g & \lambda v & \lambda l & \lambda i & \lambda u \\ (1)^{\frac{2}{3}} & (\frac{8}{9})^{\frac{2}{3}} & (\frac{5}{6})^{\frac{2}{3}} & (\frac{3}{4})^{\frac{2}{3}} & (\frac{2}{3})^{\frac{2}{3}} & (\frac{3}{5})^{\frac{2}{3}} & (\frac{9}{16})^{\frac{2}{3}} & (\frac{1}{2})^{\frac{2}{3}} \end{array} \quad . \; . \; . \; (2.)$$

The extensions of the colours upon the circumference of the coloured circle, according to Newton, are proportional to the following differences:—

$$\varphi r = 1 - \frac{Xo}{Xr}, \; \varphi a = 1 - \frac{Xr}{Xa}, \; \varphi g = 1 - \frac{Xa}{Xg}, \text{ \&c.,}$$

whence we have

$$\begin{array}{ccccccc} \varphi r & \varphi a & \varphi g & \varphi v & \varphi l & \varphi i & \varphi u \\ \frac{1}{9} & \frac{1}{16} & \frac{1}{10} & \frac{1}{9} & \frac{1}{10} & \frac{1}{16} & \frac{1}{9} \end{array} \quad . \; . \; . \; (3.)$$

If the circumference of the circle be divided in the proportion of these numbers, the lengths of the curves or, ra, &c. would be

$$\begin{array}{ccccccc} or & ra & ag & gv & vl & li & iu \\ 60^\circ 45'; & 34^\circ 11'; & 54^\circ 41'; & 60^\circ 46'; & 54^\circ 41'; & 34^\circ 11'; & 60^\circ 45'. \end{array}$$

The spectrum represented by this, when rectilinearly extended, as in fig 4, Plate II., would form Newton's normal spectrum. The centre of this spectrum would be in the middle of the green;

the colours would be symmetrically distributed on both sides and the lengths of the fits of the rays, which correspond to two colours equidistant from the centre of the spectrum, would pretty nearly satisfy the condition which was first noticed by Blanc, that its product was constant and equal to $(\frac{1}{2})\pi$ this leads to the equation

$$\lambda\phi = 511.6\,(\tfrac{1}{2})^{-\frac{1}{3}\frac{\phi}{\pi}}$$

which gives the length of the fits λ_ϕ corresponding to the arc ϕ counted from the middle and assumed as positive towards the red end, in millionths of a millimetre

The three series (1.), (2.), (3.), which we have here combined, comprise in a single point of view the simple relations by means of which Newton ingeniously attempted to ascertain the different elements of the spectrum

Article XII.

Memoir on the Nocturnal Cooling of Bodies exposed to a free Atmosphere in calm and serene Weather, and on the resulting Phænomena near the Earth's surface. By M. Melloni*.

[Read to the Royal Academy of Sciences of Naples on the 23rd of February, and 9th and 16th of March 1847.]

Wilson was the first who observed the cold produced in bodies exposed during the night, in the open country and under a clear sky and calm atmosphere. His observations were performed towards the end of the year 1783 by means of two thermometers, one placed on the snow, the other freely suspended at the height of 4 feet. On one of these nights, the lower thermometer, under a perfectly clear sky, marked $-21^{\circ}\cdot7$, the upper thermometer -15°. The difference of six degrees diminished rapidly when clouds appeared on the horizon, and entirely vanished when the sky was completely covered: the two thermometers had then descended to $-13^{\circ}\cdot9$†.

Some years later, Six found that a thermometer placed on the grass of a meadow during calm and clear nights continued at several degrees lower than another perfectly similar thermometer suspended at the height of 5 or 6 feet, the difference between the two amounting sometimes to $7^{\circ}\cdot5$‡.

At the beginning of the present century Wells instituted a long series of experiments analogous to those of Six, but more extended and diversified, by placing thermometers in contact with the ground and leaves of plants, or by enveloping them with wool, cotton, and other substances. These thermometers, placed at a small distance from the earth's surface, in calm and serene weather, gave a fall of $4^{\circ}\cdot5$ and even $7^{\circ}\cdot8$ below a thermometer without any envelope suspended at the height of 4 feet§.

All these indications became more nearly equal to each other, and sometimes became absolutely equal when the wind blew, or

* Translated from the *Annales de Chimie et de Physique* for February 1848, by Mr. A. W. Hobson, B.A., St John's College, Cambridge.

† Edinburgh Philosophical Transactions, vol. i. p. 153.

‡ Six's Posthumous Works. Canterbury, 1791.

§ *Ann. de Chimie et de Phys.* 3rd series, vol. v. p. 183.

when the sky was covered with clouds, or when a screen was stretched horizontally at the distance of some feet from the thermometers, so as to intercept entirely the view of the celestial vault

The experiments of Wells have been repeated by several observers and in particular by M Pouillet This philosopher inserted one of the thermometers into swansdown contained in a vessel placed on the ground and left the other suspended freely at the height of 4 feet, in the same way as Wells and Wilson The lower thermometer, on certain nights descended eight or nine degrees below the upper one*

The differences of temperature between the two thermometers employed in these various experiments are evidently owing to the calorific radiation towards the upper regions of the atmosphere and the simple fact of the quickness with which they diminish or entirely cease on the appearance of clouds or under the mere influence of an obstacle opposed to the exchange of heat between the thermometers and the sky is a sufficiently evident proof of it

Nevertheless if we examine them with attention, it is not difficult to convince ourselves that these differences do not represent the excess of radiation of the lower thermometer above that of the upper one

And, in fact, we know that the temperature of the air at different distances from the ground is not constant, but variable with the height In general, the heat increases during the day as we approach the terrestrial surface, but the contrary occurs in calm and serene nights This latter fact, which was first observed by Pictet towards the end of the last century, and afterwards confirmed by Six, Maveix and other experimenters leads evidently to the consequence, that in the experiments above mentioned, the thermometers nearest the ground acquire, *by the mere contact of the medium in which they are plunged*, a temperature lower than that of the upper thermometers, and that, consequently, the difference between the two temperatures is not entirely owing to radiation Again, as glass is endowed with a very great emissive power, naked thermometers cool quite as much as the most radiating bodies and do not indicate, under a serene atmosphere, the true temperature of the air

Hence to obtain comparable results and to judge how much a thermometer covered or enveloped with a given substance falls

* Pouillet *Elements de Physique* 4th edition 1844 p 610

during the night below the surrounding temperature, we must find out a method of neutralizing, or at least diminishing as far as possible, the radiation of the thermometer which measures the temperature of the air, and it is absolutely indispensable that the two instruments should be kept during the observations in the same horizontal stratum of the atmosphere

The well known fact of the radiating power of metals being less than that of all other bodies, led several observers to cover the thermometer used in measuring the atmospheric temperatures with leaves of gold, silver or tin, but these envelopes scarcely satisfy the required conditions, in consequence of the extreme difficulty of adapting exactly the metallic leaves on the glass, without forming wrinkles or leaving some portion of the bulb exposed And then, since the thermometer enveloped in metallic leaf was employed solely to measure the temperatures of the air, and that in order to obtain the effect of radiation of different substances, they continued to use naked thermometers, there ensued a new source of error in consequence of the different sensibilities of these two species of thermometers, the former being necessarily rather more sluggish (*paresseux*) than the second Both of these inconveniences may be easily avoided by preparing in the following manner all the thermometers which it is intended to use in researches on nocturnal cooling

Procure in the first place a small cylinder of cork of fine texture, whose form and dimensions are about the same as those of a common cork, let it be pierced in the direction of its axis by a small hole, in which is to be introduced the extremity of a thermometer graduated on its tube, and having gently pushed the cork to within 5 or 6 millimetres distance from the bulb, fix it firmly in this position with mastic and some small wedges of wood or cardboard The thermometer-tube is afterwards to be applied to a sheet of paper to copy the scale, which is then to be engraved on a very slight strip of ivory This moveable ivory scale may then be adjusted to the thermometer by means of an incision made in the upper part of the cork, and securely fixed by the help of two small pegs, when a perfect coincidence has been obtained between its divisions and those of the glass The small strip of ivory is then fixed on the tube, as is usually done on thermometers with moveable scale In the instruments thus prepared, the extremity of the column of mercury and the corresponding degree on the scale are distinctly visible at a glance

when looked at in a dark place by means of a light placed behind the strip of ivory, a circumstance of great importance in nocturnal observations. But a quality still more precious in these thermometers constructed with cork, consists in the great facility which they afford for comparison between the temperatures of the air and of bodies which radiate towards the celestial space. For this purpose a small vase of silver or brass is taken, similar to a common sewing thimble, whose surface is to be smooth and polished, and its dimensions sufficient to receive the bulb of a thermometer, and then fit on with friction to the lower extremity of the small cork cylinder. The thermometer having thus its reservoir protected by a metal armature, and the tube by an envelope of the same nature, loses almost completely its emissive power, as we shall soon see, and consequently furnishes the true temperature of the stratum of air in which it is plunged. Also if we cover the exterior surface of the armature with lamp black or a varnish, the emissive power of the apparatus is raised to its maximum, and the thermometer being properly placed in free air, descends below the surrounding temperature by radiation towards the upper regions of the atmosphere. All this is clearly manifested by the following experiments.

On the 17th of last September (1846) the weather was fine and calm in the valley named *La Cava*, situated between the cities of Naples and Salerno. At 9 o'clock in the evening I exposed on a terrace raised 15 metres above the ground, three thermometers sensibly equal, armed in the manner I have just mentioned. Two had their armature polished, that of the third was coated with lamp black. These thermometers were placed horizontally, and each of them had its reservoir placed at the bottom of a vessel formed of tin plate, and of the shape of a truncated cone inverted, the radius of its lower end being 2½ centimetres, and that of the upper end 7 centimetres. These vessels, which were 8 centimetres high, were supported by tripods 50 centimetres in height, formed of slender tin plate tubes, which, in addition to their firmness, possess the advantage of having but little matter in their transverse section; and hence, by affording very little communication of heat with the ground beneath them, almost completely isolate the bodies they support.

In order to introduce the thermometer horizontally into the recipients and keep them in this position, each vessel had a lateral opening made close to the bottom, and furnished on the inside

with a metallic tube which covered half of the cork of the thermometer. The stems of the thermometers and their ivory scales were enclosed in cases of thin tin-plate, which fitted on to the other half of the cork, passing through the exterior side of the vessel, and could be taken away and replaced at pleasure, to observe the indications of the instruments, and to preserve them from the moisture of the atmosphere, and especially from the effects of the cooling of the ivory scales and thermometer-stems*. The openings in the vessels were in the first place closed by discs of tin-plate

After being exposed for half an hour, and consequently at 9^h 30^m, the three thermometers marked the same temperature, $17^\circ{\cdot}61$, or to speak more accurately, they only differed from each other by the same fractions of a degree (0·05 and 0·09) marked by the three instruments when plunged uncovered into a large vessel of lukewarm water. At 10 o'clock the thermometers were observed again, and all three were seen to mark a temperature of $17^\circ{\cdot}3$: at 10^h 30^m the three thermometric columns indicated $17^\circ{\cdot}1$.

* If the tubes of the thermometers were left exposed to the free air, the cold resulting from their radiation towards the sky might interfere with the action of the different substances applied to the bulbs, to such a degree that it would often be impossible to recognise the difference in their emissive powers And it is easy to perceive the cause of this confusion, if we reflect that in the vertical position in which the thermometer is usually held, the radiation takes place from all points of the surface of the tube The cold produced on the superficial layers is propagated along the sides to the bulb, and in a transverse direction as far as the middle. The liquids of the thermometric column which contract, descend and are replaced by a corresponding portion of particles from the bulb, and a circulation is formed whose cooling effect is added to that produced by the direct contact between the tube and bulb, so that a considerable part of the cold produced by the radiation of the stem is communicated to the whole mass of the thermoscopic liquid and to the reservoir of the thermometer

† When the experiments are made in the midst of fields on calm and serene nights, the air surrounding the thermometer is always very moist we shall hereafter perceive the cause of this great humidity Admitting it for the present as a certain fact, it evidently follows that in this case the cold transmitted by the stem to the bulb of the thermometer will cause a precipitation of aqueous vapour on whatever substance covers it Now water being endowed with a great emissive power, will begin to radiate itself, and immediately to cool down the thermometric reservoir, and this cold will become sensibly equal to that of the most radiating bodies, so that two thermometers with uncovered tubes, one of which has its reservoir coated with lamp black, and the other gilt or silvered, if exposed to the free air on calm and serene nights after having presented a difference of cold in favour of the former substance, will ultimately indicate the same degree of heat.

This is the reason why some experimenters have not observed any appreciable difference between the nocturnal cooling of a series of thermometers, *whose stems were uncovered*, placed at the same height, although the reservoirs of these thermometers were covered with different substances, or put into communication with plates of different nature sustained by *glass cylinders*

These initial observations already proved clearly that the metal armatures with which the first two thermometers were furnished, and the similar metallic sides of the three receiving vessels, were not sensibly cooled by radiation; for otherwise the thermometers furnished with polished metallic armatures would have marked a higher temperature than that of the thermometer whose armature was blackened. But this conclusion became still more evident on taking away the covers of the two latter vessels, and leaving one only of the two thermometers with polished armature in the same condition as before. The blackened thermometer now began to descend rapidly; ten minutes afterwards it had attained its lowest point, and marked 3°.1 less than the thermometer contained in the other uncovered vessel, and this latter indicated the same temperature as the thermometer in the closed vessel. Hence the immobility of the metallic thermometer in the open vessel, and the identity of its temperature with that of the metallic thermometer in the closed vessel, incontestably prove,—1st, that the cooling of the blackened thermometer is owing to radiation, and not to the contact of the external air; 2nd, that the cold produced by the radiation of the metallic thermometer is nothing, or at least so feeble as to escape direct observation.

The first conclusion is in perfect accordance with what we know of the great radiating power of lamp black; but the second is opposed to the views hitherto entertained as to the relative emissive powers of lamp black and metallic surfaces. In fact, if we denote by 100 the calorific radiation of lamp black, gold, silver, tin, and brass will have their radiating power expressed by 12, according to the experiments of Leslie, inserted in all treatises on physics. The degree of cold acquired by the black thermometer in consequence of its being freely exposed to the sky being 3°.1, the uncovered metallic thermometer ought to be cooled $\frac{12}{100}$ (3°.1), *i. e.* 0°.11, a very appreciable quantity on an instrument whose scale was divided to fifths of a degree; nevertheless the experiment indicated no clearly appreciable variation in the column of the uncovered metallic thermometer.

On the other hand, if metallic surfaces are not endowed with that capacity of calorific emission usually attributed to them by experimenters, it certainly could not hence be inferred that they are absolutely deprived of all radiating power.

I have therefore endeavoured to repeat with the greatest pos-

sible accuracy, experiments of comparison between the radiation of lamp black and of metals exposed to the nocturnal influence of a serene sky.

Those who have had occasion to compare with much accuracy the movement of several thermometers placed in the same circumstances, will no doubt have been convinced that, whatever be the skill of the maker, or the nature of the methods employed to determine the points of comparison, it is with difficulty that an identity can be obtained in the indications of two thermometers. This defect would be of very little importance in researches which, like ours, have for their object the determination of merely relative values; for we might determine the differences existing between them at a given temperature, and thus render the observations identical by a simple addition or subtraction. But experience shows that the difference between the indications of two thermometers does not generally remain invariable between points of the scale at a distance from each other, and that it is frequently variable within points which are near each other, according as the two instruments are subjected to a more or less abrupt variation of temperature.

To overcome these difficulties, I in the first place chose from my collection the three *atmospheric* thermometers which agreed best with each other, and after having armed them in the manner previously mentioned, I introduced them into their closed conical recipients, and exposed them on the 9th of October*, at 8 o'clock in the evening, the weather being very calm and serene. For greater distinctness, the three thermometers are denoted by the letters A, B, C. Half an hour afterwards we began to observe the instruments, the indications of which, noted every three minutes, gave the following results:—

* On seeing an interval of twenty two days between these experiments and the preceding, it must not be inferred that in all the intermediate nights the weather was not favourable for observations as any one may convince himself by the successive dates, but rather that the experiments in question did not follow each other in the order adopted in this memoir. The ideas and facts have not always proceeded with all the regularity desirable. Nevertheless the work being finished, I have endeavoured to arrange the materials gathered in the best way that I could.

Time of observation	Indications of the thermometers		
	A	B	C
h m 8 30	17.80	17.88	17.90
8 33	17.79	17.83	17.81
8 36	17.77	17.80	17.83
8 39	17.7[illegible]	17.7[illegible]	17.80
8 42	17.73	17.7[illegible]	17.7[illegible]
	88.82	88.98	89.11
Means	17.764	17.796	17.822

Hence the differences referred to the minimum value (that of A), that is to say to the mean indication of this instrument, were—

$$17^\circ.796-17^\circ.764=0^\circ.032 \text{ for B}$$
$$17^\circ.822-17^\circ.764=0^\circ.058 \text{ for C}$$

Afterward the metallic plates which covered the recipient vessels were removed, and at $9^h\ 15^m$ the indications of the thermometers were again noted down every three minutes. The following are the results of these latter observations.

Time of observation	Indications of the thermometers		
	A	B	C
h m 9 15	14.16	14.20	14.24
9 18	14.15	14.17	14.23
9 21	14.13	14.17	14.1[illegible]
9 24	14.12	14.13	14.18
9 27	14.10	14.12	14.16
	70.6[illegible]	70.79	70.98
Means	14.13[illegible]	14.158	14.196

Taking the differences between B, C and A, we have—

$$14^\circ.158-14^\circ.132=0^\circ.026 \text{ for B}$$
$$14^\circ.196-14^\circ.132=0^\circ.066 \text{ for C}$$

which differences are sensibly equal to the preceding, within the limits of errors which may be incurred in appreciating fractions less than tenths of a degree which were marked on the scale, and yet left a considerable interval between the divisions.

The means of these two pairs of differences are 0.030 and 0.062: it may then be admitted that *between fourteen and eighteen degrees, and temperatures but little differing from these*

two limits, 0°·03 must be subtracted from the thermometer B, and 0°·062 from the thermometer C, in order to obtain the temperature of thermometer A. After having determined the corrections to be made *in the actual circumstances of the experiment*, the blackened armatures of the thermometers A and B were removed and replaced by well-polished armatures of silver. The recipients of these two thermometers were then closed, as well as the third recipient containing the blackened thermometer C.

The weather continued calm and serene, and the temperature of the air had not altered much on the terrace where the apparatus was placed. At ten o'clock A indicated 17°·65, B 17°·69, C 17°·70. Applying to the two latter their respective corrections, we have 17°·66 for B, and 17°·638 for C, so that the three thermometers were at the same temperature. The covers of B and C were then removed, leaving A enclosed, and at half-past ten o'clock the observations were resumed, of which the following Table contains the results:—

Time of observation	Temperatures of the thermometers in their recipients		
	Closed	Open	
	A	B	C
	Metallic	Metallic	Blackened
h m 10 00	17°·55	17°·50	14°·07
10 3	17·55	17·46	14·05
10 6	17·53	17·41	14·05
10 9	17·51	17·41	14·03
10 12	17·52	17·42	14·03
10 15	17·53	17·41	14·02
10 18	17·51	17·39	14·00
10 21	17·50	17·37	13·95
10 24	17·50	17·36	13·92
10 27	17·40	17·31	13·92
Means	175·22 17·522 0·000	174·13 17·413 0·030	140·03 14·003 0·062
Corrected Means	17·522	17·383	13·941

Subtracting from the mean of A the corrected means of B and C, we obtain the differences 0°·108 for B and 3°·581 for C, and consequently if we make 3·581 = 100, the value of the emissive or radiating power of the silver forming the armature of the thermometer B will be $\frac{108}{3581} \cdot 100 = 3\cdot026$, a value which may be regarded as exact within a limit of error of fifteen or twenty

thousandths, which we may conclude from the nature of the instruments employed and from the great regularity of the different series of observations. Hence laminated silver radiates about four times less than experimenters have hitherto supposed.

This result approaches very near to that obtained lately by MM. de la Provostaye and Desains with regard to the emissive power of this same metal by the aid of the thermo-multiplier and Leslie's cube. In fact, if we continue to denote by 100 the radiating power of lamp-black, silver chemically precipitated on copper will according to them have its radiating power expressed by 5·37 in its natural state, and by 2·10 when polished; the emissive power of ordinary silver would be 2·94 when it first comes out of the flattening apparatus (*laminoir*), and 2·38 after having been burnished.

As early as the year 1838, I had deduced from some observations, that the difference of emissive power (as determined by the well-known experiment of Leslie) of a rough surface and a polished one of the same metal did not arise, as was then generally supposed, from the greater or less degree of polish of the surface, but in reality from the difference of density produced, *in metallic substances*, by the scratches on the metal made in order to change the smooth surface into a rough one, which scratches, in the ordinary cases of laminated metals, laid bare the interior part more tender and radiating than the surface; so that these changes of density sufficed to explain the phænomenon hitherto observed, even in the case where the metal is not oxidable. This proposition appeared to me to be incontestably demonstrated by the two following facts:—

1st. Silver melted and cooled slowly in the moulds, polished with oil and charcoal, and afterwards scratched with the point of a diamond so as to compress and condense the bottom of the furrows, diminishes instead of increasing its radiating power in passing from the state of polish to roughness.

2nd. This same kind of polished silver loses its emissive power as well, when smartly hammered on an anvil or passed through the flattening apparatus*.

It is hence easy to perceive that the same principle is involved in the experiments of the two French philosophers; for silver chemically precipitated in copper being much less hard and dense than polished silver, the emissive powers of these dif-

* *Bibliothèque Universelle de Genève*, année 1838.

ferent sorts of silver follow exactly the inverse ratio of the densities. The differences which I have investigated were due solely to the radiating power of the metal in its state of greatest activity, whilst the experiments of MM. de la Provostaye and Desains determine the emissive power of silver and of other metals with regard to lamp-black; whence probably has arisen the slight historical error contained in the introduction of their memoir. According to them the ratio hitherto admitted between the emissive power of metals and that of lamp-black would result as well from the experiments of Leslie as from the labours of Petit and Dulong and from my researches. It is very true that Petit and Dulong have obtained results but little differing from those of Leslie*, but no researches on this subject have been published by me. The only questions which have appeared to me sufficiently cleared up by experiment to merit the attention of philosophers, are, first, the influence of inequalities of surface, which we have just mentioned, and the influence of colour, questions which have each received a negative solution. I next examined the influence exercised by the thickness of the radiating stratum of the heated body on the phænomena of radiation, the sole cause, in my opinion, of the enormous differences observed between the emissive powers of different substances. As to the numerical determination of the radiation of metals referred to that of lamp-black, any one may easily convince himself that no mention has been made of this subject in the different memoirs which I have published on radiating heat.

I will add, lastly, that experiments analogous to those of MM. de la Provostaye and Desains had already caused me to suspect the error announced by these skilful experimenters. But the difference between the new and the old value was so great, that I was tempted to attribute it to some faults of construction in the thermo multiplier which I had employed. At present the agreement of the results obtained by such different methods as the radiation from the cubes of Leslie on the thermo-multiplier and the radiation of thermometers with metallic surfaces towards a serene sky, appears to me to remove all doubts. Let us hope that these observations will be repeated and completed by experimenters, and that before long the very inaccurate values of the emissive powers of gold and silver, of copper, tin and brass

* *Recherches sur la Mesure des Temps.* Paris, 1818, p. 75.

deduced from the researches of Leslie, Petit and Dulong, will be effaced from scientific works and replaced by measures of greater accuracy.

The parallel movement of the thermometers with polished armatures in the closed and open vessels, and above all the extremely slight difference in their indications show that in both cases these instruments give the true temperature of the stratum of air in which they are plunged, and consequently the closing of the vessel is useless, when the nature of the experiments does not require an extreme degree of precision in the measures. It is moreover quite evident that if the presence of the vessel (the sides of which tend to increase the cooling of the thermometer, by preserving it from the radiation of the ground, by reflecting towards the sky the heat radiated by the instrument towards the earth's surface, and by maintaining a certain calm in the air surrounding it) does not sensibly alter the indication of the thermometer relatively to the temperature of the stratum of air in which the instrument is plunged, this temperature will be given with so much the greater accuracy by the thermometer provided with a simple metallic case, and freely suspended by long metallic threads or placed on a support formed of tubes of tin plate, as we have above described.

The means of obtaining the true temperature of the air being known, nothing is easier than to determine the different degrees of cold, that is to say, the depressions below the temperature of the air, produced by the nocturnal radiations of different substances. In fact, it is sufficient to apply these substances on the armatures of a certain number of thermometers, introduced into their respective conical recipients and exposed to the free air during calm and serene nights, together with the thermometer armed throughout with polished metal, which gives at each instant the temperature of the air, and which, for greater perspicuity, we shall call the *atmospheric thermometer*. All the thermometric reservoirs ought to be at the same height. Each thermometer being compared, at several periods, with the atmospheric thermometer, the *frigorific action* of the substance which envelopes it will be equal to the difference between the two instruments, when this difference is preserved invariable during two or three consecutive observations.

The following are some of the results obtained by this method on the night of the 17th of October 1846:—

Name of radiating body	Temperatures		Differences	Ratios
	Of the body	Of the air		
Lamp black	14°·21	17°·61	3°·40	100
Carbonate of lead	13·94	17·30	3·36	99
Varnish	14·10	17·12	3·30	97
Isinglass	13·67	16·93	3·26	96
Glass	13·63	16·79	3·16	93
Plumbago	13·60	16·52	2·92	86

In these experiments, as well as in every other measure relating to the comparison of nocturnal cooling resulting from a difference of emissive power, it is necessary to operate *at a distance from the ground and in dry weather*; for if the air is very moist, and the aqueous vapour is precipitated by a slight degree of cold, the differences marked by thermometers whose armatures are coated, after showing themselves at first, gradually disappear.

The reason of this fact, which has already been noticed in the note to page 457, is easily perceived, if we reflect that in periods of great humidity bodies of greater or less radiating power become quickly covered with drops of water, and acquire upon the whole the degree of emissive power belonging to this liquid.

If the substances whose emissive power we wish to find cannot be applied on the metallic armature of the thermometer, such as different kinds of sand, earths, wood and leaves of plants, the armature is then left polished, and the materials* are introduced into the bottom of the conical recipient in such quantities that when heaped up they nearly cover the reservoir. The cold resulting from their radiation is propagated, as in the preceding experiments, to the thermoscopic body, which eventually marks a depression of temperature greater or less according to the serenity of the sky, the tranquillity of the air, and the nature of the radiating body.

Seven vessels prepared in this manner, the first with lamp-black in powder, the second with grass, the third with the leaves of the elm and poplar, the fourth with vegetable earth, the fifth with siliceous sand, the sixth with poplar sawdust, the seventh

* "Metals" in the French translation—evidently a mistake.

with mahogany sawdust, furnished the following results on the night of the 27th of September —

Names of radiating bodies	Temperature		Difference	Ratio
	Of the bodies	Of the air		
Lamp black	17 50	20 10	2 90	100
Different glasses with smooth leaves	17 21	20 23	2 91	103
Leaves of elm and poplar	17 17	20 10	93	101
Poplar sawdust	17 51	20 38	2 87	99
Mahogany sawdust	17 05	19 80	2 75	95
Siliceous sand	17 [illegible]	20 15	2 70	93
Vegetable earth	17 02	19 69	2 67	92

The observations were made between 8 o'clock and 11½ About midnight some clouds appeared on the horizon on the side towards Naples; in a few minutes the sky was completely covered All the thermometers marked 20 nearly at 25 minutes past 12

Here the cold is rather less, because a part of the effect due to the reflexion of the sides is no longer exerted on the thermometer, as in the preceding case but *the radiation and the consequent cooling are manifested clearly in the sand earths, wood and leaves of plants, as well as in the lamp black* And although a rigorous comparison cannot be made between these results on account of the differences of conducting power and mass of the substances submitted to experiment, the emissive power of these earths and vegetable substances did not seem to differ much from that of lamp black

Moreover, no one can doubt that the different degrees of cold observed in these experiments arose from the emission of heat into space or the elevated regions of the atmosphere by the different substances which envelope the thermometric armature, since they are seen to diminish at the very instant when clouds come and station themselves like immense diaphragms above the bodies, thus intercepting all communication between the sky and the earth, and are at length completely reduced to zero

Another proof, not less conclusive, of the same truth, is the cessation of the difference between the one and the other coated thermometer, and the facility with which they all ascend to the temperature of the thermometer whose armature is free, when the openings of the recipients are closed by the metallic discs

As to the portion of the sky which acts with the greatest intensity in this class of phænomena, it is easy to convince oneself that it consists in a certain circular space which has for its centre the zenith of the place of observation In fact, if, whilst experi

menting in serene weather, the sky becomes slowly covered with clouds, the differences between the thermometer enveloped in radiating substances and the thermometer with metallic surface, diminish very little so long as the clouds remain separated by 30° or 35° from the vertical drawn through the place of observation. As soon as this limit is passed, these differences decrease very rapidly, and totally disappear when the angular space of 35° round the zenith is completely covered with clouds.

But without waiting for these changes in the weather, which are unfortunately more frequent than the observer occupied with experiments on nocturnal radiation would wish, we may arrive at the same conclusion by a very simple artifice. In fact, it is sufficient to incline the vertical axis of the conical vessel which contains the bulb of a thermometer with blackened or varnished armature, to perceive that during this period of calm and serenity the thermometer preserves sensibly the same degree of cold until the inclination to the vertical becomes 30° or 35°; beyond this limit the temperature of the thermometer approaches that of the atmosphere, and only differs from it by a small fraction of a degree when the axis of the recipient is brought into the horizontal position

The law of the proportionality of heat to the sine of the angle formed by the radii with the normal to the element of the radiating surface, led Fourier to the consequence that calorific radiation does not proceed solely from the surface of bodies, but also from a certain depth; a result which followed most clearly from the direct observations of Leslie and Rumford.

To verify this fact relatively to nocturnal radiation, prepare two thermometers with corks, so that one may have its armature painted with only a single coat of varnish, the other with eight or ten; and after having exposed the two instruments to the free air during the night, or still better in their open conical vessels, it will be seen that the first constantly maintains itself higher than the second. At 7 o'clock in the evening on the 19th of September I placed on my terrace two of these varnished thermometers, and a third whose armature was polished. One hour afterwards the three instruments indicated 19°·4, 18°·9, 16°·5; the first indication being that of the metallic surface, the second that of the surface covered with only a single coat of varnish, and the third that of the surface which had ten coats of the same substance superimposed. The thermometers, when observed at 9 o'clock, gave 18°·2, 17°·7, 14°·8. We hence conclude that

here, as in the experiments of Leslie and Rumford, a portion of the radiation which produces the cooling of the thermometers arises from points situated at a certain depth below the surface.

This property of heat, which reveals itself during the transformation of ordinary heat into radiating heat, perfectly accounts for a phænomenon which some observers consider sufficient to completely overthrow the theory of Wells as to the formation of dew. It is generally seen at the present day, that, according to the English philosopher, dew is a necessary consequence of nocturnal radiations which give rise, in plants and other bodies exposed to the free air, to the cooling necessary to precipitate the transparent and invisible aqueous vapour pervading the atmosphere. Now if we admit as true the tendency of bodies to become colder under a serene sky, say the adversaries of Wells' theory, this tendency will be compensated by the heat of the surrounding air, especially when the body is very thin, and consequently whose mass is very small compared with the extent of its surface.

Thus the spiders' webs so profusely scattered in the country during certain seasons of the year can scarcely descend below the surrounding temperature, and ought to remain sensibly dry during the whole night; and yet precisely the opposite is observed, since, other circumstances being the same, these little corpuscles are more abundantly bathed in dew than any other substance. But the objection implies an absolute ignorance of the fact which we have just mentioned, and of the elements of physics.

In fact, as the contact of the air takes place only at the surface, and radiation occurs not merely at the surface but also at points situated at a certain depth, bodies which radiate towards a clear sky during the night may be compared to a vessel full of water the bottom of which is pierced with a number of holes, whilst, *to compensate* the loss sustained, we caused to arrive, by means of a second recipient of the same form and dimensions, water flowing from a single opening of equal diameter to one of the preceding, supposing it even the largest of all. The water would enter without ceasing into the vessel, and yet nevertheless the level of the liquid would necessarily fall. It is in this manner that the temperature diminishes in a body radiating towards the sky in spite of the heat communicated to it by the contact of the air.

Suppose now that a given quantity of matter is successively formed into discs of larger and larger size and diminishing thick

ness, the surface of contact with the air will certainly go on increasing; but the underlying stratum, which freely radiates its heat, will increase in the same proportion; so that at first sight it does not appear that after a certain interval of time there would be any difference between the cooling of very thin bodies and that of bodies having a certain thickness. But a moment's reflection is sufficient to convince ourselves that the spiders' webs are more apt to become cool than bodies of a larger volume. In fact, the threads of which these webs are composed being excessively fine, *radiate from all points of their mass*, and receive but little or no heat from the stems, leaves or earth by which they are supported*, for the conductibility, being in the inverse ratio of the diameters, becomes sensibly nothing for cylinders of extreme thinness. Therefore the nocturnal cooling of a spider's web will be quicker and more intense than that of other bodies; and since, according to the hypothesis adopted, dew depends on the degree of cold produced, the abundance of it on spiders' webs is favourable, and not the contrary, to the theory of Wells; and those who pretend to find in it so formidable an objection to the ideas received in science, only show their incapacity to judge soundly of such scientific questions.

We shall presently see analogous conclusions from other facts and other observations, which these incompetent judges regard as contrary to the explanation of dew founded on the principle of nocturnal radiation. But let us for the present remark, that, according to this principle, *the cooling of bodies must necessarily precede the precipitation of dew on their surfaces.* This fundamental proposition of Wells' principle may be easily demonstrated by means of our apparatus.

It is, in fact, sufficient to expose to the free air two of the conical vessels of tin-plate which we have described, one of which contains a thermometer armed with polished metal, and the other a thermometer armed with varnished metal. The thermometers in the closed vessel will both indicate the same temperature, but if the covers be taken off, and after having verified the fact already mentioned of the immobility of the metallic thermometer and the fall of the varnished one, we observe attentively the surface of the latter instrument, it will be seen to be at first very brilliant, then slightly clouded, afterwards more and more dull, and finally covered with drops of dew. Every other radiating

* There must be a mistake here, either in the original Italian or the French translation.—[Tr. N.]

substance will present analogous phænomena, for example, glass placed, as before described, on the bottom of the vessel.

The comparison of what takes place with leaves of gold, silver or copper, cut into strips of the same size as the glass, and superposed in the same way round the metallic armature of the thermometer, becomes then very interesting and decisive, for in this latter case there is never either cold or dew, in the former, on the contrary, the dew either does not show itself at all, or is always deposited after the thermometer has indicated a fall of some degrees below the temperature of the air. The interval between the cooling indicated by the thermometer and the precipitation of the dew is always very sensible even in the most humid weather: it frequently amounts to one or two hours when the atmosphere has its mean degree of moisture, and in periods of great dryness the glass remains dry during the whole of the night. We can also produce at will one or other of these phases, and render longer or shorter the interval of time between the indication of cooling by the thermometer and the appearance of dew, by experimenting at greater or less elevations above the level of the ground, on terraces and roofs of different heights for instance, for during calm and clear nights the moisture of the atmosphere increases rapidly in approaching the earth's surface, in consequence of certain actions and reactions of temperature between plants and the surrounding air, which we shall presently examine.

We have previously said that the experiments of Wells and others do not give the value sought of the cold produced by the radiation of a given substance, because the thermometers used by these observers having their surface of glass and being placed at different heights, we cannot deduce from their indications the true temperature of the stratum of air in which the radiating body was plunged.

In fact, our thermometers with metallic armatures in contact with any substance being compared with a thermometer of the same nature isolated in its conical vessel, have indicated to us degrees of cooling very much inferior to those resulting from the experiments of Wells, and nevertheless the recipients which surrounded these thermometers increased by reflexion and by the calm of the enclosed air, the radiation of the bodies towards the sky. If these recipients be taken away, the differences of temperature become still less, as results clearly from the following series of observations:—

Name of radiating body	Temperatures		Differences
	Of the body	Of the air	
Lamp black	14°·1	15°·4	1°·3
Varnish	14·0	15·3	1·3
Glass	14·1	15·3	1·2
Plumbago .	14·0	15·1	1·1

The emissive power of lamp-black, which has shown itself here, as elsewhere, to be one of the most radiating bodies, has been afterwards measured under many other circumstances; for to each apparatus used in the experiments were added always a couple of free thermometers, one with metallic armature, the other with blackened armature, and the difference between these two instruments never surpassed 1°·8, whatever were the clearness of the sky and the calm of the atmosphere.

There are nevertheless cases where a body freely suspended may fall 4° or 5° below the temperature of the surrounding air. In fact, the temperature of a thermometer enveloped in wool or cotton[*] is always more or less inferior to that of a blackened thermometer. But this excess of cold by no means results from a superiority of emissive power in wool or cotton above that of lamp-black; and to convince ourselves of this, it is sufficient to compare the effects of different thermometers clothed with tufts of cotton or wool more or less compact, and with tissues more or less fine and hairy, of the same substances uncoloured.

In the following table will be found collected the experimental data necessary for making these comparisons:—

Envelope of thermometer	Depression below the temperature of surrounding air		
	First series	Second series.	Third series
Cotton, carded and loose, 6 centimetres in diameter, including the reservoir of the thermometer ...	3°·8	4°·5	4°·7
The same quantity of cotton reduced to 2 centimetres, with six rounds of cotton thread . ..	2·9	3·4	3·7
Cotton cloth, commonly called *fustian*..	2·5	3·2	2·9
Fine cloth of cotton doubled .	1·9	2·2	2·4
Wool, carded and loose, 6 centimetres in diameter, including the bulb of the thermometer .	3·1	3·6	3·8
The same quantity of wool reduced to 2 centimetres, with six turns of woollen thread .	2·5	3·0	3·2
Thick flannel	2·0	2·3	2·5
Finer flannel	1·8	2·1	2·3

[*] "Carton" in the French—evidently misprint for "coton."

Thus, between the limits of these observations the cold becomes greater in proportion as the cotton and wool which envelope the thermometer become less dense and more voluminous. Now if the action increases in proportion as the matter is less dense, the cause of the *excess of cold* observed evidently does not lie in the emissive or radiating power of this matter. Whence arises then the frigorific superiority of wool and of cotton compared to lamp black? The solution of the question will become simpler if we examine the different circumstances which concur in the formation of the dew in meadows.

And entering now on the investigation of this interesting phænomenon, let us in the first place call to mind that the grass and kinds of earth and sawdust introduced into our open vessels were cooled nearly as much as lamp black in powder, and that a free thermometer coated with this latter substance does not descend more than 1·8 below the temperature of the air.

But if we wish to assure ourselves directly that nocturnal radiation lowers the temperature of the vegetables which cover the soil, we have only to place, during calm and clear nights, one of our thermometers supported by its metallic tripod, in contact with the under surface of the grass or leaves of any plant, for if the instrument be then observed, it will be constantly found lower than a thermometer with metallic armature freely suspended alongside of it in the same horizontal stratum of the atmosphere. But the difference between the two thermometers will never exceed 2°, whatever be the vigour and isolation of the plant. And the feebleness of the action manifested must not be attributed to the small mass of the leaf compared to the mass of the apparatus, for thermometers of very small size may be employed, of cylindrical or spherical form, of greater or less degrees of thinness or flattening, without any sensible variation of the final temperature indicated, although the calorific equilibrium is the more speedily established the smaller the reservoir of the thermometer, as might be easily foreseen. Now this fact, whilst completely removing all doubts of the truth of Wells' principle as to the nocturnal radiation of bodies, leads necessarily to the consideration of the theory of dew under a different aspect to that generally taken by writers on physics.

In order to incur no error by general allusions, let us take a particular example chosen from one of the best modern treatises. In the last edition of the *Traité de Physique* of M. Pouillet

we read the following.—"For dew, it is sufficient to remark, that the temperature of the air being, for example, 15° at a certain period of the night, there will be bodies at 14°, others at 13°, and the most radiating will even be at 7° and 6° or 5°, if suitably situated. Then if the air is very moist, that is if the dew-point is near 15°, nearly all the bodies will have dew upon them, the warmest in small quantity, and the colder in greater proportion. If the air is less humid, if, suppose, the dew-point is at 10°, those bodies which are higher than 10° will remain dry, those at less than 10° will be more or less covered with dew. Lastly, if the air is extremely dry, if the dew-point is below 5°, all the bodies will remain dry, the coldest as well as the warmest." After the experiments and considerations preceding, it does not appear to me to be any longer allowable to say that plants and bodies ordinarily moistened by dew are cooled 8° and 10° below the temperature of the air. The observations of Wells, Wilson and Pouillet are accurate, and certain bodies placed near the surface of the earth may indeed descend 8° or 10° below a thermometer situated at a height of 4 or 5 feet; but we cannot conclude from these observations that the differences obtained indicate the depression of temperature of the radiating body below the medium in which it is plunged; for the nocturnal cold of vegetable leaves, as we have just said, does not exceed the 2° of the Centigrade thermometer, and is therefore four or five times less than the cold admitted in works on physics and meteorology.

This great reduction in the difference of temperature between the air and the radiating bodies by no means implies that the principle of the condensation of atmospheric vapour in consequence of simple nocturnal radiations is erroneous; and to prove it, the following observation of Saussure is sufficient, which I have several times had occasion to verify in the course of my numerous researches. When the dew begins to appear, the hair-hygrometer introduced into the stratum of air contiguous to the soil marks from 90° to 98°. Hence the surrounding space is very nearly in a state of saturation; consequently it is not necessary that plants and bodies of all kinds situated near the surface of the earth should attain any very great degree of cold in order to precipitate the aqueous vapour; but the cooling of 1° or 2°, acquired by plants under the influence of a clear sky, feeble as it is compared to the 8° or 10° hitherto supposed, will

suffice to condense on the leaves a portion of the elastic and invisible vapour which pervades the surrounding air

It must be added, that the hypothesis of cooling from 1 to 10° according to the position of the body leads directly to the consequence admitted by M Pouillet himself, namely, that in many cases where the atmospheric moisture is but little certain plants ought to be covered with dew, and others preserve their habitual state of dryness and those who have had occasion to cross the fields when the sun is below the horizon will no doubt have remarked that there is absolutely no dew, or else it is found distributed in nearly equal proportions on all low plants, whatever be the position of their leaves with regard to the sky Now the fact of an absolute absence of dew or of its general diffusion in a series of surfaces situated at all sorts of inclinations, indicates clearly that the hypothesis of an extreme humidity of the air is the only one which has place in nature when there is a formation of dew And the cause of this fact, which results so evidently from the presence of dew on all low plants as well as from hygrometric observations, is connected, if I am not mistaken, with another fact, which has long been known to philosophers, but which has hitherto remained isolated in science, in spite of its great importance in phænomena of nocturnal radiation

Wilson was the first to show that the effect of the radiation of bodies towards the sky is sensibly the same at all temperatures, so that in nights equally calm and serene, the same substance is always cooled to the same extent whatever may be the temperature of the atmosphere

Snow, for instance, would, according to the experiments of Parry and of Scoresby, be cooled by about 9°, when the temperature of the air descends to −1°, or to −2, or to −21°, or to −22° so that the bulb of a thermometer introduced into the first layer of the snow mantle which covers the soil of the northern regions during the greatest part of the year would mark −10 or −11 in the first case, and −30° or −31° in the second I have never as yet found myself in circumstances favourable to the verification of the observations of these two celebrated navigators, but I have been able to convince myself of the truth of the principle of Wilson, by experimenting with thermometers surrounded with radiating substances, in the same manner as M Pouillet has done Nevertheless as the mean values obtained by me do not comprise so extended a range of temperature as that em

braced by the beautiful experiments of M. Pouillet, I shall bring forward his series of observations, which will serve us as a starting-point in conceiving clearly how it happens that the stratum of air situated near the earth's surface always descends nearly to the point of greatest moisture before the dew begins to appear.

These observations, inserted at page 610 of the second volume of his *Traité de Physique* (4th edition), have reference to the cold produced by the radiation of swansdown, suitably isolated by means of an apparatus which the author designates by the name of *actinometer*.

Putting aside everything superfluous to the object with which we are at present occupied, we shall have the two reduced tables here placed by the side of each other:—

Table of Nocturnal Cooling under different Temperatures of the Air.

Days of observation.	Hours	Temperature of air	Temperature of actinometer	Differences.	Days of observation	Hours	Temperature of air	Temperature of actinometer	Differences.
	h m	°	°	°		h m	°	°	°
April 11	5 30 A M	5.0	−3.0	8.0	May 5	5 0 P M	25.5	19.0	5.0
11	6 0 A M	5.5	−2.3	7.8	5	6 0 P M	25.1	17.5	7.6
14	7 0 P M	8.5	−0.8	7.7	5	7 0 P M	23.1	15.0	8·1
14.	8 0 P M	7.0	−0.5	7.5	5	8 0 P M	22.9	13.9	9.0
14	9 0 P M	5.8	−1.6	7.1	5	9 0 P M	21.5	12.5	9.0
14	10 0 P M	5.0	−2.4	7.4	5	10 0 P M	17.5	10.0	7.5
15	4 30 A M.	1.0	−6.0	7.0	6	1 0 A M	12.1	5.0	7.1
15.	5 0 A M	1.0	−6.0	7.0	6	1 30 A M	12.1	5.0	7.1
15	6 0 A M	1.6	−5.2	6.8	6	5 0 A.M	12.0	6.0	6.0
20	8 0 P M	5.6	−0.8	6.4	June 23	7 0 P M	20.0	12.0	8.0
20.	9 0 P M	4.5	−2.0	6.5	23	8 0 P M	17.8	10.5	7.3
20	10 0 P M	3.6	−3·0	6.6	23	9 0 P M	17.6	10.7	6.9
21	4 30 A M	0.0	−7.0	7.0	23	10 0 P M	16.3	9.2	7.1
21	5 0 A M	0.0	−7.0	7.0	24	1 0 A M	11.3	5.3	6.0
21	5 30 A M	0.1	−6.5	6.6	24	4 30 A M	11.5	5.6	5·9
		54.2	52.5	106.7			266.3	158.1	108.2
Means		3.6	3.5	7.11	Means		17.75	10.54	7.21

The data contained in these two tables prove that swansdown is cooled about 7° (mean value) under the nocturnal influence of a clear sky, whether the temperature of the air fall to nearly zero or rise to 20° or more degrees.

It is almost needless to add, that the observations of M. Pouillet, and the determination by Parry and Scoresby of the

nocturnal cooling of snow, have been obtained by a method which we believe to be inaccurate, in consequence of the unequal circumstances in which the thermometers used to measure the temperatures of the air and of the radiating bodies were placed. If properly measured the cooling of the snow and of the swans down would certainly be less; but it matters little what the absolute value is here, since we are only considering the constancy of the effect under variations of atmospheric temperature.

The results which we have announced prove therefore the truth of the proposition above asserted namely, that *a body exposed during the night to the influence of a sky of equal clearness and calmness is always cooled to the same extent, whatever may be the temperature of the air.*

This fact alone, thoroughly established by experiment, will lead us to a clear and complete explanation of all the phænomena which precede and accompany the formation of dew.

Article XIII.

On the Excitation and Action of Diamagnetism according to the Laws of Induced Currents. By Prof. W. Weber*.

[From Poggendorff's Annalen, Jan. 7, 1848.]

THE repulsion of bismuth by a magnet, first observed by Brugmans in 1778, had remained almost unknown until Faraday discovered it anew and examined it more carefully, and thus laid the foundation for the new doctrine of *diamagnetism*, the further development of which has become an important physical problem. To solve this question little can be expected from the more delicate processes of measurement, owing to the feebleness of the diamagnetic forces of bodies, even when very powerful electro-magnets act upon them, and it is therefore the more to be expected that we shall become acquainted with the nature of diamagnetism from the *various modifications* of its effects, the discovery of which is possible even in the case of the most feeble forces. The object of the following experiments is to establish with greater certainty and precision, from some peculiar modifications of the diamagnetic effects, a hypothesis already advanced by Faraday to explain the diamagnetic phænomena, and then to deduce this hypothesis required for the explanation of diamagnetic phænomena from known laws.

Diamagnetic bismuth repels both the north and south pole of a magnet, and is repelled by them. This *indifferent* repulsion of opposite poles might appear of little importance if the origin of the magnetic force were to be sought for in the unvarying metallic particles of the bismuth itself; for we are accustomed to assume generally of the ponderable bodies that they oppose without distinction equal resistance to the movements both of the two opposite magnetic as well as of the two electric fluids. But the action *at a distance* might appear more surprising than this indifferent effect, were we to admit that the diamagnetic force has its origin in the unvarying metallic particles of the bismuth itself, because it would be the first case in which the action of a ponderable upon an imponderable body *at a distance* had been observed. It appears therefore above all things im-

* Translated by W. Francis, Ph.D., F.L.S.

portant to decide the question, whether the source of the diamagnetic force acting at a distance is to be found in the unvarying ponderable constituents of bodies or whether it arises from an *imponderable* constituent, and is connected with a certain *distribution* thereof

To decide this question the experiment made by M Reich* is of the highest importance according to which both north and south poles when they act at the same time on the same side of a piece of bismuth, by no means repel it with the sum of the forces which they would individually exert, but only with the difference of these forces

From this single experiment it might be concluded with the greatest probability, that the origin of the diamagnetic force is not to be sought for in the never changing metallic particles of the bismuth but in an imponderable constituent moving between them, which, on the approach of the pole of a magnet, is displaced and distributed differently according to the difference of this pole The origin of the diamagnetic force is thus placed in the reciprocal action of two imponderable bodies instead of in the reciprocal action between ponderable and imponderable bodies at a distance, and the similar effect upon opposite poles is then explained by the different distribution of the imponderable constituent in the bismuth which is produced by the antithesis of the poles The simultaneous approach of two opposite poles on the same side must however have for result, that the imponderable constituent in the bismuth can neither assume the one or the other distribution upon which depends the appearance of the diamagnetic force, whence the disappearance of the diamagnetic force in this case is self evident

But if it be now further asked, what is the nature of the imponderable constituent which is distributed in such a different manner in the bismuth on the approach of a north or south pole, and then with this distribution constantly reacts with a repulsive force upon the approached pole, there present themselves only the two magnetic fluids, or the two electric fluids in the form of molecular currents At all events, before any other assumption can appear admissible, the impossibility of explaining the phænomena in question by the known relations of the above imponderables must be shown

From this it will be seen that Reich's experiment may be

* Philosophical Magazine for February 1849 p 127

employed to establish more firmly a view already advanced by Faraday (Experimental Researches, Art 2429, 2430) Faraday there states that "Theoretically, an explanation of the movements of the diamagnetic bodies, and all the dynamic phænomena consequent upon the actions of magnets on them, might be offered in the supposition that magnetic induction caused in them a contrary state to that which it produced in magnetic matter, i e that if a particle of each kind of matter were placed in the magnetic field both would become magnetic, and each would have its axis parallel to the resultant of magnetic force passing through it, but the particle of magnetic matter would have its north and south poles opposite, or facing towards the contrary poles of the inducing magnet, whereas with the diamagnetic particles the reverse would be the case, and hence would result approximation in the one substance, recession in the other

"Upon Ampère's theory, this view would be equivalent to the supposition, that as currents are induced in iron and magnetics parallel to those existing in the inducing magnet or battery wire, so in bismuth, heavy glass and diamagnetic bodies, the currents induced are in the contrary direction This would make the currents in diamagnetics the same in direction as those which are induced in diamagnetic conductors at the *commencement* of the inducing current, and those in magnetic bodies the same as those produced at the *cessation* of the same inducing current No difficulty would occur as respects non conducting magnetic and diamagnetic substances, because the hypothetical currents are supposed to exist not in the mass, but round the particles of the matter"

I shall now submit this ingenious view, first proposed by Faraday, and which has obtained greater probability from Reich's experiment, to a still more direct criticism by the following experiments, which, in my opinion, scarcely leave a doubt of its correctness

All the diamagnetic forces hitherto observed have exhibited a repulsive, never an attractive action, but from Faraday's assumption, it follows that diamagnetic forces must likewise occur which act *attractively* upon the pole of a magnet, and such cases may easily be determined more accurately and tested by experiment

But for this purpose we must not observe the force which the diamagnetic bismuth exerts upon that pole by which it has been rendered diamagnetic, but those forces which this bismuth

exerts upon other magnet poles at a distance, and which have no influence upon its diamagnetic condition.

Now if a piece of bismuth is placed in the plane which is bisected at right angles by a small magnet needle suspended by a silk thread and symmetrically magnetized, it is evident that the poles of the small needle can have no influence, or at least no perceptible influence, upon the diamagnetic state of the distant piece of bismuth, according to Reich's experiment. In fact it is easily seen that the needle experiences not the slightest deflection by the bismuth.

But if we arrange a powerful horse-shoe magnet of iron, so that the locality previously occupied by the bismuth is situated in the free space between its two poles, and the magnet is at the same time brought into such a position that its magnetic axis prolonged bisects the needle, this powerful magnet will exert a very great momentum of rotation upon the needle. But if this rotatory action excited by the horse-shoe magnet is compensated by another equally powerful but opposite rotatory action of a bar magnet brought to bear upon the needle from the opposite side, we can cause the needle to re-assume its original position and its original vibratory power (sensitiveness), so that with respect to the needle it is just the same as if no magnet acted upon it.

Now if, after these preliminary arrangements, the same piece of bismuth which previously had no action upon the needle is brought to the same position as before, *i. e.* between the two poles of the horse-shoe magnet, a very perceptible and measurable effect is exhibited, viz. a deflection of the needle, owing to one pole being repelled and the other attracted.

If the poles of the magnets, the effects of which upon the needle are compensated, be reversed, and the experiment repeated, it is found that the same piece of bismuth brought to the same spot and in the very same position, now produces exactly the opposite deflection.

If, lastly, a piece of iron is substituted for the bismuth, it is found that the deflection produced by the latter is the opposite of that produced by the former.

These experiments may be variously modified, but in every case the force of the bismuth must be observed upon other magnet poles than that which determines the diamagnetic condition of the bismuth; they all confirm the assertion that bismuth

constantly acts upon such poles in an opposite manner to iron in its place, that it consequently repels where iron attracts, and attracts where iron repels; in short, that at other magnet-poles than that which diamagnetizes the bismuth, we as frequently observe attractive as repulsive forces of the bismuth. For instance, if the one extremity of the bar of bismuth was brought near the north end of a powerful magnet, while its other extremity was approached to the north end of the magnet-needle, the latter was attracted; but if the same extremity of the bar of bismuth was brought near to the south end of the powerful magnet, the north end of the magnet-needle was repelled by the other extremity of the bar of bismuth approached to it.

We may hence regard Faraday's supposition as proved, at least in so far as it places the origin of the diamagnetic force, not in the unvarying metallic particles of the bismuth itself, but in a variable distribution which occurs in the bismuth, and acts upon other magnets in the same manner as a definite distribution of magnetic fluids.

In order, lastly, to remove every doubt as to its being really nothing else than the magnetic fluids, or their equivalent, Ampère's currents, which are subject to this variable distribution in the bismuth, it may be required to be shown by experiment, not merely that the effects connected with the *presence* of the diamagnetic and of a certain magnetic state are equal, but likewise that the effects connected with the *origin* of the two states are so.

It is well known that, according to the laws of induction discovered by Faraday, the motion of the magnetic fluids in a body, or the rotation of the molecular currents of Ampère, is connected with an electrical action at a distance upon neighbouring conductors, owing to which an electric current is excited or induced in the latter.

Consequently, if the two magnetic fluids, or their equivalents, Ampère's currents, are really present in the diamagnetic bodies, which are set in motion or rotated under the influence of a powerful magnet, they must induce an electric current in a neighbouring conductor at the moment this change takes place.

Now to observe this induced current itself, it is requisite that no other current be induced in the same conductor, for instance by the powerful magnet to which the bar of bismuth is approached. For this purpose therefore the force of this magnet must be retained unaltered during the experiment, which pre-

supposes in an electro-magnet a constant galvanic current. But on the other hand, the conductor upon which the bismuth is to act must have a fixed immutable position to that magnet, so that it incloses the space in an annular form, in which the bar of bismuth has to be brought in order to produce in it the diamagnetic distribution by the influence of the magnet. That, lastly, the current induced by the bismuth can be observed by continuing the two extremities of the above annular conductor, and connecting them with the ends of the multiplier of a sensitive galvanometer, requires no further explanation.

But with respect to the power of this current induced by the bar of bismuth, it may readily be estimated *a priori* how small it will be if we consider how feeble the diamagnetic forces are in comparison to the magnetic forces of the iron substituted for the bismuth. On further examination, it results that the induced current must be so feeble that it can no longer be observed if all the conditions do not act together most favourably for the object.

The following arrangements were made to attain this end, viz. *to induce galvanic currents in a neighbouring conductor by the diamagnetic action of the bismuth, and thus actually to observe the induced currents.* An iron nucleus 600 millimetres in length, coated several times with thick copper wire, was used as electro-magnet. To the circular terminal surface, 50 millimetres in diameter, of this iron nucleus was fixed the annular conductor, which consisted of copper wire 300 metres long and $\frac{2}{3}$ millimetres thick, well spun with silk and coiled upon wooden cylinders. The space included in this annular conductor, in which the bar of bismuth was to be placed, was 140 millimetres in length and 15 millimetres in breadth; the bar of pure precipitated bismuth was somewhat thinner. The extremities of the annular conductor were connected with a commutator, as were also the extremities of the multiplier of a very sensitive galvanometer, the magnet-needle of which was provided with a mirror in which the image of the distant scale was observed by a telescope directed towards it. The galvanometer was moreover provided with so effective a damper that it was scarcely possible to observe any vibration of the needle.

Now whilst a very powerful and constant galvanic current passed through the thick wire of the electro-magnet, the bar of bismuth was withdrawn from the annular conductor in which it was situated, the commutator changed, and the bar of bismuth

again inserted, the commutator again changed, and the bar of bismuth withdrawn, &c. During this experiment, continued for about 1 minute, the state of the galvanometer was read off at intervals of about 10 seconds.

A second series of experiments was now made, but with this difference, that the commutator assumed that position on withdrawing the bar of bismuth which it had occupied in the first series on inserting the bismuth, and *vice versâ*.

The third series was an accurate repetition of the first, and so forth.

Previous to commencing each series the state of the galvanometer was observed, without however waiting until the needle had attained a perfect state of rest. Each series was begun by withdrawing the bismuth.

In the following table the whole of the readings made on the galvanometer are arranged together. The different series are distinguished by Roman numbers; the two states of the commutator which occurred in the different series on the withdrawal of the bar of bismuth are distinguished in the heading by A and B. The state of the galvanometer before commencing each series is also noticed in the heading.

I *A* 512·3	II *B.* 517·4	III *A* 515·0	IV *B* 517·2	V *A* 517·0	VI *B.* 523·0	VII *A.* 524·7
513·3	513·0	519·5	517·1	518·2	522·0	526·0
514·1	512·9	520·7	517·5	518·7	519·0	528·0
511·6	512·8	519·1	516·2	525·0	518·5	530·0
515·3	514·2	519·2	516·7	525·1	519·0	530·7
515·6	515·2	518·3	517·7	523·0	521·0	530·0
516·7	516·0	515·5	—	—	—	528·5
514·92	514·02	518·72	517·01	522·00	519·00	528·87

Now if we compare the states of the galvanometer in the odd alternate series, where the commutator occupied the position A on withdrawing the bismuth from the annular conductor, with the mean value in the bottom line, it is seen that the latter is always somewhat *greater*. For instance, the mean values are—

1 Series 514·92=512·3+2·62
3. " 518·72=515·9+2·82
5. " 522·00=517·0+5·00
7. " 528·87=524·7+4·17

The same comparison yields for the even series, where the commutator occupied the position B on removing the bismuth from the annular conductor, always a somewhat *smaller* mean value

2	Series	514.02 = 517.4 − 3.38
4	"	517.04 = 517.2 − 0.16
6	"	519.90 = 523.0 − 3.10

It should be borne in mind that the state of the galvanometer observed before the commencement of each series was not exactly that of rest. To avoid the uncertainty arising from this, the reading made previous to each series may be wholly excluded from the calculation, and the comparison restricted to the mean values of the several series. The comparison of the mean value of the 2nd to the 6th series, with the mean from the immediately preceding and succeeding series, then gives the following results:—

2	Series	514.02 = 516.82 − 2.80
3	"	518.72 = 515.53 + 3.19
4	"	517.04 = 520.36 − 3.32
5	"	522.00 = 518.47 + 3.53
6	"	519.90 = 525.43 − 5.53

We see then also from this that in the uneven series in which the commutator occupied the position A while the bismuth was withdrawn from the annular conductor, the state of the galvanometer was constantly somewhat higher, and that the reverse occurred in the even series in which the commutator had the position B on the removal of the bar of bismuth. The differences are somewhat greater for the last than for the first series, which is easily explained from the change of induction being gradually accelerated.

Corresponding experiments were now made for the purpose of direct comparison, the bar of bismuth being exchanged for a slender bar of iron. The induced current was then so powerful that no repetition could be made as in the case of the bismuth, and that only the extreme end of the iron bar could be inserted in the annular conductor. And even then the induced current was so powerful that the deviation of the needle could not be observed on the galvanometer, but merely the direction, whether the position of the galvanometer rose, *i. e.* went from lower to higher divisions of the scale, or the reverse.

First Experiment.

Position of the commutator A.

Increasing numbers on inserting the iron bar in the annular conductor.

Decreasing numbers on withdrawing the iron bar from the annular conductor

Second Experiment.

Position of the commutator B.

Decreasing numbers on inserting the iron bar in the annular conductor.

Increasing numbers on removing the iron bar from the annular conductor.

The position of the commutator A, and the case in which the iron bar was removed from the annular conductor, for which consequently a *decrease* in the deflection of the galvanometer was observed, will serve to compare this experiment made with iron with the former relative to bismuth. In the above experiments with the bismuth, this case corresponds to the uneven series, for which a *higher* state of the galvanometer resulted with the induction continued in the same direction. It results consequently that the bismuth induced a positive current under the same conditions that iron induced a negative one, and *vice versa*.

Hence the induction of electric currents by the diamagnetization of the bismuth is proved, and it is at the *same time* evident that the direction of these currents is constantly the reverse of those induced by iron under the same circumstances, precisely as it should be if bismuth contained magnetic fluids or their equivalent, Ampère's currents, which are set in motion or rotated under the influence of powerful magnets in exactly an opposite direction to that in iron. The view advanced by Faraday appears therefore to be placed beyond all doubt.

Now although a rule has been found according to which the variable diamagnetic conditions of bodies are determined for all cases in such a manner that the collective effects appear as a necessary consequence according to magnetic and electro-dynamic laws, the *cause* of this rule remains still unknown and unexplained according to magnetic and electro-dynamic laws. For if magnetic fluids are really contained in the diamagnetic bodies, on the approach of a magnet-pole, the one fluid must be attracted, the other repelled; and the direction of the separation

of the two fluids is, according to this, necessarily determined by magnetic laws. But this direction is exactly the reverse of that stated in the above rule. Exactly the same however, obtains upon the other assumption, which presupposes the existence of Ampère's molecular currents in diamagnetic bodies instead of the magnetic fluids, which on the approach of a pole of a magnet should be rotated in a direction determined by electro-magnetic laws. But this rotation is exactly the reverse of that indicated by the above rule. There exists consequently a contradiction between the above rule of *excitation* and the laws of the *activity* of the diamagnetic condition. Until this contradiction is removed, all the diamagnetic conditions of bodies continue to form a group of isolated facts without any connection with other phænomena, just as those of rotation magnetism formed a similar group until Faraday gave the key to their solution by his discovery of induction.

In the preceding observations which referred to the *effects*, it was indifferent whether separate magnetic fluids or Ampère's molecular currents of the same direction constitute the excited diamagnetic state of bodies. This is no longer the case in the following considerations which relate to the *causes*, i. e. to the forces exciting the diamagnetic state of bodies. For if it were a certain distribution of the magnetic fluids which constituted the diamagnetic condition of bodies, no account, as above shown, could be given of the forces producing them, at least this distribution could not be explained from the known *magnetic* forces which act upon those fluids. But the case is different if the diamagnetic condition of bodies is constituted by molecular currents of like direction; for a system of molecular currents of like direction can obtain in a two-fold manner. In the *first* place, it is possible that the molecular currents existed previously in the bodies, and that only one force acted upon these already existing currents which communicated the *same direction* to them; but, *secondly*, it is also possible that the currents of like direction, which form the diamagnetic condition of bodies, did not previously exist, but first *originated* or were *induced* on diamagnetizing the body. Now if one of these two possible cases falls to the ground for the same reasons as that of the above considered distribution of magnetic fluids, the other possible case for the molecular currents still remains, according to which they have been *produced by induction*.

Hitherto it has never been a question of *induced molecular currents*, but solely of fixed invariable molecular currents, according to Ampère's definition, to whom indeed the origin of currents by induction was unknown. But it is evident if the existence of molecular currents be admitted, we must further allow that their intensity may be increased or diminished, and that even new currents of this kind may be produced by the very forces which produce currents in larger circuits.

If we go back to *induction* in order to explain diamagnetism, it might at first sight be doubted whether it is really necessary to admit induced molecular currents for this purpose, or whether the currents induced in large circuits are not of themselves sufficient. These currents would, it is true, be able to produce all diamagnetic phænomena if they were *permanent*; but as these currents, which are subject to Ohm's laws, are not permanent, but instantly disappear with the inducing force, and can only be maintained by continued induction, they can for this reason alone not serve to explain diamagnetism.

But if the rapid disappearance of these currents is the sole reason of its being impossible to deduce thence the diamagnetic condition of bodies, there appears to be no reason why the persistent diamagnetic state of bodies should not be ascribed to *induced molecular currents*, as these must behave in all other respects like those currents, and differ only in possessing that *permanency* which is wanting in the others. For the difference between those currents which move through conductors in large circuits and these molecular currents, consists solely in the circumstance that the circulating electricity of the former is so quickly deprived of its active force in passing to the molecules of the conductor, that it would come to rest in an immeasurably small time if the loss it sustained were not constantly replaced by continuous electro-motive forces, whence it results that currents of this kind are, according to the laws of Ohm, constantly proportional to the existing electro-motive force, and instantly disappear with the electro-motive force. The reverse applies to the molecular currents which do not pass through a conductor from molecule to molecule, but circulate around a single molecule, to which consequently the above reason, deprivation of their active force, does not apply. These currents therefore persist of equal intensity without any electro-motive force.

Now admitting an *inducing force* which acts upon the elec-

tricity of a conductor, the latter is set in motion, and this motion distributes itself according to laws in proportion to the capacity for conduction between all the paths which the conductor presents; consequently a portion of the motion must likewise take its course around the individual molecules of the conductor and form *induced molecular currents*, which because they find no resistance in their course around the molecules, by which they might be retarded, must continue in their full strength until in consequence of a new opposite induction, other induced molecular currents are added which neutralize the previous ones.

If therefore, with Ampère, we admit *molecular currents* in the doctrine of electro-magnetism, we must at present, as a necessary consequence after the discovery of induction, adopt *induced molecular currents* in the doctrine of magneto-electricity, and must ascribe permanence to all, whether they have always existed or been first produced by induction. Assuming this, it results that all bodies in which diamagnetic effects have been observed, must have been acted upon by forces which must have induced molecular currents, and indeed such as produce the effects designated by the name of diamagnetic.

The latter follows from the fact, that a magnetic force tends to give such a direction to an *existing current* that its course is exactly opposed to that of a *current induced by the increase of that magnetic force*. Consequently, if this induced current is a molecular current which is persistent, it will likewise have permanently the opposite effects of another molecular current which existed (for instance in iron) independently of the increase of that magnetic force, but has acquired its present direction by means of that force. The opposite behaviour of the diamagnetized bismuth and of the magnetized iron follows according to this from known laws. The essential difference between bismuth and iron would then be this, that molecular currents, whose direction however is not unalterable, exist in iron independently of any external excitation, but subject to the influence of external forces, which is not the case in bismuth. However, bismuth and iron may in so far be rendered equivalent as a decreasing or increasing magnetic force induces in both fresh persistent molecular currents which however must be much weaker in the iron than those existing in it already, independently of such induction.

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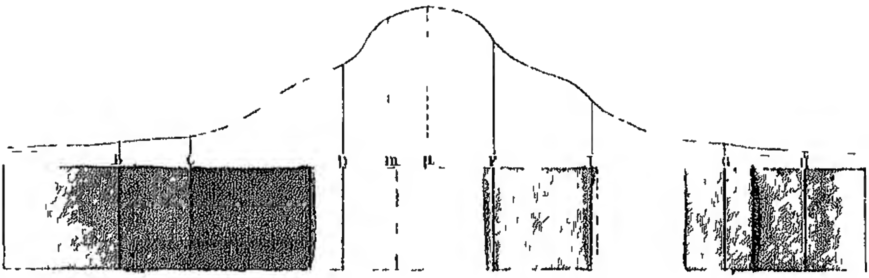

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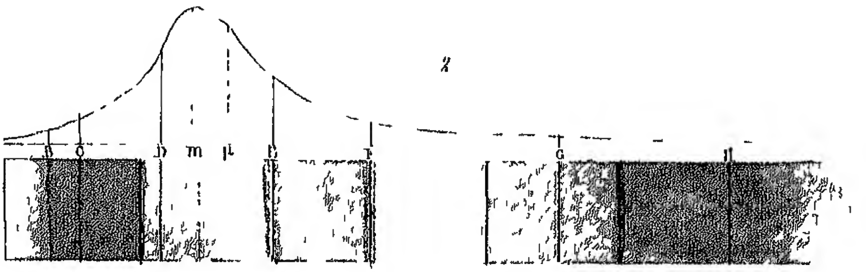
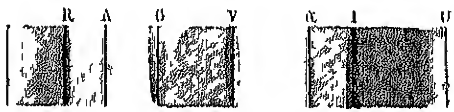
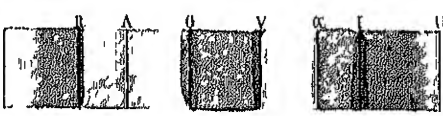






SCIENTIFIC MEMOIRS.

VOL V.—PART XX.

ARTICLE XIV.

On the Measurement of Electro-dynamic Forces. *By* W. WEBER.

[From Poggendorff's *Annalen*, vol. lxxiii p. 193, January 1848.]

A QUARTER of a century has elapsed since Ampère laid the foundation of electro-dynamics, a science which was to bring the laws of magnetism and electro-magnetism into their true connexion and refer them to a fundamental principle, as has been effected with Kepler's laws by Newton's theory of gravitation. But if we compare the further development which electro-dynamics have received with that of Newton's theory of gravitation, we find a great difference in the fertility of these two fundamental principles. Newton's theory of gravitation has become the source of innumerable new researches in astronomy, by the splendid results of which all doubt and obscurity regarding the final principle of this science have been removed. Ampère's electro-dynamics have not led to any such result; it may rather be considered, that all the advances which have since been really made have been obtained independently of Ampère's theory,—as for instance the discovery of induction and its laws by Faraday. If the fundamental principle of electro-dynamics, like the law of gravitation, be a true law of nature, we might suppose that it would have proved serviceable as a guide to the discovery and investigation of the different classes of natural phenomena which are dependent upon or are connected with it; but if this principle is not a law of nature, we should expect that, considering its great interest and the manifold activity which during the space of the last twenty-five years that peculiar branch

of natural philosophy has experienced, it would have long since been disproved. The reason why neither the one nor the other has been effected, depends upon the fact, that in the development of electro-dynamics no such combination of observation with theory has occurred as in that of the general theory of gravitation. Ampère, who was rather a theorist than an experimenter, very ingeniously applied the most trivial experimental results to his system, and refined this to such an extent, that the crude observations immediately concerned no longer appeared to have any direct relation to it. Electro-dynamics, whether for their more secure foundation and extension, or for their refutation, require a more perfect method of observing, and in the comparison of theory with experiment, demand that we should be able accurately to examine the more special points in question, so as to provide a proper organ for what might be termed the spirit of theory in the observations, without the development of which no unfolding of its powers is possible.

The following experiments will show that a more elaborate method of making electro-dynamic observations is not only of importance and consideration in proving the fundamental principle of electro-dynamics, but also because it becomes the source of new observations, which could not otherwise have been made.

Description of the Instrument.

The instrument about to be described is adapted for delicate observations on, and measurements of, electro-dynamic forces, and its superiority over those formerly proposed by Ampère depends essentially upon the following arrangement.

The two galvanic conductors, the reciprocal action of which is to be observed, consist of two thin copper wires coated with silk, which, like multipliers, are coiled on the external part of the cavities of two cylindrical frames. One of these two coils incloses a space which is of sufficient size to allow the other coil to be placed within it and to have freedom of motion.

When a galvanic current passes through the wires of both coils, one of them exerts a rotatory action upon the other, which is of the greatest intensity when the centres of both coils correspond, and when the two planes to which the convolutions of the two coils are parallel form a right angle with each other. The common diameter of both coils is the axis of rotation. This respective position of the two coils constitutes the normal position, which

they obtain in the instrument. Hence also the common diameter of the two coils, or their axis of rotation, has a vertical position, in order that the rotation may be performed in a horizontal plane.

That coil which is to be rotated, to allow of the onward transmission and return of the current, must be brought into connexion with two immoveable conductors; and the main object of the instrument is to effect these combinations in such a manner that the rotation of the coil is not in the least interfered with even when the impulse is the least possible, as occurs when these connexions are effected by means of two points dipping into two metallic cups filled with mercury in which the two immoveable conductors terminate, as in Ampère's arrangement. Instead of these combinations, which on account of the unavoidable friction do not allow of the free rotation of the coil, in the present arrangement two long and thin connecting wires are used, which are fastened at their upper extremities to two fixed metallic hooks, in which the two immoveable conductors terminate, and at their lower extremities to the frame of the coil, and are there firmly united to the ends of the wires of the coil. The coil hangs freely suspended by these two connecting wires, and each wire supports half the weight of the coil, whereby both wires are rendered equally tense.

These two connecting wires thus effect the transmission of the galvanic current from one of the immoveable conductors to the coil, and back to the other immoveable conductor, and they effect this without the least friction interfering with the rotation of the coil.

These connecting wires are also of service, because each rotation of the coil through a certain angle corresponds to a definite rotatory momentum, which tends to diminish this angle, and is proportional to the sine of the angle of rotation, whence a standard is formed for all rotatory momenta, by the aid of which any other rotatory momentum acting upon the coil may be measured. This is effected according to those simple laws which Gauss has developed in the case of the bifilar magnetometer. Lastly, this measure may be made more or less delicate at pleasure, or as occasion may require, by the approximation or separation of the two connecting wires. This method of suspension not being accompanied with any friction, allows of increase in the weight of the suspended coil, which may be any amount provided it is not more than the connecting wires are capable of supporting. Hence a very long wire may be wound

many times around the coil, and thus a very strong multiplication of the galvanic force be obtained. Moreover, this rotating coil may without injury be loaded with a speculum which also rotates, and here as in Gauss's magnetometer may be used for the delicate measurement of angles; for provided friction be excluded, the application of delicate optical instruments in this case also does not form any impediment. Regarding the construction of the instrument in detail, as this has been described very perfectly by M. Leyser, the instrument maker in Leipzic, I shall insert the explanation which he has given, and which refers to the figures sketched by him, Plate III. fig. 1–10. The instrument is called an *Electro-dynamometer*.

Description of the Electro-dynamometer.

Fig. 1, Plate III. represents the little frame for supporting the reel which vibrates in the multiplier, seen diagonally. This frame consists of two round ivory discs, *a a* and *a a*, which are riveted to two ivory plates, *b b'* and *b b'*; their distance apart is regulated by a small ivory roller, *c*. The latter is hollow, so that a metallic rod can be passed through it, and by means of a screw each of the discs with its plate can be fixed to the ends of the roller, and thus a reel is formed for the reception of the wire. One end of the wire to be coiled passes through the small hole *d*, and projects from it. When the wire is placed upon the reel and the end fixed by means of silk, the metallic supports, *e e e* and *e e e'*, of the reel are fixed to the ends of the plates above mentioned; thus, one support, *e e e'*, to which the speculum *f f* is screwed at *g*, is riveted at *b' b'*, whilst the other support *e e e*, to which the counterpoise *h h* is fixed by the screw *i*, is fastened by screws at *b b*, so that this support, near the screws *b b*, may be thrown back in the direction *b b'*, in order that the entire reel may be conveniently placed in the multiplier. The commencement of the reel, which was left projecting through the hole at *d*, is now placed lengthwise along a portion of the plate *b b'* towards *b'*, until the circumference of the reel admits at *l* of its being again placed within the frame, and then ascending to the support of the speculum, where by means of a small screw *m'* above the point at which the speculum is fixed, it comes into metallic contact with the support. The end of the reel is also brought into metallic contact with the other support by means of the screw *m*; this end must however be long enough not to stand in the way

of the support when it is thrown back. When the speculum ff is now placed at g, and its counterpoise hh at i, the reel is prepared for suspension in the multiplier by the metallic threads. For this purpose both the supports of the reel terminate at c and c' in hooks or pieces in the form of T, and the bifilar metallic threads are furnished below with a small ivory beam, ll, which towards each end terminates in a metallic plate, and this again in a small metallic cylinder; the latter fit into the above hooks or upsila of the support, and thus receive the reel. The bifilar metallic threads no and $n'o'$ are united to the cross beam ll in the following manner. The commencement n of the thread no is fastened by means of a screw to the metallic plate i, proceeds a short distance towards l, and then returns through a small hole at the end of the plate beneath the beam ll to its centre p, where it runs through a small hole again above the beam, and can then be continued to o and further. The thread $n'o'$ is arranged in the same manner, its direction however being reversed; in the centre p of the beam ll each has a separate aperture, through which it passes; these lie very near each other, but are separated and kept isolated by the ivory. The index qq is placed upon the centre of the beam before the metallic threads no and $n'o'$ are inserted.

Fig. 2 exhibits the lateral view of the vibrating reel, with the coil as placed upon the beam, and the mirror and counterpoise adapted and vibrating on the bifilar metallic threads. Only the very narrow anterior portion of the index is perceptible.

Fig. 3 represents the reel seen at right angles to the surface of the speculum; the hooks or upsila, as also the index vibrating above the scale-plate ee, are very distinctly seen.

Fig. 4 presents the view from above, in which the beam and the index form a right angled cross.

Fig. 5 serves to illustrate the further course of the bifilar metallic thread to its termination; for the sake of distinctness it is represented of twice the size of the other figures, and as seen in a vertical section. The bifilar metallic threads continue to ascend from o and o', inclosed in a brass tube; they are wound round the moveable rollers a and a', and are finally fixed to the ivory roller B at b and b' round rotating pegs. The threads can be wound up or unwound on these pegs or small rollers by means of a small key, according as the weight of the vibrating reel may render this requisite; the small rollers a and a' are also necessarily turned round at either of these operations. The ivory

cylinder itself B, with the prong and the screw *e e*, can also be screwed up or down by means of the nut *ff* and thus the vibrating reel may be arranged in the proper position as regards the multiplier in the centre of which it should oscillate. At the same time the roller B which is moveable in the prong *e e* round the peg *m* assumes a state of equilibrium as soon as the vibrating reel is suspended freely from the bifilar metallic wires since these wires act at *b* and *b′* as it were at the ends of a lever the centre of motion of which is at *m*. Thus the load of the vibrating cylinder is equally divided between the two threads.

To allow of the approximation or separation of the two bifilar wires the rollers *a* and *a′* are set in broad prongs, which as seen in the figure terminate in screws by means of which they can be approximated or separated between two metallic plates (indicated by the lines engraved perpendicularly) with the nuts *c c* and *c′ c′*. The latter are fitted into a kind of case, indicated in the figure by lines drawn obliquely in which they are fixed by a pin but are not impeded as regards their rotation. The roller *a*, with its prong and screw plate and nut *c c*, is isolated from the roller *a′*, with its prong and screw plate and nut *c′ c′* because the circular discs *d d* and *d′ d′* which are perforated in the centre and which connect them above and below are made of ivory. To allow of the bifilar metallic wires being brought out conveniently, the nuts *c c* and *c′ c′* terminate in trumpet shaped projections, as shown in the figure, from which hangs a wire *g g* and *g′ g′* thrice wound round. Hence a galvanic current takes the following course:—If it enters at *g* it ascends to *g*, is communicated to the nut *c c* and the roller *a* (it also ascends to *b* but as *b* is isolated it returns) and runs down the threads to *o*; from *o* it proceeds (fig. 2) further down through the centre *p* of the transverse beam, then to its extremity *i* where by the metallic contact with the support it runs down it and at *m* enters the extremity of the reel itself through the coils of which it continues, again making its exit at *n*, but again passing to the other support at *m′* through *k* from *i′* along the transverse beam to its centre, and from this up to *o′*, from *o′* the current (fig. 5) again runs over the other roller *a′* into the nut *c′ c′*, and finally arrives at the other conducting wire *g′ g′*. Thus the current, to arrive at one conducting wire *g′ g′* from the other *g g*, must necessarily run through the vibrating reel inasmuch as the wire from *g* to *g* is perfectly isolated. To do away with the torsion of the bifilar metallic wires the whole of the upper portion of the instrument as far as

$h\,h$ and $h'\,h'$ rotates horizontally, and is furnished with a torsion-circle and an index, as is distinctly seen in figs 6 and 7 at $h\,h'$

Figs 6 and 7 are not sectional, and fig 6 belongs to fig 2 Fig 7 exhibits the roller B with the prong and the screw $c\,c'$ of fig 5 more distinctly, $i\,i$ here represent two screws, to fix the roller B on moving the instrument, without which precaution the bifilar threads would be easily injured

We now pass to fig 8, which exhibits in a vertical section the lower part of the instrument, with the multiplier and the pedestal, which is constructed of serpentine In it we first recognise fig 2, suspended by the bifilar metallic wires o and o', also as seen on a vertical section The letters $m\,m$ exhibit a section of the multiplier, wound round a brass drum furnished with wooden sides, in the interior of which the vibrating cylinder R is placed These wooden sides support the tubes, within which the bifilar threads descend, the two scales for the index are also fixed to them

Fig 10, a view of the instrument as seen from above, exhibits more accurately the scale and the metallic plates, to which the tube is fastened The sides of this multiplier are in connexion with a strip of copper, which by means of two clip screws can be connected with the upper part $n\,n$ of the foot of serpentine This portion, $n\,n$, with its cone $z\,z$, is capable of rotation in the lower part of the serpentine foot, and by means of the metallic bolt r is kept in connexion with it by the screw t Since, as shown in fig 8, both the speculum and the counterpoise project towards the wooden sides of the multiplier, the whole is protected from the influence of a current of air by a cylindrical wooden cover, which is fixed to the upper corners of the wooden sides of the multiplier In the direction of the speculum to the counterpoise, however, this cylindrical cover is flattened, so as to allow of a free view through the cavity of the multiplier The flat side of the cover next the speculum can be opened or closed at pleasure by a wooden plate, which however, to enable us to use the mirror, is furnished with a flat parallel glass, S The whole of the other flat side of the cover, which is turned towards the counterpoise, may be closed or opened by a glass plate Thus the vibrating reel, when the sides of the cover are closed, can still be seen, and its free oscillation in the cavity of the multiplier be observed and regulated by means of the three screws in the serpentine pedestal Moreover, from above downwards, above the graduation, the cover is closed by two glass plates, which are moveable towards each other in metallic grooves, and excavated

in a semicircular form in the centre to allow the tube in which the bifilar wires are suspended to pass through them In fig 8, $v\,v$ exhibits the glass plate at the side $v'\,w$ is the wooden plate with the flat parallel glass S at the other side $v\,v'$ is one of the upper glass plates The letters $l\,l$ are loops, through which the conducting wires $g\,g$ and $g'\,g'$ in fig 6 descend these wires are fixed in these loops to avoid their lying loosely throughout their entire length they terminate in pegs, or small cylinders

Fig 9 also exhibits a vertical section but at right angles to that of fig 8 m is the multiplier and R a section of the reel vibrating within it At the side of the case we perceive four metallic knobs marked $u\,u'\,z\,z'$ These are perforated crucially, and the perforation most distant from the case is furnished with a screw on the inner side of the case it is fixed to it by another screw Two of these knobs u and u', are in metallic contact with the commencement and termination of the multiplier, so that a current from the knob u can run through the multiplier into the knob u' and vice versa The other two knobs, z and z', are perfectly isolated, but all four of the knobs are very useful for reversing the current and for effecting various combinations In this figure also we see the index vibrating above the scale plate as also in fig 3, where the case is supposed to be removed

Let us now trace the course of a galvanic current which enters the instrument at the knob u, it passes from u through the multiplier m and towards u' if the conducting wire $g'\,g'$ with its metallic cylindrical extremity be now inserted into this knob, the current ascends in $g'\,g'$ and (fig 5) towards the nut $c'\,c'$ above the roller a', then down within the tube to o', thence (fig 2) from o through the centre p of the transverse beam to $r'\,m'\,k\,d$, through the vibrating reel to $m\,r\,p\,o$, and (fig 5) to o, ascending above the roller a in the nut $c\,c$, to the second conducting wire $g\,g$ and (fig 9) through $g\,g$ down into the knob z, whence it runs into the other of the two exciting surfaces

By means of the upper rotating part of the serpentine pedestal, the instrument may be arranged in any part of a hall or room as required All the figures are drawn one fourth of the linear magnitude of the electro dynamometer excepting fig 5, which is one half the real magnitude

The wire on the vibrating reel is 200 metres in length, that of the multiplier 300, the first forms about 1200 coils, the latter about 900 The length of the bifilar wires, (which are very fine,

composed of silver, and were heated to redness,) from the transverse beam to the small rollers $a\,a'$, was half a metre.

The price of the instrument is 10 guineas.

Observations demonstrating the fundamental principle of Electro-dynamics.

The following observations were not made with the instrument which has just been described. However, it is unnecessary to describe separately the instrument made use of on this occasion, because it merely differs from the former in minor points of arrangement, which were less convenient than those in the latter. One important modification only requires to be mentioned, viz. that the multiplier, which in the above description assumes an invariable position, in which its centre coincides with the centre of the bifilarly suspended reel, was left moveable, so that it could be placed in any position as regards the vibrating reel, for the purpose of extending the observations to all relative positions of the two galvanic conductors, which act upon each other. Now as these two conductors form two coils, one of which can enclose the other, and in the instrument described above the inner and smaller coil was suspended by two threads, to serve as it were as a galvanometer needle, whilst the outer and larger coil was fixed and formed the multiplier, it was requisite for the object in question to reverse the arrangement, and to suspend the outer and larger coil by two threads so as to use the inner and smaller coil as a multiplier, because it was only by this means that the position of the multiplier could be altered at pleasure without interfering with the bifilar suspension. It is at once seen that the external reel, on account of its size, has a greater momentum from inertia, which produces a longer duration of its vibration; this influence however may be easily compensated for when necessary by altering the arrangement of the bifilar suspension.

As regards the observations themselves, it remains to be remarked, that to render the results comparable, the intensity of the current transmitted by the two conductors of the dynamometer was, simultaneously with the observation on the dynamometer, accurately measured by a second observer with a galvanometer. This was requisite, because no reliance can be placed upon the constancy of the intensity of the current during a continued series of experiments, even when the so called constant battery of Grove or Bunsen is used.

The first experiment was made by passing three currents of different intensity, i.e. from 3, 2 and 1 of Grove's elements, through the two conductors of the dynamometer, and observing the simultaneous deflections of the dynamometer and galvanometer. After making the necessary reductions, the following means were obtained as the deflection:—

Number of Grove's elements.	Deflection.	
	Of the Dynamometer.	Of the Galvanometer.
3	440.038	108.426
2	198.55	72.398
1	50.915	36.632

These observations are reduced so that the former furnish a measure of the electro-dynamic force with which the two conductors of the dynamometer act upon each other, when currents of equal intensity are transmitted through them, whilst the latter furnish a measure of this intensity itself.

If we denote the dynamometric observations by δ, and the galvanometric observations by γ, we obtain

$$\gamma = 5.15534 \cdot \sqrt{\delta},$$

for if we calculate the values of γ from the values found by observation for δ according to this formula, we obtain in the order of the series,

$$\begin{array}{r} 108.144 \\ 72.589 \\ 36.786, \end{array}$$

which exhibit less differences from the values of γ found by observation than could be anticipated, thus

$$\begin{array}{r} -0.282 \\ +0.191 \\ +0.151. \end{array}$$

The electro-dynamic force of the reciprocal action of two conducting wires, through which currents of equal intensity are transmitted, is therefore in proportion to the square of this intensity, which is exactly what is required by the fundamental principle of electro-dynamics.

A more extended series of experiments was then made for the purpose of ascertaining the dependence of the electro-dynamic force, with which the two conducting wires of the dynamometer

react upon each other, upon the relative position and distance of these wires.

For this purpose the arrangement was effected in such a manner, that one conducting wire, *i. e.* the multiplier, could be placed in any position as regards the other, *i. e.* as regards the bifilarly-suspended coil, the latter forming the larger coil, which inclosed the former smaller one.

Both coils were always placed in such a position that their axes were in the same horizontal plane, and formed a right angle with each other.

The distance of the two coils was determined by the distance of their centres from each other, and was thus assumed as $= 0$ when the centres of the two coils coincided.

When the latter was not the case, in addition to the magnitude of the distance of the two centres, it was also requisite to measure the angle which the line uniting the two central points formed with the axis of the bifilarly-suspended coil, whereby the direction in which the centre of the multiplier was removed from the centre of the bifilarly suspended coil was defined. For this purpose the four cardinal directions were selected at which the former angle had the value 0°, 90°, 180° and 270°, *i. e.* when the axis of the bifilarly suspended coil, like the axis of the needle of a magnet, was arranged in the magnetic meridian, the centre of the multiplier was removed from the centre of the above coil, sometimes in the direction of the meridian, *north* or *south*, and sometimes in the direction at right angles to the magnetic meridian, *east* or *west*. In each of these different directions the multiplier was placed successively at different distances from the suspended coil.

This arrangement of different positions and distances of the two conducting wires of the dynamometer accurately corresponds, as is seen at a glance, to the arrangement of different positions and distances of the two magnets, upon which Gauss based his measurements, in demonstrating the fundamental principle of magnetism. The bifilarly-suspended coil here occupied the place of Gauss's magnetic needle and the multiplier the place of Gauss's deflection-rod. The only important difference is, that the mutual action of the magnets could only be observed from a distance; consequently in the magnetic observations that case was excluded in which the centres of the two magnets coincided; whilst in the electro-dynamic measurements of which we are now speaking, the system could

moreover be rendered complete by the case, in which the centres of the two coils coincided

Simultaneously with the observations made on the dynamometer the intensity of the current which was transmitted through the two coils of the dynamometer was measured by another observer with a galvanometer By these auxiliary observations I was enabled to reduce all the observations made on the dynamometer in accordance with the law shown above, (that the electro dynamic force is in proportion to the square of the intensity of the current,) to an equal intensity of the current and thus to render the results obtained comparable

The following table gives the reduced mean values which were obtained in the different instances The first vertical column shows the distance of the two coils of the dynamometer, above the other columns, the direction formed by the line uniting the two centres with the axis of the bifilarly suspended coil directed towards the magnetic meridian is given —

D[illegible] t	N[illegible] th 0	E[illegible] t 90	S[illegible] th 180	W[illegible] t 27[illegible]
0	2[illegible]60	2[illegible]00	2[illegible]000	[illegible]
300	77 16	18[illegible] 4	77 03	1[illegible]0[illegible]
100	31 78	77 [illegible]1	[illegible] 77	77 8
500	18 17	[illegible] 37	18 10	30 16
[illegible]00		2 [illegible]3		2 38

It is at once seen that when the centres of the two coils of the dynamometer coincide, or their distance apart is = 0, the difference dependent upon the change of the direction in which the multiplier is removed from the bifilarly suspended coil, vanishes The result obtained in this case therefore could only be repeated in the above table in the various columns

Moreover the above table shows that the results obtained for an equal distance in opposite directions varying 180 , agree together as far as the observations could be depended upon

These values, when reduced by taking their means, after converting the divisions of the scale into degrees, minutes and seconds, yield the following table —

R						
0 3	0	19	22	0	20	3
0 4	0	20	8	0	0	2
0 5	0	10	12	0	1	11
0 6	0	5	50			

in which the same notation is adopted as used by Gauss in his *Intensitas Vis Magneticæ*, &c. in the comparison of the magnetic observations.

According to the fundamental principle of electro-dynamics, we should be able to develope the tangents of the angle of deflection v and v' according to the diminishing odd powers of the distance R, and we should have

$$\tan v = a\,\mathrm{R}^{-3} + b\,\mathrm{R}^{-5}$$
$$\tan v' = \tfrac{1}{2}\,a\,\mathrm{R}^{-3} + c\,\mathrm{R}^{-5},$$

where a, b and c are constants to be determined from the observations. If now in the present instance we make

$$\tan v = 0{\cdot}0003572\ \mathrm{R}^{-3} + 0{\cdot}000002755\ \mathrm{R}^{-5}$$
$$\tan v' = 0{\cdot}0001786\ \mathrm{R}^{-3} - 0{\cdot}000001886\ \mathrm{R}^{-5},$$

we obtain the following table of *calculated* deflections, and their difference from those *found by observation*:—

R	v	Difference.	v'	Difference.
0·3	0° 49′ 22″	0″	0° 20′ 4″	−1″
0·4	0 20 7	+1	0 8 58	+1
0·5	0 10 8	+4	0 4 42	+2
0·6	0 5 49	+1		

Thus in this agreement of the calculated values with those obtained by observation, we have a confirmation of one of the most universal and most important consequences of the fundamental principle of electro-dynamics, viz. *that the same laws apply to electro-dynamic forces exerted at a distance as to magnetic forces.*

In this application of the laws of magnetism to electro-dynamic observations, that case of the latter where the centres of the two coils of the dynamometer coincide must be excluded. Moreover, in this extension of the laws of magnetism to electro-dynamic observations, the values of three constants must be deduced from the observations themselves, which is unnecessary when we have recourse to the fundamental principle of electro-dynamics itself, and calculate directly from it the results which the observations should have yielded in accordance with it. Hence from the fundamental principle of electro-dynamics—

1. In that case in which the straight line uniting the centre

of the two coils coincides with the axis of the bifilarly suspended coil

when m designates the radius of the multiplying coil, n the radius of the bifilarly suspended coil and a the distance of the centres of the two coils and for brevity we make

$$\frac{mm}{aa+nn} = vv$$

$$\frac{nn}{aa+nn} = ww$$

$$\frac{4aa+nn}{16(aa+nn)} = f$$

$$\frac{8a^4+4aann+n^4}{64(aa+nn)} = g,$$

the electro dynamic momentum of rotation which the multiplying coil exerts upon the bifilarly suspended coil, when a current of the intensity i passes through both coils is determined with sufficient accuracy to be

$$= -\frac{\pi\pi}{2} v^3\, nn\, ii\; S$$

S designating the following series —

$$\begin{aligned} S = &+ \left[\tfrac{1}{3} - ww\right] \\ &- \tfrac{3}{8}\left[\tfrac{3}{5} - ww - (3 - 7ww)f\right] vv \\ &+ \tfrac{1}{8}\left[\tfrac{5}{7} - ww - 2(5 - 9ww)f + 3(5 - 11ww)g\right] v^4 \\ &- \tfrac{5}{16}\left[\tfrac{7}{9} - ww - 3(7 - 11ww)f + 11(7 - 13ww)g\right] v^6 \\ &+ \tfrac{35}{128}\left[\tfrac{9}{11} - ww - 4(9 - 13ww)f + 26(9 - 15ww)g\right] v^8 \\ &- \&c \end{aligned}$$

If in this equation we substitute the values known from direct measurement in millimetres,

$$m = 41.1,$$
$$n = 55.8,$$

and successively

$$a = 300,\ 400,\ 500,$$

we obtain as the rotating momentum sought, the following three values to be multiplied by $\pi\pi ii$ —

$$-1.1511$$
$$-0.6517$$
$$-0.3152$$

Moreover,

2 In that case where the right line uniting the centres of both coils is at right angles to the axis of the bifilarly-suspended coil,

m, n and a having the same signification, and

$$\frac{mm}{aa+nn}=vv,$$

$$\frac{aa}{aa+nn}=f,$$

$$\frac{nn}{aa+nn}=1\,g\,vv,$$

the rotatory momentum required is

$$=+\pi v^3 nn\,ii\;S',$$

S' expressing the following series —

$$\begin{aligned}S'=&+\tfrac{1}{3}\\&-3\left[\tfrac{1}{5}-\tfrac{10}{3}fg\right]vv\\&+\tfrac{1}{8}\left[\tfrac{1}{7}+\tfrac{?}{?}(1-11f)g+12ffgg\right]v^4\\&-\tfrac{3}{16}\left[\tfrac{1}{9}+\tfrac{3}{7}(2-18f)g-\tfrac{1}{7}(1-11f)fgg-572f^3g^3\right]v^6\\&+\tfrac{?}{128}\left[\tfrac{1}{11}+\tfrac{1}{9}(3-22f)g+\tfrac{1}{?}(1-22f+143ff)gg\right.\\&\qquad\left.+\tfrac{1144}{5}(1-10f)ffg^3+\tfrac{24310}{?}f^4g^4\right]v^8\\&-\ \&c\end{aligned}$$

If in this series we substitute for m and n the given values, and successively $a=300$, 400, 500 and 600, we obtain as the rotating momentum required, the following values to be multiplied by $\pi\pi ii$ —

$$\begin{aligned}&+3.5625\\&+1.1661\\&+0.7120\\&+0.1267\end{aligned}$$

Lastly,

3 In that case where the centres of both coils coincide,—when m designates the radius of the multiplier, and n' and n'' the least and greatest radius of the bifilarly suspended coil, the rotatory momentum sought is

$$\begin{aligned}=\frac{\pi\pi m^3}{n''-n'}ii\Big[&\tfrac{1}{3}\log\text{nat}\frac{n''}{n'}\\&+\frac{9}{160}\left(\frac{1}{n''n''}-\frac{1}{n'n'}\right)mm-\frac{225}{11136}\left(\frac{1}{n''^4}-\frac{1}{n'^4}\right)m^4\\&+\frac{6125}{881736}\left(\frac{1}{n''^6}-\frac{1}{n'^6}\right)m^6+\frac{691675}{184519376}\left(\frac{1}{n''^8}-\frac{1}{n'^8}\right)m^8+\ \Big]\end{aligned}$$

If in this formula we substitute the values known from direct measurement in millimetres,

$$m = 11.1$$
$$n' = 50.25$$
$$n'' = 61.35$$

we obtain as the rotatory momentum the following value to be multiplied by $\pi\pi ii$ —

$$+ 11.711$$

This value suffers a reduction of about $\frac{1}{11}$th when we take into consideration that all the turns of the two coils do not lie in one plane, which in this case exerts greater influence on account of their proximity than in the other cases. The above value thus becomes reduced to

$$+ 10.715 \ \pi\pi ii$$

The numerical coefficients thus calculated should now be proportional to the observed values, and when multiplied by $\pi\pi ii$, the intensity of the current i being expressed according to the dimensions upon which the above measurements were based, should be equal.

In fact, when all the calculated numerical coefficients are multiplied by 53.06, and then arranged according to the analogy of the observed values, we obtain the following table of the calculated values, and their difference from those found by observation:—

[illegible]	N [illegible] 180	Diff.	[illegible] 475	Diff.
0	+ 22680.00	+ 280.00	+ 22680.00	+ 280.00
300	180.03	+ 0.90	77.17	− 0.08
400	77.70	− 0.11	31.71	+ 0.01
500	30.37	− 0.10	18.11	− 0.07
600	2.61	− 0.18		

In this comparison of theory and experiment, the single factor 53.06 was deduced from observations, and this was merely done because this factor could not be determined with sufficient accuracy by direct measurements. The direct determination of this factor is based upon the ascertainment of the proportion of that measure of the intensity of the current, upon which the scale of the galvanometer used is based, to that absolute measure to which the theoretical expression refers. The measurements

necessary for ascertaining this proportion could not all be effected with the requisite accuracy, because separate measures were not taken for this purpose. In fact, however, the above factor was provisionally, as well as circumstances permitted, determined by direct measurement, and found = 19·5. This result also exhibits an agreement with that previously deduced from the observations, which under the circumstances could not have been expected to be greater.

OBSERVATIONS TENDING TO ENLARGE THE DOMAIN OF ELECTRO-DYNAMIC INVESTIGATIONS.

A. *Observation of Voltaic Induction.*

If the bifilarly suspended coil of the dynamometer be made to oscillate whilst a current is transmitted through it, or through the coil of the multiplier, or through both simultaneously, this motion is *inductive*, and excites a current in the conductor, through which no current was passing, or alters the current passing through this conductor. This mode of excitation of the current is called *voltaic induction*. The inducing motion, i. e. the velocity of the oscillating coil, is on each occasion diminished or *checked* by the antagonism of the currents excited by the voltaic induction and those conducted through the coil. This *check* to the vibrating coil *effected* by the voltaic induction may be accurately observed; and at the same time the motion of the oscillating coil itself, which *produces* the *voltaic induction*, may be accurately determined; and this twofold use of the dynamometer affords the data necessary for the more accurate investigation of the laws of voltaic induction.

The bifilarly suspended coil closed in itself was made to oscillate to the greatest extent at which the scale permitted observations to be made, and its oscillations from 0 were counted until they became too minute for accurate observation. During the counting, the magnitude of the arc of oscillation was measured from time to time. These experiments were *first* made under the influence of voltaic induction, a current from three Grove's elements being conducted through the multiplying coil; the same experiments were *next* repeated, after the removal of the elements, without voltaic induction:—

[illegible]		[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]
0	764 10	0	[illegible] 0 80
9	[illegible] 11	14	[illegible]01 13
18	[illegible]	2	[illegible] 90
35	481 1[illegible]	[illegible]2	485 28
47	414 [illegible]0	82	409 [illegible]2
[illegible]7	3[illegible]5 50	100	3[illegible] 08
71	2[illegible]2 27	131	306 79
85	2 3 30	163	261 08
103	200 80	180	228 33
118	16 [illegible]	212	1[illegible]8 [illegible]
130	111 37	23[illegible]	178 [illegible]
143	119 33	2 [illegible]1	1 7 98
1[illegible]7	100 [illegible]	281	131 17
17[illegible]	7[illegible]	300	11[illegible] 30
19[illegible]	60 [illegible]8	328	10 2
210	50 06	369	84 08
		387	7[illegible] 15

It is evident on comparison that the diminution of the magnitude of the arc, which without the influence of induction from one oscillation to another amounted on an average to $\frac{1}{180}$th, with the cooperation of the induction rose to $\frac{1}{77}$th part

When for the multiplying coil with the current transmitted through it, a magnet equivalent in an electro magnetic point of view is substituted, the diminution of the arc is found to be equally great, i e the magnetic induction of this magnet is equal to the voltaic induction of the current in the multiplier

The velocity which the inducing motion must possess for the intensity of the induced current to be equal to that of the inducing current, may also be deduced from these experiments

B *Determination of the duration of Momentary Currents, as also its application to Physiological Experiments*

When the intensity of a *continued* constant current is to be determined both the galvanometer (the sine or tangent compass) and the dynamometer may be used, but if the current, the intensity of which is to be determined, is merely of *momentary* duration observation made with either of these instruments is not sufficient, because the deflection observed does not depend merely upon the intensity of the current, but also upon the duration itself It is therefore requisite, in experimentally investigating the intensity of the current, also to determine its duration

The two instruments, *i. e.* the galvanometer and the dynamometer, are complementary to each other, so that when the same momentary current is transmitted through both, and the deflection of both instruments thus produced is observed, both the duration and the intensity of the momentary current can be determined from these two observations. This reciprocity is based upon the circumstance that the observed deflection of both instruments depends in the same manner upon the duration of the momentary current, *i. e.* it is proportional to it, whilst it is not dependent in the same manner upon the intensity of the current, because the deflection of the galvanometer is in proportion to the intensity of the current.

Let s and ς indicate the duration of the oscillations of the galvanometer and dynamometer;

e' and ε' the deflection at which both instruments remain when the same constant current of the intensity i' is transmitted through them;

Whilst e and ε indicate the extent of the deflection which both instruments attain in consequence of a momentary current of the duration Θ and of the intensity i, the following equation then gives the *duration* Θ:—

$$\Theta = \frac{1}{\pi}\cdot\frac{s\,\varsigma}{\varsigma}\cdot\frac{\varepsilon'}{e'e'}\cdot\frac{ee}{\varepsilon},$$

and the following that of the *intensity* of the current i:—

$$i = \frac{\varsigma}{s}\cdot\frac{e'}{\varepsilon'}\cdot i'\cdot\frac{\varepsilon}{e}.$$

s, ς, e', ε', i', e and ε in these formulæ are magnitudes which can be determined by observation.

This combination of the dynamometer with the galvanometer is of special importance in physiology, to investigate accurately the excitation of the nerves by galvanic currents. For it is found that nerves of sensation especially are quickly deadened by continued currents, and hence that for such experiments momentary currents are frequently required to be used. But the observed impressions of sense depend less upon the duration of the current than upon its intensity, and it is essential to be acquainted with both.

C. *Repetition of* Ampère's *fundamental Experiment with common Electricity and measurement of the duration of the Electric Spark on the discharge of a Leyden Jar.*

It is evident from the preceding remarks that the action of a current upon the dynamometer depends more upon the intensity of the current, to the square of which it is proportionate, than upon the duration of the current to which it is simply proportional. Hence it follows that even a small quantity of electricity, when passed through the dynamometer within a very short period, so that it forms a current of very short duration but very great intensity, will produce a sensible effect. This is, in fact, the case when the small quantity of electricity which can be collected in a Leyden jar or battery is transmitted during its discharge through the dynamometer. By this means it was found that Ampère's fundamental experiment, which had previously been made only with powerful galvanic batteries, could also be made with common electricity.

When the same electricity, collected in Leyden jars, after having been transmitted through the dynamometer, was also conducted through a galvanometer and the deflection thus produced in both instruments was measured, in accordance with the above rules, the duration of the current, *i. e.* the duration of the electric spark on the discharge of the Leyden jar, and at the same time the intensity of the current could be determined, admitting that the current might be considered as uniform during its brief duration.

It is well known that in experiments of this kind the discharge of the Leyden jar is effected by means of a wet string, to prevent its taking place through the air instead of through the fine wires of the two instruments. In this manner a series of experiments was made, a battery of eight jars being discharged through a wet hempen string, 7 millimetres in thickness and of different lengths, the following results were obtained:—

Length of the string. Millimetres.	Duration of the spark. Seconds.
2000	0.0851
1000	0.0345
500	0.0187
250	0.0095

Hence the duration of the spark was nearly in proportion to

the length of the string; for the observed duration of the spark is:—

Seconds.
0·0816 + 0·0035
0·0408 − 0·0063
0·0204 − 0·0017
0·0102 − 0·0007

The first part of the duration of the spark is thus exactly in proportion to the length of the string; but the second part is so small that it may be considered as arising from error of observation, which was unavoidable.

It is thus evident that the result obtained by Prof Wheatstone, according to which the duration of the spark on discharge by simple metallic conductors is infinitely short in comparison with that ascertained in the present case, is in direct accordance with this result

D. *Application of the Dynamometer to the measurement of Sonorous Vibrations.*

When a rapid alternation of positive and negative currents ensues in a conducting wire, the continued motion of the electricity becomes converted into an *oscillation*. An oscillation of this kind cannot however be observed by means of a galvanometer (for instance, a sine- or tangent-compass), because in this case the effects of the successive opposite oscillations destroy each other.

But the case is different with the dynamometer, in the two coils of which the direction of the vibration always changes simultaneously, and in which the deflection observed is in proportion to the square of the intensity of the current; for it is self-evident that the simultaneous change of the direction in both coils can exert no influence upon the action, because in the dynamometer a negative current transmitted through both coils produces a deflection towards the same side as a positive current transmitted through both coils. The occurrence of the deflection of the dynamometer to one side or the other does not, as in the galvanometer, depend upon the direction of the transmitted current, but merely upon the mode of connexion of the extremities of the wires of both coils.

But an electric vibration may be readily produced in a conducting wire by a magnetized steel bar vibrating so as to produce

a musical sound, when one portion of the conducting wire, forming as it were the inducing coil surrounds the free vibrating end of the bar, so that the direction of the vibration is at right angles to the plane of the coils of the wire All vibrations of the bar on one side then produce positive currents in the wire, and all the vibrations on the other side produce negative currents, which follow each other as rapidly as the sonorous vibrations themselves

When the ends of the wire of the inducing coil are united to the ends of that of the dynamometer, a deflection of the latter during the vibration of the bar is observed, which can be accurately measured This deflection remains unaltered so long as the intensity of the sonorous vibrations remains unaltered, but speedily diminishes when the intensity of the sonorous vibrations diminishes, and when the amplitude of the sonorous vibrations has fallen to a half it then amounts to the fourth part only

The dynamometer thus presents a means of estimating the intensity of sonorous vibrations, which is of importance, because methods adapted to these measurements are still much required

In addition to the investigations which we have hitherto considered, and which are based on the use of the dynamometer, there are others which will be subsequently treated of, when some modifications in the construction of this instrument for special objects will also be more accurately detailed

ON THE CONNEXION OF THE FUNDAMENTAL PRINCIPLE OF ELECTRO-DYNAMICS WITH THAT OF ELECTRO-STATICS

The fundamental principle of electro statics is, that when two electric (positive or negative) masses, denoted by e and e', are at a distance r from each other, the amount of the force with which the two masses act reciprocally upon each other is expressed by

$$\frac{e e'}{r r}$$

and that repulsion or attraction occurs accordingly as this expression has a positive or negative value

On the other hand, the fundamental principle of electro dynamics is as follows —When two elements of a current the lengths of which are α and α', and the intensities i and i', and which are at the distance r from each other, so that the directions in which

the positive electricity in both elements moves, form with each other the angle ε, and with the connecting right line the angles Θ and Θ', the magnitude of the force with which the elements of the current reciprocally act upon each other is determined by the expression

$$-\frac{\alpha\alpha' i i'}{r r}\left(\cos\varepsilon - \tfrac{3}{2}\cos\Theta\cos\Theta'\right),$$

and repulsion or attraction occurs according as this expression has a positive or negative value. The expressions of the rotatory momentum excited by one coil of the dynamometer upon the other, developed at p. 502 and 503, are all deduced from this fundamental principle.

The *former* of the two fundamental principles mentioned refers to two electric masses and their antagonism, the *latter* to two elements of a current and their antagonism. A more intimate connexion between the two can only be attained by recurring, likewise in the case of the elements of the current, to the consideration of the electric magnitudes existing in the elements of the current, and their antagonism.

Thus the next question is, what electric magnitudes are contained in the two elements of a current, and upon what mutual relations of these masses their reciprocal actions may depend.

If the mass of positive electricity in a portion of the conducting wire equal to a unit of length be represented by e, and consequently the mass of the positive electricity contained in the elements of the current, the length of which is $= \alpha$, by αe, and if u indicates the velocity with which the mass moves, the product $e u$ expresses that mass of positive electricity which in a unit of time passes through each section of the conducting wire, to which the intensity of the current i must be considered as proportional; hence, when a expresses a constant factor,

$$a e u = i.$$

If now αe represent the mass of positive electricity in the element of the current α, and u its velocity, $-\alpha e$ represents the mass of negative electricity in the same element of the current, and $-u$ its velocity.

We have also, when

$$a e' u' = i',$$

$\alpha' e'$ as the mass of positive electricity in the second element of the current α', and u' its velocity, and lastly, $-\alpha' e'$ as the mass

of negative electricity, and $-u'$ as its velocity. If now for i and i', in the expression of the force which one element of a current exerts upon another, their values $i = \alpha e u$, and $i' = \alpha e' u'$ are substituted, we then obtain for them

$$-\frac{\alpha e\cdot\alpha' e'}{rr}\,aauu'\,\left(\cos\varepsilon - \tfrac{3}{2}\cos\Theta\cos\Theta'\right).$$

If now we *first* consider in this expression $\alpha e\cdot\alpha' e'$ as the product of the *positive* electric masses αe and $\alpha' e'$ in the two elements of the current, and uu' as the product of their velocities u and u', and if we denote by r the variable distance of these two masses in motion, and lastly, by $s_{\prime}$ and $s_{\prime}'$ the length of a portion of each of the two conducting wires, to which the elements of the current α and α' just considered belong, estimated from a definite point of origin and proceeding in the direction of the *positive* electricity, as far as the element of the current under consideration; we then know that the cosines of the two angles Θ and Θ', which the two conducting wires in the situation of the elements of the current mentioned form with the connecting right line $r_{\prime}$, may be represented by the partial differential coefficients of $r_{\prime}$ with respect to $s_{\prime}$ and $s_{\prime}'$, thus

$$\cos\Theta = \frac{dr_{\prime}}{ds_{\prime}},\quad \cos\Theta' = -\frac{dr_{\prime}}{ds_{\prime}'};$$

we then have

$$\cos\varepsilon = -r_{\prime}\frac{ddr_{\prime}}{ds_{\prime}\,ds_{\prime}'} - \frac{dr_{\prime}}{ds_{\prime}}\frac{dr_{\prime}}{ds_{\prime}'}$$

as the cosine of the angle ε which the directions of the two conducting wires form with each other. Moreover, if the differential coefficients above mentioned be substituted for the cosines of the three angles ε, Θ and Θ', we have

$$-\frac{\alpha e\cdot\alpha' e'}{r_{\prime}r_{\prime}}\,aauu'\,\left(\frac{1}{2}\frac{dr_{\prime}}{ds_{\prime}}\frac{dr_{\prime}}{ds_{\prime}'} - r_{\prime}\frac{ddr_{\prime}}{ds_{\prime}\,ds_{\prime}'}\right)$$

as the expression of the force with which one element of the current acts upon the other.

Secondly, if in the above expression, $-\alpha e\cdot\alpha' e'$ be considered as the product of the *positive* electric mass αe of one element of the current α into the *negative* electric mass $-\alpha' e'$ of the other element of the current α', and $-uu'$ as the product of their velocities u and $-u'$; moreover, if the variable distance of these two moving masses be denoted by $r_{\prime\prime}$ and by $s_{\prime}$ and $s'_{\prime\prime}$ the length of a portion of each of the two conducting wires, to which the elements of

the current under consideration belong, taken from a definite point of origin, and proceeding in that direction in which, in the first the *positive*, in the second the *negative* electricity runs, as far as the element of the current mentioned, we obtain in the same manner

$$\cos\Theta = \frac{d r_{II}}{d s_I}, \qquad \cos\Theta' = \frac{d r_{II}}{d s_{II}'}$$

$$\cos\epsilon = r_{II}\frac{d d r_{II}}{d s_I d s_{II}'} + \frac{d r_{II} d r_{II}}{d s_I d s_{II}'}.$$

On substituting these values, we have the following expression for the force with which one element of the current acts upon the other:—

$$+\frac{\alpha e . \alpha' e'}{r_{II} r_{II}} . a a u u' . \left(\frac{1}{2}\frac{d r_{II} d r_{II}}{d s_I d s_{II}'} - r_{II}\frac{d d r_{II}}{d s_I d s_{II}'}\right).$$

If, *thirdly*, we consider in the original expression $\alpha e . \alpha' e'$ as the product of the *negative* electrical masses $-\alpha e$ and $-\alpha' e'$ into the two elements of the current, and $u u'$ as the product of their velocities $-u$ and $-u'$, and r_{III} denote the variable distance of these two moving masses, and lastly, s_{II} and s_{II}' denote the length of a portion of each of the two conducting wires to which the elements of the current under consideration belong, calculated from a definite point of origin, and proceeding in that direction in which the *negative* electricity runs, as far as the element of the current under consideration, we have

$$\cos\Theta = -\frac{d r_{III}}{d s_{II}}, \quad \cos\Theta' = \frac{d r_{III}}{d s_{II}'}$$

$$\cos\epsilon = -r_{III}\frac{d d r_{III}}{d s_{II} d s_{II}'} - \frac{d r_{III} d r_{III}}{d s_{II} d s_{II}'}.$$

On substituting these values, we have a third expression for the force with which one element of the current acts upon the other, namely,

$$-\frac{\alpha e . \alpha' e'}{r_{III} r_{III}} . a a u u' . \left(\frac{1}{2}\frac{d r_{III} d r_{III}}{d s_{II} d s_{II}'} - r_{III}\frac{d d r_{III}}{d s_{II} d s_{II}'}\right).$$

In fine, if, *fourthly*, in the original expression we consider $-\alpha e . \alpha' e'$ as the product of the *negative* electric mass $-\alpha e$ of the element of the current α into the *positive* electric mass $\alpha' e'$ of the element of the current α', and $-u u'$ as the product of their velocities $-u$ and u'; if, moreover, r_{IIII} designate the variable distance of these two moving masses, and s_{II} and s_I' the

length of a portion of each of the two conducting wires, to which the elements of the current under consideration belong, calculated from a defined point of origin, proceeding in that direction in which in the first the *negative*, in the second the *positive* electricity runs, we have

$$\cos \theta = -\frac{d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}}, \quad \cos \theta' = -\frac{d r_{\prime\prime\prime\prime}}{d s'_{\prime}}$$

$$\cos \varepsilon = r_{\prime\prime\prime\prime} \frac{d d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime}} + \frac{d r_{\prime\prime\prime\prime}\, d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime}}$$

If now these values be substituted we have the fourth expression of the force with which one element of the current acts upon the other, viz.

$$+\frac{\alpha e \cdot \alpha' e'}{r_{\prime\prime\prime\prime} r_{\prime\prime\prime\prime}}\, a a u u' \left(\frac{1}{2} \frac{d r_{\prime\prime\prime\prime}\, d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime}} - r_{\prime\prime\prime\prime} \frac{d d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime}}\right)$$

Now at that moment in which the electric masses alluded to occur in the two elements α and α', the distances $r_{\prime}, r_{\prime\prime}, r_{\prime\prime\prime}, r_{\prime\prime\prime\prime}$ have all the same value, which is expressed by r. Hence the four expressions of the electro-dynamic force of the two elements of the current α and α' become converted into the following:—

$$-\frac{\alpha e \cdot \alpha' e'}{r r}\, a a u u' \left(\frac{1}{2} \frac{d r_{\prime}\, d r_{\prime}}{d s_{\prime}\, d s'_{\prime}} - r \frac{d d r_{\prime}}{d s_{\prime}\, d s'_{\prime}}\right), \qquad (1)$$

$$+\frac{\alpha e \cdot \alpha' e'}{r r}\, a a u u' \left(\frac{1}{2} \frac{d r_{\prime\prime}\, d r_{\prime\prime}}{d s_{\prime}\, d s'_{\prime\prime}} - r \frac{d d r_{\prime\prime}}{d s_{\prime}\, d s'_{\prime\prime}}\right), \qquad (2)$$

$$-\frac{\alpha e \cdot \alpha' e'}{r r}\, a a u u' \left(\frac{1}{2} \frac{d r_{\prime\prime\prime}\, d r_{\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime\prime}} - r \frac{d d r_{\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime\prime}}\right), \qquad (3)$$

$$+\frac{\alpha e \cdot \alpha' e'}{r r}\, a a u u' \left(\frac{1}{2} \frac{d r_{\prime\prime\prime\prime}\, d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime}} - r \frac{d d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime}}\right), \qquad (4)$$

from which we can construct the fifth expression, viz. (5)—

$$-\frac{\alpha e \cdot \alpha' e'}{r r} \cdot \frac{a a}{4}\, u u' \left[\frac{1}{2}\left(\frac{d r_{\prime}\, d r_{\prime}}{d s_{\prime}\, d s'_{\prime}} - \frac{d r_{\prime\prime}\, d r_{\prime\prime}}{d s_{\prime}\, d s'_{\prime\prime}} + \frac{d r_{\prime\prime\prime}\, d r_{\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime\prime}} - \frac{d r_{\prime\prime\prime\prime}\, d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime}}\right) \right.$$
$$\left. - r\left(\frac{d d r_{\prime}}{d s_{\prime}\, d s'_{\prime}} - \frac{d d r_{\prime\prime}}{d s_{\prime}\, d s'_{\prime\prime}} + \frac{d d r_{\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime\prime}} - \frac{d d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}\, d s'_{\prime}}\right)\right]$$

The four variable distances $r_{\prime}, r_{\prime\prime}, r_{\prime\prime\prime}, r_{\prime\prime\prime\prime}$ are now respectively dependent upon the variable magnitudes of the paths $s_{\prime}$ and $s'_{\prime}$, $s_{\prime}$ and $s'_{\prime\prime}$, $s_{\prime\prime}$ and $s'_{\prime\prime}$, $s_{\prime\prime}$ and $s'_{\prime}$ through which the moveable masses to which they refer have passed in the two given con-

ducting wires, and which consequently are again functions of the time t. On developing then complete differentials, we have

$$d r_{\prime} = \frac{d r_{\prime}}{d s_{\prime}} d s_{\prime} + \frac{d r_{\prime}}{d s_{\prime}'} d s_{\prime}',$$

$$d r_{\prime\prime} = \frac{d r_{\prime\prime}}{d s_{\prime}} d s_{\prime} + \frac{d r_{\prime\prime}}{d s_{\prime\prime}'} d s_{\prime\prime}',$$

$$d r_{\prime\prime\prime} = \frac{d r_{\prime\prime\prime}}{d s_{\prime\prime}} d s_{\prime\prime} + \frac{d r_{\prime\prime\prime}}{d s_{\prime\prime}'} d s_{\prime\prime}',$$

$$d r_{\prime\prime\prime\prime} = \frac{d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}} d s_{\prime\prime} + \frac{d r_{\prime\prime\prime\prime}}{d s_{\prime}'} d s_{\prime}',$$

moreover,

$$d d r_{\prime} = \frac{d d r_{\prime}}{d s_{\prime}^2} d s_{\prime}^2 + 2 \frac{d d r_{\prime}}{d s_{\prime} d s_{\prime}'} d s_{\prime} d s_{\prime}' + \frac{d d r_{\prime}}{d s_{\prime}'^2} d s_{\prime}'^2,$$

$$d d r_{\prime\prime} = \frac{d d r_{\prime\prime}}{d s_{\prime}^2} d s_{\prime}^2 + 2 \frac{d d r_{\prime\prime}}{d s_{\prime} d s_{\prime\prime}'} d s_{\prime} d s_{\prime\prime}' + \frac{d d r_{\prime\prime}}{d s_{\prime\prime}'^2} d s_{\prime\prime}'^2,$$

$$d d r_{\prime\prime\prime} = \frac{d d r_{\prime\prime\prime}}{d s_{\prime\prime}^2} d s_{\prime\prime}^2 + 2 \frac{d d r_{\prime\prime\prime}}{d s_{\prime\prime} d s_{\prime\prime}'} d s_{\prime\prime} d s_{\prime\prime}' + \frac{d d r_{\prime\prime\prime}}{d s_{\prime\prime}'^2} d s_{\prime\prime}'^2,$$

$$d d r_{\prime\prime\prime\prime} = \frac{d d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}^2} d s_{\prime\prime}^2 + 2 \frac{d d r_{\prime\prime\prime\prime}}{d s_{\prime\prime} d s_{\prime}'} d s_{\prime\prime} d s_{\prime}' + \frac{d d r_{\prime\prime\prime\prime}}{d s_{\prime}'^2} d s_{\prime}'^2.$$

If these differentials are respectively divided by the elements of the time $d t$, and their squares $d t^2$, and admitting that

$$\frac{d s_{\prime}}{d t} = \frac{d s_{\prime\prime}}{d t} = u, \quad \frac{d s_{\prime}'}{d t} = \frac{d s_{\prime\prime}'}{d t} = u',$$

we have

$$\frac{d r_{\prime}}{d t} = u \frac{d r_{\prime}}{d s_{\prime}} + u' \frac{d r_{\prime}}{d s_{\prime}'},$$

$$\frac{d r_{\prime\prime}}{d t} = u \frac{d r_{\prime\prime}}{d s_{\prime}} + u' \frac{d r_{\prime\prime}}{d s_{\prime\prime}'},$$

$$\frac{d r_{\prime\prime\prime}}{d t} = u \frac{d r_{\prime\prime\prime}}{d s_{\prime\prime}} + u' \frac{d r_{\prime\prime\prime}}{d s_{\prime\prime}'},$$

$$\frac{d r_{\prime\prime\prime\prime}}{d t} = u \frac{d r_{\prime\prime\prime\prime}}{d s_{\prime\prime}} + u' \frac{d r_{\prime\prime\prime\prime}}{d s_{\prime}'};$$

moreover,

$$\frac{d\,d\,r_{I}}{d\,t^2} = u\,u\,\frac{d\,d\,r_{I}}{d\,s_{I}} + 2\,u\,u'\,\frac{d\,d\,r_{I}}{d\,s_{I}\,d\,s_{I}'} + u'\,u'\,\frac{d\,d\,r_{I}}{d\,s_{I}'^2},$$

$$\frac{d\,d\,r_{II}}{d\,t^2} = u\,u\,\frac{d\,d\,r_{II}}{d\,s_{I}^2} + 2\,u\,u'\,\frac{d\,d\,r_{II}}{d\,s_{I}\,d\,s_{II}'} + u'\,u'\,\frac{d\,d\,r_{II}}{d\,s_{II}'^2},$$

$$\frac{d\,d\,r_{III}}{d\,t^2} = u\,u\,\frac{d\,d\,r_{III}}{d\,s_{II}^2} + 2\,u\,u'\,\frac{d\,d\,r_{III}}{d\,s_{II}\,d\,s_{II}'} + u'\,u'\,\frac{d\,d\,r_{III}}{d\,s_{II}'^2},$$

$$\frac{d\,d\,r_{IIII}}{d\,t^2} = u\,u\,\frac{d\,d\,r_{IIII}}{d\,s_{II}} + 2\,u\,u'\,\frac{d\,d\,r_{IIII}}{d\,s_{II}\,d\,s_{I}'} + u'\,u'\,\frac{d\,d\,r_{IIII}}{d\,s_{I}'^2}$$

From the four last equations we get immediately—

$$2\,u\,u'\,\frac{d\,d\,r_{I}}{d\,s_{I}\,d\,s_{I}'} = +\frac{d\,d\,r_{I}}{d\,t^2} - u\,u\,\frac{d\,d\,r_{I}}{d\,s_{I}^2} - u'\,u'\,\frac{d\,d\,r_{I}}{d\,s_{I}'^2},$$

$$-2\,u\,u'\,\frac{d\,d\,r_{II}}{d\,s_{I}\,d\,s_{II}'} = -\frac{d\,d\,r_{II}}{d\,t^2} + u\,u\,\frac{d\,d\,r_{II}}{d\,s_{I}^2} + u'\,u'\,\frac{d\,d\,r_{II}}{d\,s_{II}'^2},$$

$$2\,u\,u'\,\frac{d\,d\,r_{III}}{d\,s_{I}\,d\,s_{II}'} = +\frac{d\,d\,r_{III}}{d\,t^2} - u\,u\,\frac{d\,d\,r_{III}}{d\,s_{II}} - u'\,u'\,\frac{d\,d\,r_{III}}{d\,s_{II}'^2},$$

$$-2\,u\,u'\,\frac{d\,d\,r_{IIII}}{d\,s_{II}\,d\,s_{I}'} = -\frac{d\,d\,r_{IIII}}{d\,t^2} + u\,u\,\frac{d\,d\,r_{IIII}}{d\,s_{II}^2} + u'\,u'\,\frac{d\,d\,r_{IIII}}{d\,s_{I}'^2}$$

Now the differential coefficients $\frac{d\,d\,r_{I}}{d\,s_{I}^2}$, $\frac{d\,d\,r_{II}}{d\,s_{I}}$, $\frac{d\,d\,r_{III}}{d\,s_{II}^2}$, $\frac{d\,d\,r_{IIII}}{d\,s_{II}^2}$ have the same value, which is dependent merely upon the position and form of the *first* conducting wire, and which we shall denote by $\frac{d\,d\,r}{d\,s^2}$ This applies also to the differential coefficients $\frac{d\,d\,r_{I}}{d\,s_{I}'^2}$, $\frac{d\,d\,r_{II}}{d\,s_{II}'^2}$ $\frac{d\,d\,r_{III}}{d\,s_{II}'^2}$, $\frac{d\,d\,r_{IIII}}{d\,s_{I}'^2}$, all of which denote the same magnitudes, which are dependent merely upon the position and form of the *second* conducting wire, and which for brevity we shall denote by $\frac{d\,d\,r}{d\,s'^2}$ On summation bearing this in mind, we have

$$2\,u\,u'\left(\frac{d\,d\,r_{I}}{d\,s_{I}\,d\,s_{I}'} - \frac{d\,d\,r_{II}}{d\,s_{I}\,d\,s_{II}'} + \frac{d\,d\,r_{III}}{d\,s_{II}\,d\,s_{II}'} - \frac{d\,d\,r_{IIII}}{d\,s_{II}\,d\,s_{I}'}\right)$$
$$= \frac{d\,d\,r_{I}}{d\,t^2} - \frac{d\,d\,r_{II}}{d\,t^2} + \frac{d\,d\,r_{III}}{d\,t^2} - \frac{d\,d\,r_{IIII}}{d\,t^2}$$

But from the first four equations, after they have been squared, we have

$$2uu'\frac{dr_{\prime}\,dr_{\prime}}{ds_{\prime}\,ds'_{\prime}} = +\frac{dr_{\prime}^2}{dt^2} - uu\frac{dr_{\prime}^2}{ds_{\prime}^2} - u'u'\frac{dr_{\prime}^2}{ds_{\prime}'^2},$$

$$-2uu'\frac{dr_{\prime\prime}\,dr_{\prime\prime}}{ds_{\prime}\,ds'_{\prime\prime}} = -\frac{dr_{\prime\prime}^2}{dt^2} + uu\frac{dr_{\prime\prime}^2}{ds_{\prime}^2} + u'u'\frac{dr_{\prime\prime}^2}{ds_{\prime\prime}'^2},$$

$$2uu'\frac{dr_{\prime\prime\prime}\,dr_{\prime\prime\prime}}{ds_{\prime\prime}\,ds'_{\prime\prime}} = +\frac{dr_{\prime\prime\prime}^2}{dt^2} - uu\frac{dr_{\prime\prime\prime}^2}{ds_{\prime\prime}^2} - u'u'\frac{dr_{\prime\prime\prime}^2}{ds_{\prime\prime}'^2},$$

$$-2uu'\frac{dr_{\prime\prime\prime\prime}\,dr_{\prime\prime\prime\prime}}{ds_{\prime\prime}\,ds'_{\prime}} = -\frac{dr_{\prime\prime\prime\prime}^2}{dt^2} + uu\frac{dr_{\prime\prime\prime\prime}^2}{ds_{\prime\prime}^2} + u'u'\frac{dr_{\prime\prime\prime\prime}}{ds_{\prime}'^2}$$

Now the differential coefficients $\frac{dr_{\prime}}{ds_{\prime}^2}$, $\frac{dr_{\prime\prime}^2}{ds_{\prime}^2}$, $\frac{dr_{\prime\prime\prime}^2}{ds_{\prime\prime}}$, $\frac{dr_{\prime\prime\prime\prime}}{ds_{\prime\prime}^2}$ have also the same value, which shall be denoted by $\frac{dr^2}{ds^2}$, as have likewise $\frac{dr_{\prime}^2}{ds_{\prime}'^2}$, $\frac{dr_{\prime\prime}^2}{ds_{\prime\prime}'^2}$, $\frac{dr_{\prime\prime\prime}^2}{ds_{\prime\prime}'^2}$, $\frac{dr_{\prime\prime\prime\prime}}{ds_{\prime}'^2}$, which we shall denote by $\frac{dr^2}{ds'^2}$. On summation, keeping this in view, we have

$$2uu'\left(\frac{dr_{\prime}\,dr_{\prime}}{ds_{\prime}\,ds'_{\prime}} - \frac{dr_{\prime\prime}\,dr_{\prime\prime}}{ds_{\prime}\,ds'_{\prime\prime}} + \frac{dr_{\prime\prime\prime}\,dr_{\prime\prime\prime}}{ds_{\prime\prime}\,ds'_{\prime\prime}} - \frac{dr_{\prime\prime\prime\prime}\,dr_{\prime\prime\prime\prime}}{ds_{\prime\prime}\,ds'_{\prime}}\right)$$
$$= \frac{dr_{\prime}^2}{dt^2} - \frac{dr_{\prime\prime}^2}{dt^2} + \frac{dr_{\prime\prime\prime}^2}{dt^2} - \frac{dr_{\prime\prime\prime\prime}^2}{dt^2}.$$

On substituting these values in the fifth expression found for the electro dynamic force, it becomes

$$-\frac{ae\;a'e'}{rr}\;\frac{aa}{16}\left[\left(\frac{dr_{\prime}^2}{dt^2} - \frac{dr_{\prime\prime}^2}{dt^2} + \frac{dr_{\prime\prime\prime}^2}{dt^2} - \frac{dr_{\prime\prime\prime\prime}^2}{dt^2}\right)\right.$$
$$\left. - 2r\left(\frac{ddr_{\prime}}{dt^2} - \frac{ddr_{\prime\prime}}{dt^2} + \frac{ddr_{\prime\prime\prime}}{dt^2} - \frac{ddr_{\prime\prime\prime\prime}}{dt^2}\right)\right],$$

an expression which may be resolved into the four following members —

$$-\frac{ae\;a'e'}{r_{\prime}r_{\prime}}\;\frac{aa}{16}\left(\frac{dr_{\prime}^2}{dt^2} - 2r_{\prime}\frac{ddr_{\prime}}{dt^2}\right),$$

$$+\frac{ae\;a'e'}{r_{\prime\prime}r_{\prime\prime}}\;\frac{aa}{16}\left(\frac{dr_{\prime\prime}^2}{dt^2} - 2r_{\prime\prime}\frac{ddr_{\prime\prime}}{dt^2}\right),$$

$$-\frac{ae\;a'e'}{r_{\prime\prime\prime}r_{\prime\prime\prime}}\;\frac{aa}{16}\left(\frac{dr_{\prime\prime\prime}^2}{dt} - 2r_{\prime\prime\prime}\frac{ddr_{\prime\prime\prime}}{dt^2}\right),$$

$$+\frac{ae\;a'e'}{r_{\prime\prime\prime\prime}r_{\prime\prime\prime\prime}}\;\frac{aa}{16}\left(\frac{dr_{\prime\prime\prime\prime}^2}{dt^2} - 2r_{\prime\prime\prime\prime}\frac{ddr_{\prime\prime\prime\prime}}{dt^2}\right)$$

Each of these four members refers exclusively to *two* of the four electric masses distinguished in the two elements of the current, viz. the *first* member to the two positive masses αe and $\alpha' e'$ the relative distance of which is $r_{\prime}$, velocity $\frac{d r_{\prime}}{d t}$, and acceleration $\frac{d d r_{\prime}}{d t^2}$; the *second* to the positive mass αe in the first, and to the negative mass $-\alpha' e'$ in the second element, the relative distance of which is $r_{\prime\prime}$, velocity $\frac{d r_{\prime\prime}}{d t}$, and acceleration $\frac{d d r_{\prime\prime}}{d t^2}$, and so on; and in fact all four are members of the masses to which they refer, the distance, velocity and acceleration of which are composed *in exactly the same manner*.

Hence it is evident that if the entire expression of the electro-dynamic force of two elements of a current be considered as the sum of the forces, which each two of the four electric masses they contain exert upon each other, this sum would be decomposed into its *original constituents*, the four above members representing individually the four forces which the four electric masses in the two elements exert *in pairs* upon each other.

Hence also the force with which any positive or negative mass E acts upon any other positive or negative mass E', at the distance R, with a relative velocity of $\frac{d R}{d t}$, and acceleration $\frac{d d R}{d t^2}$, may be expressed by

$$-\frac{a a}{16}\,\frac{\mathrm{E E'}}{\mathrm{R R}}\left(\frac{d \mathrm{R}^2}{d t^2}-2\mathrm{R}\,\frac{d d \mathrm{R}}{d t^2}\right),$$

for this fundamental principle is necessary and at the same time sufficient to allow of the deduction of Ampère's electro-dynamic laws which are confirmed by the above measurements.

However, this new fundamental principle of electro-dynamics is in its nature more *general* than that formerly laid down by Ampère; for the latter refers merely to the special case, in which four electric magnitudes are given at the same time, subject to the conditions premised for invariable and undisturbed elements of the current; whilst such a limitation to the above conditions does *not* occur in the former. This fundamental principle, consequently, admits of application in those cases where the former is inapplicable; hence its greater utility.

If, lastly, the newly discovered fundamental principle of elec-

tro-dynamics be compared with the fundamental principle of electro-statics mentioned at the commencement, we see that each estimates a force which *two electric masses* exert upon each other; but that in the cases hitherto considered, one of the two forces disappears each time, whence the other only requires consideration. This occurs *first* in all cases which belong to electro-statics, because here the force determined by the new principle of electro-dynamics always disappears; but it also occurs, *secondly*, in all cases belonging to electro-dynamics which have yet come under consideration, where relations are constantly pre-supposed to exist, in which all forces estimated by the principle of electro-statics are mutually checked.

Thus the two principles are complementary to each other, and hence they may be combined to form a general fundamental principle *for the whole theory of electricity*, which comprises both electro-statics and electro-dynamics.

By the fundamental principle of electro-statics, a force

$$= \frac{E\,E'}{R\,R}$$

was found for two electric masses E and E' at the distance R; if this force be then added to that yielded by the new principle of electro-dynamics,

$$= -\frac{aa}{16}\cdot\frac{E\,E'}{R\,R}\left(\frac{d\,R^2}{d\,t^2} - 2\,R\,\frac{d\,d\,R}{d\,t^2}\right),$$

we obtain, as the general expression for the complete determination of the force which any electric mass E exerts upon another E', whether at rest or in motion,

$$\frac{E\,E'}{R\,R}\left(1 - \frac{aa}{16}\cdot\frac{d\,R^2}{d\,t^2} + \frac{aa}{8}\cdot R\,\frac{d\,d\,R}{d\,t^2}\right).$$

For a definite magnitude assumed for the purpose of measuring the time, in which $a = 4$, this expression becomes

$$\frac{E\,E'}{R\,R}\left(1 - \frac{d\,R^2}{d\,t^2} + 2\,R\,\frac{d\,d\,R}{d\,t^2}\right).$$

Moreover, supposing that both R and $\frac{d\,R}{d\,t}$ are functions

onsequently that $\frac{d\,R}{d\,t}$ is to be regarded as a functio

we shall denote by [R], we may also say that the *potential* of the mass E, in regard to the situation of the mass E', is

$$= \frac{E}{R}(1 - [R]^2),$$

for the partial differential coefficients of this expression, with respect to the three coordinates x, y, z yield the components of the decomposed accelerating force in the directions of the three coordinate axes.

Lastly if by the *reduced relative velocity* of the masses E and E', we understand that relative velocity which these magnitudes,—the distance of which apart at the moment supposed was R, the relative velocity $\frac{dR}{dt}$, and the acceleration $\frac{ddR}{dt^2}$, *if the latter were constant*,—would possess at that instant in which both, in accordance with this supposition, met at one point, and if V denoted this *reduced relative velocity*, the above expression,

$$\frac{EE'}{RR}\left(1 - \frac{dR^2}{dt^2} + 2R\frac{ddR}{dt^2}\right)$$

becomes converted into the following,

$$\frac{EE'}{RR}(1 - VV),$$

which may be verbally expressed as follows:—*The diminution arising from motion of the force with which two electric masses would act upon each other when they are at rest, is in proportion to the square of their reduced relative velocity.*

Thus the expressions given for the determination of the force which two electric masses exert upon one another are now confirmed:—

1st. As regards the entire domain of electro-statics;

2nd. As regards that domain of electro-dynamics the object of which is the consideration of the forces of the elements of the current when invariable and undisturbed; hence

3rdly. Its confirmation as regards all that domain of electro-dynamics which is not limited to the invariable and undisturbed state of the elements of the current, is all that remains to be desired.

Theory of Voltaic Induction.

It has already been mentioned that the principle of electro-dynamics laid down by Ampère refers merely to the special case,

where four electric masses occur under the conditions premised to exist where two invariable and undisturbed elements of a current are concerned. Under conditions where these premises do not exist, the new fundamental principle only can be applied for the *a priori* determination of the forces and phænomena, and it is exactly in this way that the greater advantage of the new principle, arising from its more general application, will be exhibited.

The case in which the principle of electro dynamics laid down by Ampère is inapplicable, thus occurs even when one element of a current is disturbed or its intensity varies, in addition to which it may also happen, that instead of the other element of the current, one element only of the conductor of a current may be present, without however any current being present in it. In fact, we know from experience that currents are then excited or *induced*, and the phænomena of these induced currents are comprised under the name of *voltaic induction*, but none of these phænomena could be predicted or estimated *a priori* either from the principle of electro statics or the principle of electro dynamics laid down by Ampère. It will now however be shown, that by means of the new fundamental principle as laid down here, the laws for the *à priori* determination of all the phænomena of voltaic induction may be deduced. It is evident that the laws of voltaic induction deduced in this manner are correct, so far only as we are in possession of definite observations.

For the purpose of this deduction the magnitudes concerned may be denoted as follows: — α and α' denote the length of two elements, the former of which, α, is supposed to be *at rest*. This supposition does not limit the generality of the consideration, because every movement of the element α may be transferred to α', by attributing the opposite direction to it in α'. The four following electric masses are distinguished in these two elements, viz —

$$+\alpha e, \quad -\alpha e, \quad +\alpha' e', \quad -\alpha' e'$$

The *first* of these masses $+\alpha e$ would move with the velocity $+u$ in the direction of the quiescent element α, which forms the angle Θ with the right line drawn from α to α'. This velocity during the element of time dt would alter by $+du$.

The *second* mass $-\alpha e$ would move, in accordance with the determinations relating to a galvanic current, in the same direc-

tion as the velocity $-u$, *i. e.* backwards, and this velocity during the element of time dt would alter by $-du$.

The *third* mass $+\alpha' e'$ would move with the velocity $+u'$ in the direction of the element α', which with the right lines drawn from α to α', and produced forms the angle Θ. This velocity in the element of time dt would alter by $+du'$. Moreover, this electric mass would itself share the motion of the element α', which takes place with the velocity v in a direction which forms the angle η with the prolonged right line drawn from α to α', and is contained in a plane lying in this right line, which with the plane running parallel with the element α through the same right line, encloses the angle γ. The velocity v would alter during the element of time dt by dv.

The *fourth* mass $-\alpha' e'$ would move, in accordance with the determinations for a galvanic current, in the direction of the element α', with the velocity $-u'$, which during the element of time dt alters by $-du'$, but, moreover, like the previous mass, would itself acquire the velocity v of the element α' in the direction already indicated.

The distances of the two former masses from the two latter, at the moment under consideration, are equal to the distance r of the two elements themselves; but since they do not remain the same, they may be denoted by r_1, r_2, r_3, r_4.

Lastly, if two planes pass through the right line drawn from α to α', the one parallel to α, the other to α', ω would denote the angle enclosed by these two planes.

Then on applying the new principle, we obtain as the *sum* of the forces which act upon the *positive* and *negative* electricity in the element α', *i. e.* as the force which moves the element α' itself, the following expression:—

$$-\frac{aa}{16}\,\frac{\alpha e\,\alpha' e'}{rr}\left\{\left(\frac{dr_1^2}{dt^2}+\frac{dr_2^2}{dt^2}-\frac{dr_3^2}{dt^2}-\frac{dr_4^2}{dt^2}\right)\right.$$
$$\left.-2r\left(\frac{ddr_1}{dt^2}+\frac{ddr_2}{dt^2}-\frac{ddr_3}{dt^2}-\frac{ddr_4}{dt^2}\right)\right\}$$

But for the *difference* of these forces, upon which the *induction* depends, we have the following expression:—

$$-\frac{aa}{16}\,\frac{\alpha e\,\alpha' e'}{rr}\left\{\left(\frac{dr_1^2}{dt^2}-\frac{dr_2^2}{dt^2}+\frac{dr_3^2}{dt^2}-\frac{dr_4^2}{dt^2}\right)\right.$$
$$\left.-2r\left(\frac{ddr_1}{dt^2}-\frac{ddr_2}{dt^2}+\frac{ddr_3}{dt^2}-\frac{ddr_4}{dt^2}\right)\right\}$$

Moreover, when, in addition to the motions of the electric masses in their conductors, the motion common to them and their conductors is taken into account, we have the following expressions for the first differential coefficients:—

$$\frac{d r_1}{d t} = -u \cos \Theta + u' \cos \Theta' + v \cos \eta,$$

$$\frac{d r_2}{d t} = +u \cos \Theta - u' \cos \Theta' + v \cos \eta,$$

$$\frac{d r_3}{d t} = -u \cos \Theta - u' \cos \Theta' + v \cos \eta,$$

$$\frac{d r_4}{d t} = +v \cos \Theta + u' \cos \Theta' + v \cos \eta.$$

Hence

$$\left(\frac{d r_1^2}{d t^2} + \frac{d r_2^2}{d t^2} - \frac{d r_3^2}{d t^2} - \frac{d r_4^2}{d t^2}\right) = -8\, u u' \cos \Theta \cos \Theta',$$

$$\left(\frac{d r_1^2}{d t^2} - \frac{d r_2^2}{d t^2} + \frac{d r_3^2}{d t^2} - \frac{d r_4^2}{d t^2}\right) = -8\, u v \cos \Theta \cos \eta.$$

We obtain the second differential coefficients when the variability of the velocity u, u', and v is also taken into account:—

$$\frac{d\, d r_1}{d t^2} = +u \sin \Theta \frac{d \Theta_1}{d t} - u' \sin \Theta' \frac{d \Theta'_1}{d t} - v \sin \eta \frac{d \eta_1}{d t}$$
$$- \cos \Theta \frac{d u}{d t} + \cos \Theta' \frac{d u'}{d t} + \cos \eta \frac{d v}{d t},$$

$$\frac{d\, d r_2}{d t^2} = -u \sin \Theta \frac{d \Theta_2}{d t} + u' \sin \Theta' \frac{d \Theta'_2}{d t} - v \sin \eta \frac{d \eta_2}{d t}$$
$$+ \cos \Theta \frac{d u}{d t} - \cos \Theta' \frac{d u'}{d t} + \cos \eta \frac{d v}{d t},$$

$$\frac{d\, d r_3}{d t_2} = +u \sin \Theta \frac{d \Theta_3}{d t} + u' \sin \Theta' \frac{d \Theta'_3}{d t} - v \sin \eta \frac{d \eta_3}{d t}$$
$$- \cos \Theta \frac{d u}{d t} - \cos \Theta' \frac{d u'}{d t} + \cos \eta \frac{d v}{d t},$$

$$\frac{d\, d r_4}{d t^2} = -u \sin \Theta \frac{d \Theta_4}{d t} - u' \sin \Theta' \frac{d \Theta'_4}{d t} - v \sin \eta \frac{d \eta_4}{d t}$$
$$+ \cos \Theta \frac{d u}{d t} + \cos \Theta' \frac{d u'}{d t} + \cos \eta \frac{d v}{d t}.$$

Consequently it becomes

$$\left(\frac{ddr_1}{dt^2}+\frac{ddr_2}{dt^2}-\frac{ddr_3}{dt^2}-\frac{ddr_4}{dt^2}\right)=+u\sin\Theta\left(\frac{d\Theta_1}{dt}-\frac{d\Theta_2}{dt}-\frac{d\Theta_3}{dt}-\frac{d\Theta_4}{dt}\right)$$
$$-u'\sin\Theta'\left(\frac{d\Theta'_1}{dt}-\frac{d\Theta'_2}{dt}+\frac{d\Theta'_3}{dt}-\frac{d\Theta'_4}{dt}\right)$$
$$-v\sin\eta\left(\frac{d\eta_1}{dt}+\frac{d\eta_2}{dt}-\frac{d\eta_3}{dt}-\frac{d\eta_4}{dt}\right)$$

and

$$\left(\frac{ddr_1}{dt^2}-\frac{ddr_2}{dt^2}+\frac{ddr_3}{dt^2}-\frac{ddr_4}{dt^2}\right)=+u\sin\Theta\left(\frac{d\Theta_1}{dt}+\frac{d\Theta_2}{dt}+\frac{d\Theta_3}{dt}+\frac{d\Theta_4}{dt}\right)$$
$$-u'\sin\Theta'\left(\frac{d\Theta'_1}{dt}+\frac{d\Theta'_2}{dt}-\frac{d\Theta'_3}{dt}-\frac{d\Theta'_4}{dt}\right)$$
$$-v\sin\eta\left(\frac{d\eta_1}{dt}-\frac{d\eta_2}{dt}+\frac{d\eta_3}{dt}-\frac{d\eta_4}{dt}\right)$$
$$-4\cos\Theta\,\frac{du}{dt}$$

The differential coefficients $\frac{d\Theta_1}{dt}$, $\frac{d\Theta'_1}{dt}$, $\frac{d\eta_1}{dt}$, &c. are easily developed according to the well known laws of trigonometry and we thus obtain the following expressions, viz.—

$$r_1\frac{d\Theta_1}{dt}=+u\sin\Theta-u'\sin\Theta'\cos\omega-v\sin\eta\cos\gamma,$$
$$r_1\frac{d\Theta'_1}{dt}=-u'\sin\Theta'+u\sin\Theta\cos\omega-v\sin\eta\cos(\omega+\gamma),$$
$$r_1\frac{d\eta_1}{dt}=-v\sin\eta+u\sin\Theta\cos\gamma-u'\sin\Theta'\cos(\omega+\gamma),$$
$$r_2\frac{d\Theta_2}{dt}=-u\sin\Theta+u'\sin\Theta'\cos\omega-v\sin\eta\cos\gamma,$$
$$r_2\frac{d\Theta'_2}{dt}=+u'\sin\Theta'-u\sin\Theta\cos\omega-v\sin\eta\cos(\omega+\gamma),$$
$$r_2\frac{d\eta_2}{dt}=-v\sin\eta-u\sin\Theta\cos\gamma+u'\sin\Theta'\cos(\omega+\gamma),$$
$$r_3\frac{d\Theta_3}{dt}=+u\sin\Theta+u'\sin\Theta'\cos\omega-v\sin\eta\cos\gamma,$$
$$r_3\frac{d\Theta'_3}{dt}=+u'\sin\Theta'+u\sin\Theta\cos\omega-v\sin\eta\cos(\omega+\gamma),$$
$$r_3\frac{d\eta_3}{dt}=-v\sin\eta+u\sin\Theta\cos\gamma+u'\sin\Theta'\cos(\omega+\gamma),$$

$$r_4 \frac{d\,\Theta_4}{d\,t} = -u \sin\Theta - u' \sin\Theta' \cos\omega - v \sin\eta \cos\gamma,$$

$$r_4 \frac{d\,\Theta'_4}{d\,t} = -u' \sin\Theta' - u \sin\Theta \cos\omega - v \sin\eta \cos(\omega + \gamma),$$

$$r_4 \frac{d\,\eta_4}{d\,t} = -v \sin\eta - u \sin\Theta \cos\gamma - u' \sin\Theta' \cos(\omega + \gamma).$$

Now since for the moment under consideration $r_1 = r_2 = r_3 = r_4 = r$, we thus get

$$r\left(\frac{d\,\Theta_1}{d\,t} - \frac{d\,\Theta_2}{d\,t} - \frac{d\,\Theta_3}{d\,t} + \frac{d\,\Theta_4}{d\,t}\right) = -4\,u' \sin\Theta' \cos\omega,$$

$$r\left(\frac{d\,\Theta_1}{d\,t} + \frac{d\,\Theta_2}{d\,t} + \frac{d\,\Theta_3}{d\,t} + \frac{d\,\Theta_4}{d\,t}\right) = -4\,v \sin\eta \cos\gamma;$$

again:

$$r\left(\frac{d\,\Theta'_1}{d\,t} - \frac{d\,\Theta'_2}{d\,t} + \frac{d\,\Theta'_3}{d\,t} - \frac{d\,\Theta'_4}{d\,t}\right) = +4\,u \sin\Theta \cos\omega,$$

$$r\left(\frac{d\,\Theta'_1}{d\,t} + \frac{d\,\Theta'_2}{d\,t} - \frac{d\,\Theta'_3}{d\,t} - \frac{d\,\Theta'_4}{d\,t}\right) = 0,$$

lastly:

$$r\left(\frac{d\,\eta_1}{d\,t} + \frac{d\,\eta_2}{d\,t} - \frac{d\,\eta_3}{d\,t} - \frac{d\,\eta_4}{d\,t}\right) = 0,$$

$$r\left(\frac{d\,\eta_1}{d\,t} - \frac{d\,\eta_2}{d\,t} + \frac{d\,\eta_3}{d\,t} - \frac{d\,\eta_4}{d\,t}\right) = +4\,u \sin\Theta \cos\gamma.$$

These values by substitution become

$$r\left(\frac{d\,d\,r_1}{d\,t^2} + \frac{d\,d\,r_2}{d\,t^2} - \frac{d\,d\,r_3}{d\,t^2} - \frac{d\,d\,r_4}{d\,t^2}\right) = -8\,u\,u' \sin\Theta \sin\Theta' \cos\omega,$$

$$r\left(\frac{d\,d\,r_1}{d\,t^2} - \frac{d\,d\,r_2}{d\,t^2} + \frac{d\,d\,r_3}{d\,t^2} - \frac{d\,d\,r_4}{d\,t^2}\right) = -8\,u\,v \sin\Theta \sin\eta \cos\gamma,$$

$$-4\,r \cos\Theta \,.\, \frac{d\,u}{d\,t}.$$

With these values, the *sum* of the forces which act upon the *positive* and *negative* electricity in the element α' is

$$= -\frac{\alpha\,\alpha'}{r\,r} \,.\, a\,e\,u \,.\, a'\,e'\,u' \left(\sin\Theta \sin\Theta' \cos\omega - \tfrac{1}{2} \cos\Theta \cos\Theta'\,\eta\right).$$

If in this equation the angle which the directions of the two elements α and α' form with each other be denoted by ε, and, as

in p. 511, i and i' be substituted for aeu and $a'e'u'$, the above sum, with slight transposition, becomes

$$= -\frac{\alpha\alpha' i i'}{rr}(\cos\varepsilon - \tfrac{3}{2}\cos\Theta\,\cos\Theta'),$$

the same expression at which Ampère arrived where the elements of the current are invariable and undisturbed, *i.e.* the electro-dynamic force acting upon the entire element α' is determined in the same manner when the conductors are in motion and the intensities of the current variable, as when the intensities of the current remain invariable and the conductors undisturbed. Hence Ampère's law is of general application in the determination of the forces, which act upon the entire element of the current when the position of the elements of the current and the intensities of the current are given. The application of this law merely requires that the intensities of the current when variable, as also the position when variable, be given *for each individual moment*, and further the intensities of the currents, including that part added at each moment in consequence of induction.

But as regards the *difference* of the forces which act upon the *positive* and *negative* electricity in the element α' by which these two electricities are separated from each other, and move in the conductor in opposite directions, this now becomes

$$= -\frac{\alpha\alpha'}{rr}\,aeu\cdot ae'v\,(\sin\Theta\sin\eta\cos\gamma - \tfrac{1}{2}\cos\Theta\cos\eta)$$

$$-\tfrac{1}{2}\frac{\alpha\alpha'}{r}\,aaee'\cos\Theta\,\frac{du}{dt},$$

or, because $aeu = i$ and $ae\,du = di$,

$$= -\frac{\alpha\alpha'}{rr}\,i\,(\sin\Theta\sin\eta\cos\gamma - \tfrac{1}{2}\cos\Theta\cos\eta)\cdot ae'v$$

$$-\tfrac{1}{2}\frac{\alpha\alpha'}{r}\,ae'\cos\Theta\,\frac{di}{dt}.$$

The force thus determined then tends to separate the *positive* and *negative* electricity in the induced element α' in the direction of the right line r. When the conductor is linear, however, separation cannot occur in this direction, but only in the direction of the induced linear element α' itself, which forms the angle Θ' with the produced right line r. By thus decomposing the whole of the above separating force in this direction, *i.e.* by multiplying the above value by $\cos\Theta'$, we find the force, which effects the true separation,

$$= -\frac{\alpha\alpha'}{r r} i (\sin\Theta \sin\eta \cos\gamma - \tfrac{1}{2}\cos\Theta\cos\eta) \,.\, a e' v \cos\Theta'$$
$$- \tfrac{1}{2}\frac{\alpha\alpha'}{r} a e' \,.\, \cos\Theta\cos\Theta' \frac{d i}{d t}$$

This expression, divided by e', gives the *electromotor* force exerted by the inducing element α upon the induced element α', in the ordinary direction,

$$= -\frac{\alpha\alpha'}{r r} i (\sin\Theta \sin\eta \cos\gamma - \tfrac{1}{2}\cos\Theta\cos\eta) \,.\, a v \cos\Theta'$$
$$- \tfrac{1}{2}\frac{\alpha\alpha'}{r} a \cos\Theta\cos\Theta' \,.\, \frac{d i}{d t}.$$

This is therefore the *general law of voltaic induction, as found by deduction from the newly laid down fundamental principle of the theory of electricity.*

If we now, *first*, take the case *in which no alteration occurs in the intensity of the current,* thus

$$\frac{d i}{d t} = 0,$$

we have *the law of the induction exerted by a constant element of a current upon the element of a conductor moved against it, i. e.* the *electromotive* force becomes

$$= -\frac{\alpha\alpha'}{r r} i (\sin\Theta \sin\eta \cos\gamma - \tfrac{1}{2}\cos\Theta\cos\eta) \,.\, a v \cos\Theta',$$

or, when ε denotes the angle which the direction of the inducing element of the current forms with the direction in which the induced element itself is moved, by a transformation which is readily made it becomes

$$= -\frac{\alpha\alpha'}{r r} i (\cos\varepsilon - \tfrac{3}{2}\cos\Theta\cos\eta) \,.\, a v \cos\Theta'.$$

The induced current is *positive* or *negative* according as this expression has a *positive* or *negative* value; by a positive current being understood one, the positive electricity of which moves in that direction of the element α', which with the produced right line r forms the angle Θ'.

Now if *e. g.* the elements α and α' are parallel to each other, and if the direction in which the latter is moved with the velocity v is contained within the plane of these two parallels, and at right angles to their direction, we have, *when α' by its motion recedes from α,*

$$\Theta = \Theta', \quad \cos\eta = \sin\Theta, \quad \cos\varepsilon = 0;$$

consequently the *electromotive* force is

$$= +\tfrac{3}{2}\frac{\alpha\alpha'}{r r} i \sin\theta\cos^2\theta \quad a v$$

This value is always *positive*, because we must consider $\theta < 180°$ and this *positive* value here denotes an induced current of the same direction as the inducing current in conformity with that which has been found by experiment for this case

Under the same conditions, with the difference merely that *the element α' by its motion becomes approximated to the element α*, we have

$$\theta = \theta', \quad \cos\eta = -\sin\theta, \quad \cos\epsilon = 0,$$

consequently the *electromotive* force becomes

$$= -\tfrac{3}{2}\frac{\alpha\alpha'}{r r} i \sin\theta\cos^2\theta \quad a v$$

The *negative* value of this force denotes an induced current, in the opposite direction to that of the inducing current, also in conformity with that found by experiment for this case

As is well known voltaic induction may be produced in two essentially different ways, for currents may be induced by *constant* and by *variable* currents It is produced by *constant* currents either when the conductor through which the current passes is moved towards that conductor in which a current is about to be induced or *vice versâ* It may be induced by *variable* currents even when the conductor through which the variable current passes remains undisturbed as regards that conductor in which a current is about to be induced

Just as the particular law of the first kind of voltaic induction was at once found from the *general laws of voltaic induction* deduced above by the conditional equation

$$\frac{di}{dt} = 0,$$

so we also find the peculiar law of the latter kind of voltaic induction by the conditional equation

$$v = 0$$

Thus if we take, *secondly*, the case in which *no motion of the conductors as regards each other takes place*, or where $v = 0$, the *law of the induction of a variable current upon that element of a current which is not moved as regards it*, or the value of the *electromotive force* becomes

$$= -\tfrac{1}{2}\frac{\alpha\alpha'}{r} a \cos\theta\cos\theta' \frac{di}{dt}$$

Hence the induction, during the element of time dt, *i. e.* the product of this element of time into the acting *electromotive force*, becomes

$$= -\frac{a}{2} \cdot \frac{\alpha\alpha'}{r} \cos\Theta \cos\Theta' \cdot di,$$

consequently the induction for any period of time in which the intensity of the induced current increases by i, whilst r, Θ and Θ' remain unchanged, is

$$= -\frac{a}{2} \frac{\alpha\alpha'}{r} i \cos\Theta \cos\Theta'.$$

The *positive* value of this expression denotes a current induced in the element α' in the direction of α', which with the produced right line r forms the angle Θ'; the *negative* value denotes an induced current in the opposite direction.

When the two elements α and α' are parallel, and $\Theta = \Theta'$, the above expression, when the intensity of the current is *increasing*, or where the value of i is positive, has a *negative* value, *i. e.* when the intensity of the current is on the increase in α, a current is excited in α' in an opposite direction to that of the inducing current. The reverse applies when the intensity of the current diminishes. Both results agree with well-known facts. The proportionality of the induction to the variation of the intensity i of the inducing current is also in accordance with experiment.

Lastly, if we return from the consideration of these two distinct kinds of *voltaic induction* to the general case, where at the same time the intensity of the inducing current is variable and the two conductors are in motion as regards each other, the *electromotive force* exerted by the variable element of a current upon the *moved* element of a conductor is found to be simply as the *sum of the electromotive forces* which would occur—

1. If the element of the conductor *were not in motion* at the moment under consideration;

2. If the element of the conductor were in motion, but the *intensity of the current* of the induced element did *not* alter at the moment under consideration.

Article XV

Memoir on the Nocturnal Cooling of Bodies exposed to a free Atmosphere in calm and serene Weather, and on the resulting Phænomena near the Earth's surface (Second Memoir). By M. Melloni*

[Read to the Royal Academy of Sciences of Naples on the 23rd of February and 9th and 16th of March 1817.]

THE experiments described in the First Memoir (p. 153) tended to prove—

1. That the emissive power of metals is much weaker than has been hitherto supposed, and that a thermometer contained in a tin or copper case, exposed at night in the middle of the fields at a distance from substances which radiate heat strongly, indicates very nearly the true temperature of the stratum of air in which it is plunged, whatever be the state of the sky and the calm of the atmosphere.

2. That two thermometers, armed with their metallic cases, one of which is polished and the other covered with lamp black, suspended in the free air by threads or tubes of metal, at the same height, and during calm and clear weather, always mark different temperatures, the blackened thermometer being constantly lower than the polished one.

3. That the difference between the two radiations disappears under the influence of a strong wind, or of a sky covered with clouds, and is consequently the result of the unequal radiation of the thermometers towards space, as has been admitted in physics with reference to the nocturnal cooling of plants, since the labours of Wells on the subject of dew.

4. That the effect of the radiation of lamp black is nevertheless greatly inferior to that which is generally attributed to vegetable substances, for instead of 7° or 8°, it is 1°.5 or 1°.7 in the most favourable circumstances, which cannot be ascribed to an inferiority in the emissive power of lamp black as compared with vegetables, but rather to the faulty method employed for deter-

* Translated from the *Annales de Chimie et de Physique* for April 1818 by the Rev. A. W. Hobson, M.A., St John's College, Cambridge. The first Memoir will be found at page 153 of the present volume.

mining the temperatures of the air and of the plants; in fact, if we substitute a vegetable leaf for the lamp black in the arrangement adopted in our experiments, the cold produced on the thermometer is no more than from 1° to 2°, as in the observations above named.

5. That cotton and woollen stuffs communicate to the thermometers degrees of cold three or four times as great as those obtained by means of lamp-black and vegetable leaves; that such excess is diminished by condensing the matter round the bulb of the thermometer, and is reduced to the fraction of a degree in the case of cotton and woollen stuffs of fine and close texture, whence it follows, that the greater energy of these substances arises scarcely from their greater radiating power, but from the air interposed between the threads of which they are formed.

6. That the degree of cold due to the nocturnal radiation of bodies, does not vary with the varying temperature of the atmosphere

We shall now endeavour to prove that certain nocturnal differences of heat, humidity, and aqueous precipitation, do not arise, as is tacitly admitted in Wells's theory, from the direct action of the cold due to the radiation of plants and from the exposed portions of the ground, and that almost all the facts which precede and accompany the formation of dew, result from the presence (of shorter or longer duration) of the air around the radiating surfaces. Consider, in the first place, a large and fertile meadow, well furnished with grass, where the phenomenon of dew is developed in all its glory. Suppose the air to be calm, the sky pure and clear. In order to make the reasoning clearer, omit the consideration of the higher regions of the atmosphere, and let us divide the rest into two strata,—the *lower*, which scarcely rises above the grass of the meadow; the *higher*, which extends upward from this limit 30 or 40 metres. And although experience has shown us that the cold due to the nocturnal radiation of plants, that is, the lowering of their temperature below that of the surrounding medium, sometimes reaches 2°, let us suppose it to be only 1°, and let it not be forgotten, *that this degree of cold is always the same, whatever be the temperature of the atmosphere.*

If the air is at 20°, the higher portions of the grass will descend to 19° a few minutes after sunset; the air in contact with

them will be cooled will descend into the interior of the meadow, and reach the ground. This movement of descent along the leaves and stems will necessarily restore to it a portion of the lost heat, and will force it to reascend towards the higher part of the meadow where it will undergo a fresh cooling, which will cause a second descent, and so on: so that, the air of the meadow, or of our *lower* stratum, impelled by two opposite influences will soon take a circulatory motion, entirely analogous to that observed in the water of a vessel placed on the fire. The cold produced at the surface of the meadows will be gradually transmitted, by this aerial circulation, to the lower parts, which will also be cooled; and on the other hand, both by radiation and by their contact with the superior portion of the stems, the temperature of the whole mass of air which is put in motion in the interior of the meadow, will fall. Suppose it sunk to 19°.5. Now, according to the law which we have just referred to, the grass ought to maintain itself constantly 1° below the surrounding air: it will then have acquired half a degree of cold, and have sunk from 19° to 18°.5.

By repeating the same reasoning in these new conditions of temperature, it is evident that the air will fall to 19° and the grass to 18°. After that, the air arriving at 18°.5, the grass will descend to 17°.5, and so on in succession; so that by the *action* of the grass on the air and by the *reaction* of the air on the grass the temperature of the *lower stratum* will be gradually lowered several degrees and the space encumbered by the herbage of the meadow preserving all its vapour, will necessarily approach the state of saturation. Then, the thermometer introduced into this space will mark a temperature much lower than that of the *higher stratum*: the hygrometer will there be kept near its maximum of humidity, and the slightest degree of cold will suffice to precipitate the aqueous vapour on the bodies which are immersed therein.

Before studying the distribution of the dew and of the cold at different depths of the meadow, let us remark, that the extraordinary lowering of temperature presented in the preceding experiments by the thermometers enveloped with cotton or wool, compared with varnished or blackened thermometers (see the first part of this memoir), is the result of an action entirely analogous to that we have just been examining. In fact, the air, cooled by contact with the higher portion of these envelopes,

penetrates into the interior, and tends to fall towards the ground in virtue of its greater specific gravity; but the mechanical obstacles, and the attraction of this multitude of interlacing threads, hold it suspended for some time in the neighbourhood of the parts which radiate towards the sky, there is thus produced a series of actions and reactions similar to that we have just examined, and the mixture of air and of cotton or wool is much more cooled than the simple stratum of varnish or lamp-black applied to the thermometer. It is for the same reason that, *cæteris paribus*, those plants whose leaves are hairy acquire a rather lower temperature than those whose leaves are smooth and free from pubescence, and consequently are covered with a greater quantity of dew. But to return to the meadow. In order to indicate the portions of grass which are the most cooled by virtue of radiation, we have just now employed constantly the term "*higher*" instead of "summit," because, on examining the facts with a little attention, it is quickly seen, that if the first impression of greatest cold is produced at the outset at the superficial portion of the meadow, the minimum of temperature soon quits the surface, and is transferred to the interior.

Suppose, in fact, our "*lower stratum*" divided into three subdivisions or elementary strata, the first composed of the air which envelopes the summit of the herbage; the second, formed of the subjacent part, where the leaves are more numerous, and more or less exposed to the aspect of the zenithal region (which, according to preceding observation, is the most active of all in the phænomena of nocturnal radiation) (see First Memoir); finally, the third, composed of the air which embraces the stems and leaves, entirely shut out from the aspect of the sky. The summits of the grass certainly are placed in the most favourable conditions for radiating their heat freely into space; but the leaves are few there, and exposed to atmospheric disturbances, so that the small quantity of air which is cooled by contact with them scarcely produces any sensible effect on the rest of the stratum. The middle portion of the meadow, being more copiously provided with leaves and more sheltered, without being withdrawn from the so-powerful influence of the zenithal region, still further cools the corresponding air. As to the lower portion, which is totally shut out from the aspect of the celestial vault, it can only transmit to the surrounding air the cold derived from the communication of the stems, or from its radiation

towards the upper leaves, and consequently the temperature of the last elementary stratum will, at first, be the highest of all. But the air of the two upper strata will descend by virtue of its greater specific gravity, and will at the same time react on the radiating portions of the grass; this reaction will be the more energetic the more slowly the movement takes place. Now the obstacles are less numerous in the first stratum than in the second; the air, therefore, will react more strongly in the latter case; and having thus caused a greater depression of temperature in the middle portion of the grass, it will itself participate in this excess of cold through contact, and in its descent will communicate it to the upper portion of the third subdivision, which, again, will itself finally acquire a lower temperature to that of the former.

Thus the solid portions comprised in the three strata into which we have supposed the grass of the meadow divided, commence by a cooling proportioned to the quantity of heat which each of them can vibrate freely towards space; but the reaction of the surrounding medium soon disturbs this order of things to such an extent as to render coldest the leaves and stems, which are much less exposed to the aspect of the sky than the summits of the herbage. The thermometer then ought to maintain itself lower, when plunged to a certain depth in the grass, than when placed in contact with the surface; which is in accordance with experiment*. This distribution of cold, and the greater humidity which prevails in the midst of the grass in consequence of the evaporation from the ground, the transpiration of plants, and the difficulty of renewing the air, will necessarily render the precipitation of vapour more prompt in the interior than at the surface of the meadow. But the descending motion of the air continuing constantly in consequence of the cold due to the upper portions of the grass, and the ascending motion in consequence of the heat of the soil; if this is not too moist, its surface will soon be dried.

Then, *the cold air which descends will itself become dried,*

* If the earth had no atmosphere the *minimum* of temperature would always be found in the parts most exposed to the aspect of the sky; a thermometer imbedded in the interior of the meadow would at every hour of the night mark a higher temperature than that of a second thermometer placed in contact with the summits of the grass. By this we see how much the presence of the air modifies the effects of nocturnal radiation, and how great has been the error of neglecting the *reaction* of this fluid in the theory of dew.

being *heated by contact with the terrestrial surface*, and may easily, on its again ascending, evaporate the first drops of water deposited on the lower portions of grass, and again allow them to be afresh precipitated on the higher leaves. This successive transportation of dew will never occur in wet or very moist soils; and the lower portions of grass will there preserve the water condensed on their surface. But both in one case and in the other, the first appearance of the phænomenon will take place at a short distance from the soil, and will afterwards extend itself to portions of the plants more and more elevated, just as if the dew rose out of the ground and gradually rose in the atmosphere. Such, in fact, was the opinion of the ancient philosophers, generally adopted by those of the last century, and such is still the fundamental idea of the hypothesis maintained at this day by certain experimenters, who consider the phænomena which we have been describing as altogether contrary to the explanation of dew derived from the cooling produced by radiation.

Another fact, which, according to the same experimenters, also supports this alleged contradiction, is, the abundance of dew in perfectly calm weather. It is very true that great *tranquillity in the atmosphere is remarkably favourable to the deposition of dew; that the least wind diminishes it, instead of increasing it, as has been wrongly maintained of late; and that frequently the water deposited amounts to a much greater quantity than could arise from the elastic vapour contained in the small quantity of air placed in contact with the leaves and other radiating substances.*

But we have seen that the *lower stratum* of the atmosphere (as we have termed it) loses its state of equilibrium *in consequence of the nocturnal radiation of vegetables, and takes a rotatory movement, which commences in the first place by cooling the whole fluid mass of which it consists, and afterwards continues when the air deposits the vapour it contains; so that the fluid in contact with the leaves changes at every instant, becomes cooler and cooler, and, by fresh precipitations, increases the liquid drops scattered over the surfaces of bodies.*

Let us add, that the quantity of water deposited *does not depend solely on the vapour disseminated through the atmosphere, but also, and principally, on the humidity of the soil; and that it is most copious when the ground is saturated with water, as any one may easily convince himself in countries where artificial*

irrigation is practised. The air, in this case, becomes completely saturated every time it comes in contact with the soil; the quantity of vapour which it deposits by superficial contact on substances cooled by radiation, is much greater than in the case of a dry or nearly dry soil; and since these effects always ensue in virtue of the circulation established in the *lower stratum* of the atmosphere, we see that the air of this stratum forms a sort of vehicle, by means of which the liquid spread over the surface of the earth is successively carried to the surface of plants and other bodies cooled by nocturnal radiation. Now, it will be understood, that in order for this transportation to go on regularly the atmosphere must be calm: the slightest breeze disturbs it, and it is entirely destroyed by strong winds, which moreover (as Wells had already observed) introduce another cause of disturbance into the process, by communicating their own temperature to plants and thus causing that slight difference between the temperature of solid bodies and that of the surrounding medium to disappear on which, in fine, the phenomenon of dew depends.

Some have pretended to discover proofs of the existence of *a current of warm vapour exhaled by the earth*, and an objection against the principle of nocturnal radiation, in the different proportions of water deposited during calm and clear nights, on the two surfaces of a bell glass inverted on the ground; for it often happens that the dew is more copiously formed on the inside than on the outside of the vessel. But this fact by no means justifies the conclusion; for the phænomena of circulation and aqueous precipitation just described with reference to the air and grass of a meadow, are also produced in the interior of the vessel, the sides of which are cooled by radiation: these actions become even more intense in this case because the imprisoned air is sheltered from the least atmospheric disturbance; and we have just seen that the quantity of water condensed on the outside depends, on the contrary, on the degree of calm in the atmosphere. Hence, the slightest degree of wind will suffice to render more abundant the precipitation on the interior of the bell glass, without leading to the conclusion of an increase favouring the pretence of an exhalation of vapour from the earth, and contrary to the theory of dew founded on the cold produced by nocturnal radiation.

Nothing then is simpler now than to comprehend why a

radiating body, such as a piece of wood or stone, placed on a moist soil, towards sunset, is abundantly covered with dew on its lower side before a single drop of liquid appears on the upper surface The body submitted to the frigorific action of the sky is in contact with two masses of air,—the one, at rest and humid, because it is sheltered and situated close to the earth's surface, the other, less humid, and exposed to the changes of the atmosphere. The former then will be more disposed than the latter for the precipitation of vapour, and the dew ought to show itself first on the side turned towards the soil; it may even exist only on this surface, if the air has but little moisture or is agitated by wind. Hence the experiment of a plate covered with waxed cloth, which, being placed on the grass, was found sometimes to be moistened only on its lower surface, by no means proves that the *dew is exhaled from the ground, like those clouds of vapour which are seen to arise from a vessel full of hot water.*

Neither does the humidity which sometimes appears, towards the end of the night, on the surface of a dry soil, constitute an argument favourable to this hypothesis and contrary to the principle of nocturnal radiation, as some have maintained. In fact, two causes may contribute, either conjointly or separately, to the production of the phænomenon. Every one knows, that, when a moist soil becomes dry by means of wind or solar radiation, the water which has penetrated to a certain depth ascends by capillary action, and again moistens the surface when the permanent cause of the drying has ceased. Moreover, the uncovered soil is itself endowed, like the grass, with a proper radiation of its own, capable of cooling and of bringing upon it the deposition of the atmospheric vapour, especially in the long and humid nights of autumn, during which the cold engendered by the radiation of the surface penetrates more deeply, and can no longer be compensated by the heat of the internal strata.

The details into which we have entered are more than sufficient to prove that the reproach which has been cast several times on the partisans of Wells's theory, that they have neglected to take into account the moisture of the soil, is altogether unfounded; these philosophers, on the contrary, in accordance with the vulgar notions in this respect, refer the whole atmospheric humidity by which dew is caused, to the water spread over the surface of the earth In fact, vapour, *in its elastic and invisible state*, penetrates the atmosphere, not only by the means of rain,

but also by the evaporation, more or less abundant, of the seas lakes and rivers, the winds afterwards transport it, and spread it even to those countries where water is scarcest If the air and soil are impregnated with moisture which is the case in regard to calm and clear nights that succeed a season of rain, the dew shows itself everywhere in the greatest profusion But when the weather is extremely dry and the air calm, the local action predominates especially during the night, when the equilibrium of the atmosphere is not disturbed by the presence of the sun and in this case the atmospheric humidity is in proportion to the proximity of the sources Now in order to cause the air to deposit its vapour, it is necessary that there be a fall of temperature, more or less considerable according to the degree of humidity prevailing the precipitation of the atmospheric vapour, therefore, will be more slow and scanty in proportion as we remove further off from the reservoirs of water, and will cease entirely at a certain distance if the air be sufficiently dry, whatever may be the degree of cold which bodies acquire under the influence of a pure and calm sky This is the reason why, in seasons of great dryness, dew no longer shows itself except on plants situated in marshy or watered places, along the borders of lakes ponds and rivers

The nocturnal frigorific action excited by vegetables on the surrounding air, and the reaction of this fluid on the vegetables, can never cease until the heat communicated by the earth to the plants is equal to the heat lost by the radiation and the contact of the air And this state of equilibrium in a system of bodies so heterogeneous appears to require a considerable time, for if the sky be clear and the air calm during the whole of the night, the temperature goes on decreasing at the earth's surface, even till sunrise Hence in calm and clear weather, the lower strata of the air ought to be the more humid in proportion as the night is the more advanced It is for this reason that, *cæteris paribus*, the dew is precipitated in greater abundance, and penetrates more deeply into the interior of the tufts of plants, hedges, and groves, towards morning than in the earlier hours of the night, and that the phænomenon shows itself more copiously in autumn than in summer when, in consequence of the short absence of the sun the radiation of plants and the circulating movement of the surrounding medium do not last long enough to produce any great humidity in the lower region of the atmosphere

Every one has doubtless remarked that the dews are less copious in the earlier part of spring than in the equally long nights of the latter part of autumn. To see clearly the cause of this difference, it will be sufficient for us to observe, that the leaves, whence arises the greatest portion of the cold manifested at night in the lower strata of the atmosphere, are few and small at the beginning of the former season, large and numerous at the end of the latter; so that the cold, and consequently the increased degree of humidity, being greater in the latter case, the precipitation of dew is also more abundant. Add to this, that the quantity of elastic vapour existing in a given space increases more rapidly than the temperature; and since the diurnal heat is generally greater in autumn than in spring, we see that under the influence of the same radiation, there ought to be a greater quantity of vapour precipitated in the former season. The thickness of the stratum of air cooled at night by the contact of plants, will evidently depend on the nature and on the luxuriance of the vegetation; it will be large in meadows abundantly clothed with long grass of thick and vigorous growth, less in those where the herbage is low and poor, and still less on naked soils. The same theory will hold with regard to the position of the minimum of temperature, which will be found quite close to the soil in naked places, and can scarcely exist, as we have just seen, either at the base or at the summit of the grass, and will maintain itself near the numerous and compact leaves which are subject to the action alone of the zenithal part of the heavens.

These direct consequences of the theory have been perfectly confirmed by those persons even who deny the origin of dew founded on nocturnal radiation, and who think to explain the phænomenon by the exhalation from the soil. In fact, these gentlemen have found the *maximum* of cold at the height of 7 inches in a meadow covered with a luxurious vegetation, at the height of 2 inches in a meadow recently mown, and at a fraction of a line above the soil beaten down and entirely deprived of grass. Their thermometers, badly prepared for these sort of observations, being placed in contact with the leaves of different species of plants, gave indications sometimes equal, sometimes lower, *and scarcely ever higher*, to those of thermometers freely suspended at the same elevation above the soil. And in spite of results so little conformable to their views, they have persisted in maintaining that the depression of temperature

observed at night in plants does not arise from their radiation, but *from the presence of a thin stratum of cold air, which at sunset suddenly appears near the terrestrial surface*; thus substituting the effect for the cause, and therefore falling into one of the greatest errors with which observers of nature can be reproached. The cold produced by the radiation of vegetable leaves, of the soil, or of any other substance exposed to the nocturnal influence of a calm and clear sky, always precedes, as we have said, the fall of dew. The condensation of the vapour at first communicates to the radiating substance the heat disengaged in passing from the aeriform to the liquid state; but this heat is soon destroyed by virtue of the great emissive power of water; so that the moistened body, always preserving a temperature lower than that of the surrounding medium, continues to envelope itself with dew. All this may easily be verified in the fields by means of observation and of our thermometer prepared with the coil and metallic armature.

It must nevertheless be remarked, that in certain cases the nocturnal temperature of plants, under a clear sky, may equal, and even surpass for a few instants, the temperature of the surrounding air, when, in the midst of the calm, and the phænomena of cold and of dew which thence result, a sudden breeze comes and carries off from the radiating body the air which surrounded it, and substitutes in its place that of other bodies placed in conditions more favourable to cooling. For instance, the grass under a tree, enveloped suddenly in the air carried off from the neighbouring meadow, will at first show itself warmer than the surrounding medium, and will come at length to acquire the same temperature, if the action of the wind be sufficiently prolonged. But these anomalies are rare, and easily recognised, because of the wind which must necessarily precede or accompany them.

We have seen in the former memoir, that two of our thermometers with metallic armature, one of which was polished and the other covered with lamp-black varnish, sawdust or leaves of plants, marked the same temperature in the free air when care had been taken to shut them out from the aspect of the sky by means of metallic vessels closed on all sides; but that they indicated different temperatures the instant that the covers of the receptacles were removed; for then the former remained nearly immoveable, whilst the second descended 3° or 4° in a few

minutes. This experiment is sufficient to explain the small quantity of dew which is remarked under trees, in the interior of hedges, and in all places where the calorific communication between the sky and the earth is more or less intercepted; the radiating substances in these cases remain more or less dry, because the cold resulting from their nocturnal radiations is nothing, or less decided than in open places, as we may easily prove moreover directly by aid of the thermometer.

It would be needless to add, that the influence of the clouds on dew, and the cold which precedes and accompanies it, is perfectly analogous to that of trees, or of any other obstacle which intercepts more or less the view of the celestial vault from the radiating body. The upper clouds diminish, the lower ones completely destroy, the difference between the temperature of plants and that of the surrounding medium, and with it the gradual cooling, the increasing humidity, and the precipitation of vapour.

It is well known that the dew is less abundant on shrubs than on herbaceous plants, and that scarcely any traces of this nocturnal phænomenon are found on the summit of trees of a certain height. The explanation of this fact presents itself at once, if we consider that, in spite of their great emissive power, the leaves of lofty plants cannot become so much cooled as the grass of the meadow, nor precipitate the same quantity of water:—

1. Because they are more exposed to the action of winds than the leaves of plants nearer the ground.

2. Because the atmospheric stratum which envelopes them is less moist than that in contact with the soil.

3. Because the air which becomes cooler and condenses itself around them traverses the mass of foliage, and falls to the ground without the power of reascending, as in the case of the grass, towards the upper leaves, or of reacting on it or sufficiently lowering its temperature, and thus acquiring the degree of moisture necessary to a copious precipitation of dew.

The currents of air which descend from the top of the trees, must, like every other agitation of the atmosphere, disturb the actions and reactions between the neighbouring bodies and the medium which surrounds them, and thus render less intense the degree of cold which these bodies would acquire in a calm atmosphere. Consequently the grass situated close to trees will be less cooled and less moistened by dew than that which is in the middle of

the meadow, not only because its radiation into space is wholly or partly intercepted, but also because the medium surrounding it is less tranquil and the union of these two causes will produce the marked difference which is found, during calm and clear nights, in passing from the open field to a wood or from the wood to the fields

When we reflect on the numerous inequalities of temperature resulting as well by night as day, from the nature, the form, the exposed state or the culture of the soil, we soon become convinced that absolute equilibrium never exists in the atmosphere, what we call a *calm atmosphere* is, properly speaking, only a less violent agitation of it It is in consequence of this incessant perturbation of the atmosphere, that the stratum of air cooled by contact with plants and the soil gradually mingles itself with the upper strata even in the seasons of greatest apparent calm, moreover the quantity of air condensed by contact with plants will go on increasing upon the soil as the night advances, and will attain greater and greater heights Hence the origin of the two facts discovered by Peclet and Dufay, namely, the nocturnal inversion of the atmospheric temperature, which in calm and clear weather diminishes instead of increasing (as it does in the day time) on approaching the soil, and the precipitation of dew becoming retarded on a substance isolated or surrounded by plants in proportion as its distance from the earth's surface is increased Hence the limits which we have supposed between the two strata, "*upper*" and "*lower*," will never be very distinct, and the cold and humidity will diminish by insensible degrees as we rise in the atmosphere It will nevertheless be understood, that the transition will be more or less abrupt according to the nature of the soil and the time of observation, and if the air were coloured so as to be perceptible in the dusk, we should see at night this colour become more marked near the earth's surface up to a certain height, greater in proportion as the night is further advanced, and thus form a kind of zone, of greater or less magnitude and distinctness, which would follow the general distribution of vegetation, attaining its *maximum* of intensity on meadows and fields clothed with low plants, growing thick and close, and spreading on all sides as far as the furthest boundary of the horizon

From all that precedes, it follows that Wells's principle with regard to the formation of dew in virtue of the radiation of bodies,

may be completely defended against the violent attacks to which it has been subjected of late, that, nevertheless, the radiating substances are cooled much less than was supposed, in consequence of the bad arrangement of the instruments formerly employed in these kind of researches. On the other hand, both Wells, his partisans and his opponents, appear to have paid no attention to the important part played in this phænomenon by the well-known fact of the invariable difference between the temperature of the air and that of the radiating body, so that everybody has completely overlooked the reaction of the medium, which exerts so remarkable an influence over the distribution and the intensity of the cold produced by the nocturnal radiation of the soil. We have endeavoured to supply this deficiency; and, taking for our point of departure the feeble degree of cold which is incontestably produced in vegetables and every other radiating substance exposed to the free air in a calm and clear night, we have arrived at a clear explanation of,—1st, the great difference of temperature between the air which envelopes the low plants of meadows and fields, and the superimposed air; 2nd, the greater degree of cold in the interior than at the surface of meadows, 3rd, the great humidity which always prevails in the stratum of air wherein the plants are immersed, from the first moment of the precipitation of dew, 4th, the favourable influence of a perfect calm in the atmosphere, 5th, the accumulation of dew during the whole of the night; 6th, the more copious formation of dew from midnight to daybreak, than from sunset to midnight; 7th, its abundance on plants which have smooth leaves; 8th, its small quantity on trees, in comparison with what is deposited on the grass; 9th, its transportation, or progressive invasion from a lower to a higher region, 10th, its different proportions in different seasons, 11th, and finally, all the circumstances, without exception, which precede and accompany, at any period whatever of the year, the appearance of dew on the earth's surface.

The principle of the invariable lowering of the temperature of bodies exposed to the free air during calm and clear nights, below the temperature of the atmosphere, constitutes therefore the fundamental base on which rests the theory of the phænomenon which we are studying.

Let us recapitulate. Dew is not an immediate effect of the cooling produced by the nocturnal radiation of vegetables on the

vapour of the atmosphere, as most treatises on physics and meteorology assume, but the result of a series of actions and reactions between the cold due to the radiation of plants and the cold transmitted to the surrounding air. The glass is cooled but little below the temperature of the air, but it very quickly communicates to it a portion of the acquired cold; and since the difference of temperature between the radiating body and the surrounding medium is independent of the absolute value of the prevailing temperature, the glass surrounded by colder air still further lowers its temperature and communicates a new degree of cold to the air, which reacts in its turn on the glass, and compels it to acquire a temperature still lower, and so on in succession. Meanwhile the medium loses its state of equilibrium and acquires a sort of vertical circulation, in consequence of the descending motion of the portions condensed by the cold of the upper foliage and the ascending motion of the portions which have touched the surface of the earth. Now the gradual cooling and the contact of the soil evidently tend to augment the humidity of the stratum of air and thus bring it by degrees towards the point of saturation. Then the feeble degree of cold produced directly by the radiation of bodies, suffices to condense the vapour contained in the air which surrounds them; and since the causes which give rise to the circulating movement and to the humidity of the air continue through the whole of the night, the quantity of water deposited on the leaves increases indefinitely.

The greatest part of the nocturnal cooling is due to the development of the leaves which presents to the sky an immense number of thin bodies having large surfaces, and almost completely isolated: this is the reason why the dews are so feeble in winter and less copious in the nights of the early part of spring than in the equally long nights of autumn. Dew is also more abundant in autumn, because the days being then warmer than in spring, and the vapour increasing more rapidly than the temperature, the same degree of cold (such as the invariable depression of the temperature of plants below that of the atmosphere) condenses a greater quantity of vapour. The slightest breath of wind disturbs the circulation of the lower atmospheric stratum, and necessarily diminishes the accumulation of dew. A strong wind impedes its formation by bringing fresh supplies of heat, and especially by renewing incessantly the stratum of

air comprised between the summit of the plants and the surface of the earth, and thus taking away from it the possibility of gradually acquiring that high degree of humidity necessary to the precipitation of the vapour, by reason of the small degree of cold which the plants contract with regard to the surrounding medium.

The differences in the quantity of dew on different substances all arise, either from their difference of emissive power, or from the diversity of their situation with regard to the heavenly vault, or from the hygrometric condition of the surrounding space, or from the greater or less obstacles which retard the descent of the air, and thus more or less favour its frigorific reaction; or, lastly, from the proximity of the soil, which permits the return of the air on the radiating substances, and gives rise to that aerial circulation, whence result the gradual cooling and successive augmentation of humidity in the lower stratum of the atmosphere.

To complete the study of our subject, it now only remains for us to examine the intensity of the nocturnal radiation and the distribution of dew in the different regions of the globe.

Many observations have been made to determine the diurnal temperature in different parts of the world, but very few with the object of determining the nocturnal heat, so that we are almost entirely ignorant as to what are the true proportions between the temperatures of day and night in different latitudes and seasons of the year. In accordance, however, with the preceding remarks, it is seen that in calm and clear seasons the difference between the temperature of the day and of the night ought to be so much the greater as the vegetation is richer and the night longer; and we have already observed, that in the nights of the early part of spring, vegetation being but little developed, the temperature is less lowered than in the latter part of autumn, when the plants still preserve a part of their foliage. We shall now add, that in those countries where the foliage is generally narrow and vertical like that of New Holland, the nocturnal temperature ought to be less diminished, relative to the diurnal temperature, than in places of the same latitude covered with plants analogous to those which grow in other countries.

But, laying aside everything depending on the alternations of the seasons in our temperate climates, and on the differences of vegetation in countries situated under the same latitude, it is

easy to convince ourselves that the greatest difference between the temperature of the day and that of the night will occur under the torrid zone and that there also the dews will, in general, be more abundant than in any other part of the globe In fact in cold and temperate countries the two principal elements of nocturnal radiation proceed (so to speak) in opposite directions, since the night is long when the earth is destitute of vegetation, and short when the plants are richly clothed with foliage But under the equator vegetation never fails, the night is always long and almost entirely without twilight and in the neighbouring countries forming the torrid zone properly so called when the night time slightly exceeds the period of daylight, the rain falls in torrents, and plants are more richly clothed with leaves than at any other season of the year The greatest difference, then between the temperature of the days and that of calm and clear nights, will occur in the equatorial regions a short time after the rainy season, and as there will then prevail in the atmosphere a high degree of humidity, the dew itself also will be very abundant at this season On the other hand since the torrid zone possesses the highest known atmospheric temperature, the nocturnal cooling ought to precipitate there a larger quantity of water than in any other country, by reason of the divergence above mentioned between the progression of the vapour and that of the temperature In fact, the dews are so copious in the equatorial regions, that M de Humboldt does not hesitate to compare their effect with those of rain itself

A curious fact, and one not much known, which seems at first sight to contradict what we have been saying, is the extreme feebleness or the absolute non existence of dew, in that extensive assemblage of small islands in the torrid zone, generally fertile and more or less rich in plants, which geographers denominate *Polynesia*

But, with a little attention, it will soon be seen that this apparent anomaly affords one of the most striking confirmations of the truth of the theoretical views unfolded in the course of this memoir In fact whatever may be the humidity of these small islands, scattered here and there in the vast ocean like oases in a desert and their tendency to the cooling produced by the long nights and luxuriant vegetation, the small extent of their territories renders the atmospheric column superincumbent on each

of them easily permeable even to its centre, by the air of the surrounding sea. This invasion is, moreover, favoured by the trade-winds which prevail constantly in those latitudes. Now we know that the air in the midst of vast seas preserves a nearly uniform temperature. The stratum of air cooled by the contact of the soil will, then, be warmed by mixing with the air which is constantly reaching it from the sea, and the difference betweeen the temperatures of the day and night being extremely small, dew can scarcely be formed at all, or at any rate, in very slight quantity.

Perfectly analogous causes prevent the formation of dew on ships which traverse the vast solitudes of the ocean. But what is truly singular, is the appearance of the phænomenon on board these same ships on arriving afterwards in the neighbourhood of *terra firma*. Thus, the navigators who proceed from the straits of Sunda to the Coromandel coast, know that they are near the end of their voyage when they perceive the ropes, sails, and other objects placed on the deck, become moistened with dew during the night. (Le Gentil, *Voyages*, tome i. page 625.) The reason of this strange phænomenon will readily be seen, if we start from the fact (well established by experience), that, in the equatorial regions, the sea-air preserves, not only a nearly constant temperature by day and night, but also an hygrometric state considerably removed from the point of saturation; and that the reverse is the case with regard to the air on land, which in the day-time is drier than the air of the sea, but which in the night may readily acquire, in countries sufficiently abounding in water, or near enough to the coast, a much greater humidity in consequence of the frigorific actions and reactions of which we have before spoken. Now the land-wind, which always blows by night on the borders of tropical countries when the sky is clear, transports this humid air to a certain distance out at sea. Then, the feeble degree of cold acquired by substances freely exposed on the deck, totally unable, as it is, to condense the vapour of the sea-atmosphere, is nevertheless sufficient to precipitate that of the air which has been in nocturnal contact with the soil.

We conclude that dew, feeble or non-existent towards the poles by reason of the extreme brevity of the summer nights, becomes more and more abundant as we approach the equator; that, notwithstanding, the general course of the phænomenon

is very much modified by the extent, the nature and the position of the land, according as it is more or less surrounded by the sea, more or less covered by mountains, ravines, lakes, meadows, marshes, or running streams. The borders of Egypt, of the Red Sea, of the Persian Gulf, of Chili and of Bengal, are celebrated for the richness of their dews, (See the *Voyages de Volney*, t. i. p. 51, of Burckhardt, p. 423, of Niebuhr, p. 10, of Ker Porter, t. ii. p. 123, of Le Gentil, t. i. p. 621, of Ruppel, p. 186.) the deserts of Central Africa, and the interior provinces of Bahia, of Fernambouc, Urmia and Mazanderan, in Brazil and Persia, by the almost total absence of this nocturnal phænomenon. (Voyages of Spix and Martius, t. ii. p. 624, of Olivier in Persia, t. i. pp. 123 and 145, of Ker Porter, t. ii. pp. 63 and 69.)

The appearance of dew may serve, in certain cases, to make known the proximity of a mass of water concealed from the eye of the observer. Thus, the dew which is almost completely wanting in certain sterile valleys traversed by the Euphrates, becomes of sufficient intensity to form visible drops of water, whilst still at a distance of some miles from the borders of this river concealed by the land. (Olivier, t. ii. p. 225.) And Major Denham says, that independently of the suffocating heat and of the intense cold that he endured during the night in his memorable journey across the Sahara, he also suffered from the extreme dryness of the air until he reached a certain distance from Tschad, where, though there was not the slightest appearance of water on any part of the horizon, the dews began to appear, feeble at first, then more and more copious, and so abundant on arriving near the banks of this great African lake, that the clothes of those persons who remained some time outside the tents were completely soaked with it. (Denham, Narrative, p. 19.)

With regard to the intense cold experienced by this intrepid traveller during the night in the desert, it is occasioned (in my opinion) neither by the extreme clearness of the sky, nor by an excess of cutaneous perspiration, but from the great nocturnal calm of this desolate region, which allows the soil to act strongly on the air and to receive with equal force the reaction of that fluid. Observe first, that a dry, flat, monotonous, horizontal and uniformly extended country, like this immense plain of Northern Africa, so well characterized by the Arabs under the name of the *sea without water (el bhar bella mda)*, presents no cause capable of disturbing during the night the equilibrium of

the air; so that this must remain in a state of almost absolute rest some time after the setting of the sun. The soil of the desert being moreover composed of dry, sandy earths, of bad conducting quality, can receive from the interior but a very poor compensation in exchange for the heat it has lost. The solid body radiating by night towards space, and the surrounding medium, will therefore be unmoving and isolated, and thus be in highly favourable conditions for reacting with energy on each other, and considerably lowering their temperature.

Another phænomenon resulting from the combination of the two frigorific actions successively excited in the radiating body and the medium which envelopes it, is the congelation of water, produced artificially in Bengal, during the calm and clear nights. It would be superfluous to repeat here the details relative to this process, a description of which may be found in all treatises on physics. It will be sufficient to call to mind, that the vessels, very shallow and uncovered, containing the liquid to be frozen, are placed at the bottom of certain excavations made in the soil, and *surrounded by a border of earth, 4 or 5 inches in height*; that the water, whose emissive power is nearly equal to that of the leaves of plants and of lamp-black, *does not descend even two degrees lower than a covered thermometer placed by its side*, and that frequently the ice is formed when the thermometer, elevated 4 or 5 feet, marks 5° or 6° above zero; which leads to the immediate inference, that the water lowers gradually its temperature down to the zero of the thermometric scale (Centigrade) by means of a series of actions and reactions perfectly similar to those which produce, under the same circumstances of calm and clearness of sky, the nocturnal cooling of any other radiating matter exposed to the free air, and the decrease of the atmospheric temperature, in proportion as we approach the earth's surface.

It is in consequence of these same frigorific actions, that the buds of plants, and the shallow waters of ditches and ponds scattered here and there over the country, often freeze during the calm and clear nights of spring, whilst the thermometer marks several degrees higher than the freezing-point.

[Note of the Editors of the *Annales de Chemie et de Physique*.]

We suppress the third and last part of the Memoir, which is devoted to the refutation of the hypothesis which ascribes the

formation of dew to the exhalation of the soil and the mysterious appearance of a stratum of cold air at the earth's surface. This refutation, however useful in Italy, where this theory of dew is still taught in certain schools placed under the protection of the Austrian government, appears to be superfluous for those persons who, after reading attentively the preceding pages, can no longer retain the shadow of a doubt as to the true cause of the phænomenon. We shall merely call the reader's attention to the two subjects treated of in the Third Part of M. Melloni's memoir, namely, the experiment by means of which the dew is forced to deposit itself on certain portions of a metallic surface, whilst the other parts are maintained in their habitual state of dryness; and the theory of that sort of dew (*serein*) or excessively fine rain, which falls sometimes on fine summer evenings when the sky is clear and free from clouds.

The experiment which is described in detail in the first of the two letters of M. Melloni to M. Arago, consists in partially varnishing one of the two surfaces of a very thin tin plate; afterwards covering a portion of the varnished surface with a sort of small detached roof of polished metal, and exposing this arrangement of two plates to the free air, so that the side which is varnished and partially sheltered under the metallic roof, may be turned towards the sky, and the surface which is entirely polished towards the ground. The dew is deposited in large quantity on the surface uncovered by varnish, and from thence in diminishing proportions, on the adjacent parts. The roof of polished metal remains entirely dry and brilliant, as also the central portions of both sides of the subjacent disc; the rest of the surface looking towards the ground is, on the other hand, quite covered with dew.

Some experiments of Wells had shown that dew does not fall from the sky; others, that it does not rise from the ground. The experiment of M. Melloni proves these two things at the same time; for there are portions of metal moistened by the dew, and others perfectly dry and brilliant above and below the system of two plates, that is, on the side looking towards the sky and on the side turned towards the ground. It proves, besides, incontestably, that metals cooled by the juxtaposition of a radiating substance, condense the elastic vapour of the atmosphere as well as the leaves of plants, and consequently, that the ordinary want of dew on the polished surfaces of these bodies,

arises neither from a peculiar repulsive force, as Leslie supposed, nor from an electric action, as Saussure believed; nor from the heat disengaged by the chemical reaction of the metal on the aqueous vapour, as M. Fusinieri has latterly maintained; but solely from their extremely feeble emissive power, which does not produce a degree of cold sufficient to cause the condensation of the elastic vapour contained in the surrounding medium. This experiment, therefore, is a sort of collective proof, which embraces in itself the fundamental principles of the theory of dew developed in the Second Part of the memoir.

As regards the theory of "*serein*," we cannot do better than give the translation of the passage in which it is pointed out.

"Several authors," says M. Melloni, "attribute to the cold resulting from the radiation of the air, the excessively fine rain which sometimes falls in a clear sky, during the fine season, a few moments after sunset. But, as no fact is yet known which directly proves the emissive power of pure and transparent elastic fluids, it appears to me more conformable to the principles of natural philosophy, to attribute this species of rain to the radiation and the subsequent condensation of a thin veil of vesicular vapour distributed through the higher strata of the atmosphere, in a manner so as not to cause any considerable alteration in the azure tint of the sky. The beautiful phenomena of colours which appear in the west when the solar rays are quitting our hemisphere, have often afforded me the opportunity of observing isolated clouds, which had lasted for some time, suddenly diminish in volume and intensity the instant they ceased to be struck by the sun's rays, and soon become completely effaced without leaving a trace of their former existence. In meditating on the causes of these vanishings, it appeared to me evident that the upper part of the cloud, no longer receiving, after the sun has set, any compensation for the heat radiated into space, is condensed into water, and the subjacent stratum takes its place, undergoing the same changes, and so on in succession, so that the whole cloud is soon reduced into liquid drops which pass into the state of elastic fluids, in falling through the space beneath *."

* It is perhaps a process of this kind which has helped to confirm an opinion held by the many, that the light of the full moon dissipates the clouds; for then the vesicular vapours are abandoned by the direct or diffused rays of the sun, and begin to be cooled by vibrating their proper heat into space, at the very moment the moon rises above the horizon. I would nevertheless observe, that

Moreover the "*serein*" always falls in summer or at the commencement of autumn at the close of warm, moist days, under a sky slightly scorched and whitish (*hâlé et blanchâtre*). There is every reason therefore for believing that the air is then saturated with humidity up to a certain elevation, and that the upper portion of this diaphanous vapour is transformed into vesicular vapour by the cold which prevails in the higher regions of the atmosphere. These vesicles which are sufficiently rare and uniformly scattered as to cause only a slight tinge of white which does not sensibly alter the proper colour of the atmosphere will therefore lose, with the last rays of the setting sun, the heat which supplied the losses caused by their radiation into space, there will be a lowering of temperature, and the formation of small drops which, traversing in their descent the lower strata of an atmosphere saturated with humidity, can suffer only a feeble degree of evaporation, and hence will even reach the surface of the earth.

dusk may very probably play a certain part in the production of this phenomenon. I make this remark because on looking at the moon through a good glass I have often observed its disc traversed by fragments of clouds whilst the stars everywhere shone brightly and the sky appeared perfectly clear.

ARTICLE XVI.

Experimental Researches on the Action of the Magnet upon Gases and Liquids. By M. PLÜCKER, Professor of Natural Philosophy in the University of Bonn.*

[From Poggendorff's *Annalen*, March 1848.]

1. AS subject of the present, which is my third communication, I have selected from my experimental researches upon the action of the magnet, two classes of phænomena—one of which relates to the magnetic or diamagnetic action of the magnet upon *liquids*, the other to its action upon *gases*. In the observation of the former I adopted a different method from that of Faraday; I observed the motions of the various liquids above the approximated poles of the magnet, and the alterations produced by the poles in the form of the surfaces. When, as in the blood, minute corpuscles are suspended in the liquid, the microscope may be advantageously called in aid to observe their motions. In all these experiments I found Faraday's results completely confirmed. Not so, however, with regard to the reactions of gaseous bodies; for in this case my experiments led me to results which completely contradict the statement of this philosopher, that bodies, in passing into the gaseous state of aggregation, become indifferent to magnetism. I communicate these results with that diffidence which becomes me in opposing so great an experimenter.

2. In my investigations I made use of the horseshoe electro-magnet, which has been described in the second paragraph of my first memoir (p. 351); this stands perpendicularly, so that the surfaces of its poles are directed upwards. To allow of the increase of the magnetic tension in different ways by the approximation of the poles, polished pieces of iron of various forms were applied; and these being united in pairs, formed the keeper. They consisted of,—*first*, two parallelopipedal halves of the keeper (A), which have been mentioned in paragraph 11 (p. 372) of the previous memoir, 27 millims. in height, 67 millims. in breadth and 198 millims. in length; *secondly*, two halves of the

* Translated by Dr. J. W. Griffith.

keeper (B), as high as the preceding, as broad as the poles of the magnet and 176 millims. in length, rounded off circularly on one side, and so narrow that the terminal surfaces formed circles 25 millims. in diameter. Upon these pieces of different shapes, such as conical apices, could be screwed. These two halves of the keeper may also be advantageously substituted for the two perforated cylindrical appendages with the inserted pointed cylinders, in the experiments described in the two preceding memoirs. Even with the most powerful magnetic excitation and the greatest approximation of the conical apices, these two halves of the keeper do not fly together. *Thirdly*, two heavier halves of the keeper (C): these were originally intended for optical purposes; 40 millims. in height, 133 millims. broad and 203 millims. in length, circularly rounded at one end, gradually tapering at the other, and prolonged into a rectangular surface of 10 millims. and 59 millims. In the middle of these two halves of the keeper, throughout their whole length, a groove is cut, 20 millims. in breadth, of the same depth, and the section of which is semicircular at the bottom. The halves of the keeper (A) and (C), when the excitation is very intense and the approximation sufficient, are attracted by each other with great force; they are kept apart by pieces of brass of different thicknesses placed between them.

In the experiments upon the free ascending gases the rectangular glass case of the torsion balance, which, when the latter is removed, has a rectangular aperture at the top in the middle, 251 millims. in length in the equatorial direction (corresponding to the breadth of the case) and 97 millims. in the axial direction, was generally placed upon the moveable table with the two round holes through which the arms of the magnet passed.

In the experiments to be described I used from five to ten Grove's cells, the larger number only being set in action when the nitric acid had been frequently used previously.

§ 1. *On the Diamagnetism of Gases.*

3. Faraday devotes the paragraphs 2400 to 2416 of the twenty-first series of his 'Experimental Researches on Electricity' to the action of magnets upon air and gases, and in the last paragraph arrives at the following conclusion:—

"Whatever the chemical or other properties of the gases, however different in their specific gravity, or however varied in

their own degree of rarefaction, they all become alike in their magnetic relation, and apparently equivalent to a perfect vacuum Bodies which are very marked as diamagnetic substances, immediately lose all traces of this character when they become vaporous."

4. Faraday performed his experiment by taking an open glass tube, which was as indifferent as possible to the magnet, and after the air had been withdrawn from it, hermetically sealing it, allowing it to oscillate, both before and afterwards, between the poles surrounded by air He found no difference, even when the tubes were allowed to oscillate in various gases, nor when filled with them. Nor did he find any difference on allowing the tube, either containing gas or not so, to oscillate in water, alcohol or oil of turpentine, nor, lastly, when a solid diamagnetic body, as heavy glass or a bar of bismuth, was made to oscillate in different kinds of, or variously compressed gases. On first perusing these experiments, it was evident to me, on mechanical grounds, that for any effect to be apparent in any of them, the gases must be magnetic or diamagnetic to a perfectly enormous extent, for the magnetic or diamagnetic force of a substance must evidently diminish with the rarefaction of the substance. We will for a moment consider, hypothetically only, this diminution and rarefaction as in proportion to each other, as is the case with attractive forces, and also, which is more to the present point, in the case of the rotation of the plane of polarization in liquids, in which, in a solution of sugar *e.g.* the amount of rotation is in proportion to the quantity of sugar in solution. A magnetically indifferent tube, when completely filled with water and suspended so as to oscillate horizontally in water, does not assume a definite position between the poles of a magnet, because the same force which urges the water contained in the tube to assume an equatorial position, is counterbalanced by the diamagnetic excitation of the surrounding water. If the tube, when oscillating horizontally, contains only half the quantity of water which it originally did, and which then for the same length has only half the section, the second force will preponderate. The tube is driven into the axial direction, and is retained in this direction with a force equal to half the diamagnetic force exerted upon the entire mass of the water originally contained in the tube, and which we shall consider as unity. If only the $\frac{1}{1000}$th of the original water remained in the tube dif-

fused equally throughout its longitudinal direction, the tube would be retained in the axial position with a force amounting to $\frac{1799}{1800}$ and this force would be equal to unity itself if the tube were entirely free from water. The difference between these two forces which amounts to only the $\frac{1}{1800}$th of the magnitude of that diamagnetic force originally acting upon the water contained *in* the tube can never be shown by the rotation of the tube even independently of the resistance in the surrounding fluid. If the mass of water $\frac{1}{1800}$ was converted into the form of vapour within the tube no alteration would be produced supposing that the action of the magnet upon the molecules of its vapour were the same as upon the molecules of water. The diamagnetism of the vapour of water could never therefore be shown in this way.

The same occurs in all the other experiments detailed at the commencement of this paragraph.

5. Faraday says:—"I have imagined an experiment with one of Cagniard de la Tour's æther tubes but expect to find great difficulty in carrying it into execution, chiefly on account of the strength and therefore the mass of the tube necessary to resist the expansion of the imprisoned heated ether."—(2435.) If Faraday's experimental skill should succeed in overcoming the difficulties of this experiment, we should obtain a direct decisive answer to the question whether diamagnetic fluids lose their diamagnetism on conversion into the state of vapour. Faraday anticipates an affirmative answer; however, on the ground of the experiment subsequently described, I confidently expect a negative answer.

6. On my own part, to decide upon the diamagnetism of the gases I first endeavoured to diminish the mass of the reservoir enclosing them, and I thus among other things, arrived at the idea of using a soap bubble as the envelope. In a preliminary experiment I laid a lamina of mica upon the two approximated surfaces of the poles of the electro magnet and placed a soap bubble of a hemispherical form upon this, but I did not find the magnet exert any action upon the form of the soap bubble; this was also equally the case when it was filled with air as when it was filled with tobacco smoke. I then completely gave it up for enclosing the gases, and the use of the *coloured* gases immediately suggested itself to me as a means of deciding upon the magnetic, diamagnetic or neutral state of the gases.

7. I placed the two halves of the keeper B (see paragraph 2.), with the conical apices screwed into them, upon the surfaces of the poles, in such a manner that the distance of the two apices from each other amounted to 3·5 millims. Beneath them I placed a previously heated thick plate of platinum, and covered the whole with the open case of the torsion balance mentioned above. Small pieces of iodine were placed upon the plate of platinum, and as soon as a very narrow column of vapour of iodine ascended perpendicularly between the apices of the poles the magnetism was excited by closing the circuit. The column which was previously ascending perpendicularly, immediately expanded above the apices of the poles so as to form a parabola in the equatorial plane; this, especially on the concave side, where the violet colour was most intense, was very accurately defined, and remained perfectly distinct for an elevation of 100 to 150 millims. We shall hereafter allude to a similar figure formed by the soot of a turpentine flame; and as a sketch of this appearance is subjoined, further notice of it will be omitted at present.

The experiment described in the above paragraph shows beyond a doubt *that the vapour of iodine is repelled by both the poles of a magnet*.

8. This result is confirmed when the two parallelopipedal halves of the keeper (A.) are applied by their broad surfaces. If the ascending vapour of the iodine is then conducted between the two surfaces of the poles of the halves of the keeper, it becomes displaced laterally (in an equatorial direction); and if it be allowed to ascend by their side, it is repelled outwards. This effect is found to be most intense when the magnetic tension is increased by again approximating the two keepers to within 3 millims. to 4 millims.

9. In interpreting the result obtained in the two preceding paragraphs, we must not forget that the iodine vapour is surrounded by air. This iodine vapour, in proportion to the repulsion which it experiences by the poles of the magnet, either becomes diamagnetic, and if so, more powerfully diamagnetic than the air, if the latter is at all so, or it exerts a neutral reaction. But for the air to become magnetic, the former, if not diamagnetic, must still be less magnetic than the latter.* The former

* The remarkable experiment of Faraday, in which he suspended a glass tube filled with a solution of protosulphate of iron in a similar solution between

of these two views, that both the iodine vapour and the air are diamagnetic, is shown to be extremely probable by the experiment detailed in paragraph 11.

10. A few drops of *bromine* were placed in a glass retort with a short neck, which was so drawn out that the aperture was not more than 3 millims. in diameter. The glass retort was then arranged with its orifice immediately beneath the apices of the poles, which were 3·5 millims. apart; it was then heated, and as soon as the vapour began to flow out the magnetism was excited. The ascending column of vapour was repelled in the equatorial plane towards that side of the apices on which it passed before the excitation of the magnetism, but not so constantly and regularly as in the case of the iodine. *Thus the vapour of bromine acts in general in the same manner as that of iodine.*

10 *a*. The vapour of chlorine, evolved from peroxide of manganese, chloride of sodium and *concentrated* sulphuric acid, was also repelled.

11. In the same retort and with the same adjustment, pieces of copper wire were placed, and nitric acid was then poured upon them; the nitrous fumes which were evolved escaped from the orifice with a varying amount of force. That they were repelled was at once perceptible; but it was subsequently found, as the vapour ascended in the form of a cylinder of about 4 millims. directly between the poles, that they expanded above the apices of the poles in the equatorial plane in the form of a parabola, the summit of which was situated somewhat above the middle of the space between the poles, and the axis of which was formed by the continuation of the direction of the original gaseous current. The latter, in the equatorial plane, retained nearly its original diameter, but sloped off at right angles to this plane to about one half. In the complete transverse section of the parabolic current the gas appeared to be nearly uniformly diffused. It ascended regularly to an elevation of from 60 millims. to 80 millims., although it was less distinctly defined than in the case of the iodine.

the poles of a magnet and allowed it to oscillate, when the tube was found to become arranged axially or equatorially, hence reacting magnetically or diamagnetically, according as the internal or the external solution was the stronger, may be easily repeated without any special precaution. For this purpose I have used tubes of thin glass 20 millims. in breadth and about 125 millims. in length, closed with calves' bladder and without a keeper.

Thus nitrous gas acts generally in the same manner as the vapour of iodine and bromine and chlorine gas.

12. The experiment described in the preceding paragraph especially claims our attention, because nitrous gas contains the same ingredients as the *air*, but in different proportions and in a state of *condensation*. If the gas were merely *condensed air*, its repulsion in the ordinary atmosphere would incontestably prove that both the air and the gas are diamagnetic; for condensed air, whether magnetic or diamagnetic, is necessarily more powerfully affected by the magnet than that in the ordinary state; and from the repulsion of the former it would follow that the action upon the air is altogether *diamagnetic*. [For exactly the same reason, a more dense stone, which contains more matter, sinks in water, whilst it would rise if the attraction of the earth were to be converted into a repulsive force. Because the force of gravity acts equally upon the matter of the stone and of the water, we conclude, from the sinking of the former, that the force of gravity exerts an attractive and not a repulsive power.]

According to a general principle which Faraday (in the case of solids and liquids) has laid down, every mechanical or chemical combination of diamagnetic bodies only is necessarily diamagnetic, whilst every compound of magnetic bodies only is magnetic. On extending this principle to gaseous bodies, the experiment in the preceding paragraph would rigidly demonstrate that, if nitrogen gas and oxygen are affected *in the same manner* by the magnet, the action of both, as also the action of the air and of the nitrous gas, must be *diamagnetic*. But if the air exerted a magnetic action, one of the gases, either the oxygen or the nitrogen, in fact that which predominates in nitrous gas in comparison with the air, hence the first, must be diamagnetic, the other magnetic. The latter supposition has not *per se* the slightest probability; on the other hand, we might admit with more certainty that the air is diamagnetic, and we shall adapt this view to our method of expression in the subsequent remarks

13 *Visible aqueous vapour*, which must in fact be regarded as nothing more than a true gas, is also repelled by the magnet. It was evolved in a vessel used for the determination of the boiling-point of water, and conducted by means of a long funnel between the apices of the poles, the distance of which apart remained the same as before. The repulsion was very distinctly

seen even without covering it with the case of the torsion balance which the apparatus for evolving steam did not permit.

When the aqueous vapour was made to ascend in the same way between the two parallelopipedal halves of the keeper (A), approximated to 3·5 millims. at the moment of the closure of the circuit, it was forced from the intermediate space between them and extended laterally in the equatorial plane.

Aqueous vapour is thus even more strongly diamagnetic than air.

13 *a*. The following proceeding was adopted for examining the magnetic relation of the *vapour of mercury*. The conducting wire issuing from the platinum end of a battery of twelve cells was conducted into a vessel containing mercury, which stood immediately under the apices of the poles, whilst the conducting wire emanating from the zinc end was coiled round the iron nucleus of the magnet, and afterwards terminated in the mercury. The circuit was opened by withdrawing the *former* conducting wire from the mercury, and again closed when the vapour of the mercury accompanying the sparks arising from the separation ascended between the apices of the pole. The anticipated repulsion then occurred. *Hence the vapour of mercury is also more strongly diamagnetic than air.*

14. Since the different kinds of flames are nothing more than gases produced by the process of combustion at a red heat, with or without an admixture of solid matters at a red heat, it became of interest to subject them also to the action of the magnet. On so doing very beautiful phænomena resulted; these exhibited great variety, according as on the one hand the form of the poles and their distance apart were altered, and on the other various kinds of flames were used. When the keeper (B), with the conical apices screwed on, was applied, most remarkable phænomena presented themselves. These I shall particularly describe*.

15. When a common *stearine candle* was placed midway be-

* M. Zantedeschi has a short time since made a communication to the Academy of Sciences at Paris, to the effect that various flames are repelled and depressed when in proximity to the poles of a magnet. I at once thought to recognize in this phænomenon a diamagnetic action, and this circumstance, in combination with the theoretical considerations developed in paragraph 1 [of] the first part of the present treatise, gave rise to ... In my experiments the two assistants of the physical cabinet, M. vom Kolke and M. Beer, aided me materially; the drawings from nature were principally made by the latter.

tween the two apices of the poles approximated to 15 millims, in such a manner that they were situated at the distance of two thirds of the height of the flame, the latter was depressed and expanded in the equatorial plane Its form is represented in the equatorial view (taken from the side of one of the poles), figured in Pl IV fig 1 Pl IV fig 1 *a* represents the perpendicular axial section of the flame

When the flame was moved out of the axial line laterally, it was repelled from that line When moved from the equatorial plane and approximated to one of the poles, it was repelled towards this plane

16 *A tallow candle, burning quietly and not smoking*, acted in the same manner Under the same circumstances, the distance of the poles apart being 15 millims, it exhibited the equatorial view figured in Pl IV fig 2 Above, the flame was extended into a sharp wedge with a rectilinear section

17 When the apices of the poles are more approximated, the appearance assumes other forms, in all the experiments which I shall describe hereafter, the constant distance between the apices of the poles, where not expressly stated otherwise, was 3 5 millims

When the above tallow candle was placed between the poles, so that the latter were at seven eighths of the height of the original flame, the equatorial view yielded the third figure Pl IV fig 3 *a* represents the corresponding axial section

When the apices of the poles were at the middle of the height of the original flame, the equatorial view yielded the fourth figure Pl IV fig 4 *a* represents the perpendicular axial section, and Pl IV fig 4 *b* is a view from above, in which the flame has the form of an elliptic ring, surrounded by a small faintly luminous margin, and enclosing a dark space

When the tallow candle was elevated as much as possible, hence when the two apices of the poles were at the same height as the upper end of the wick, and the flame cooled by the iron apex of the pole ceased to burn with perfect light, it not only perfectly re acquired its former luminosity at the moment of closing the circuit, but burned more freely, becoming depressed, and assuming in the equatorial view the form of the fifth figure. A perpendicular axial section is exhibited in Pl IV fig 5 *a*

18 As the tallow candle could not be raised higher, a thin *wax* and a *stearine candle* were taken, and instead of the conical

apices two others, which were terminated below by a triangular plane surface were screwed in When these candles, the flame of which were short were gradually raised higher, the phenomena were exactly the same, until at last, when the apices of the poles were at about the same height as the centre of the wick, the form corresponding to the fifth figure, its apices, becoming constantly more depressed, passed into the form of a small very sharply defined boat the wick representing the mast in the centre, from which the sail, which was less luminous than the boat itself was drawn down, like a tent, towards its edge

19 In the experiments with the tallow candle, we made the express condition that it deposited no soot A *very smoky tallow candle* exhibits totally different phænomena When the two apices of the poles were at three fourths of the height of the original flame the equatorial view, given in fig 6, in which the linear dimensions are reduced to a fourth was obtained on closing the circuit The ascending gray smoke, at an elevation of 7 millims expanded considerably in the equatorial plane It was sharply bounded externally by a parabola the summit of which, O coincided exactly with the middle of the space between the two poles, and retained its regularity for a considerable time and as far as an elevation of 190 millims Internally, the boundary although tending to the parabolic form, was alternately irregular and undulating More strongly luminous clouds of smoke rotated irregularly from time to time in the internal space At an elevation of more than 190 millims the smoke no longer ascended with uniformity, but like a common column of smoke The flame itself which was depressed and expanded in the equatorial plane inclined towards the external boundary of the smoke, at which becoming parabolically hollowed out and forming apices it ascended, being sharply defined, and on the outer side also was bounded by a narrow dark gaseous stripe, which at the summit of the flame passed into the column of smoke

20 Lastly I shall allude to the phænomena presented by a *turpentine flame* A narrow lemniscate shaped wick was spread in the shallow cavity of a porcelain cup, oil of turpentine was then poured on it and it was inflamed It gave a tolerably steady flame, which emanated from the entire surface of the turpentine, and deposited soot abundantly The apices of the poles

extended into the upper part of the flame, the great section of which coincided with the equatorial plane. On closing the circuit, the flame was depressed in the centre 3 millims. to 4 millims. below the height of the apices of the poles; its entire upper boundary had exactly the same form as that in the experiment last described (fig. 7.). The particles of carbon ascending with it united so as to form a black sharply-defined line, which constituted the upper boundary of the flame and ascended as a regular parabola to 180 millims., and from this spot expanding spirally, again followed the parabolic path to about the same height, and then terminated in the form of an indefinite cloud of smoke outside the case.

21. The parabola just described in the case of the turpentine flame, exhibited the same definition when a piece of *German tinder* was placed under the apices of the poles and ignited. The latter was carbonized without the production of flame, and simultaneously narrow dense columns of smoke ascended. When the circuit was closed at the moment at which one of these passed between the apices of the poles, the parabola was immediately formed.

The same also occurred, but with a somewhat less distinct outline, with the smoke arising from *a tallow candle which had been blown out.*

22. A small piece of *sulphur* was placed in a porcelain cup, which was made deep in the centre, and then ignited. After it had become fused into a mass, which was 7 millims. in diameter, it burned quietly, and formed a regular cone of flame of about 6 millims. in height, the upper part of which extended to between the apices of the poles. At the moment of closing the circuit the flame was depressed, and then merely formed a layer of fire lying upon the fused mass of sulphur. During the magnetic excitation the sulphur burnt more quickly and boiled violently.

22 *a.* When, in the experiment described in the last paragraph, a piece of phosphorus was substituted for the sulphur and ignited, the flame, which burned brightly, was in this case also depressed: this phænomenon was accompanied with the parabolic ascent of the bluish flame, which has been so frequently described before, and was here seen in remarkable beauty.

23. A *flame of alcohol,* 25 millims. in height and burning steadily with a violet-coloured flame, on closing the circuit, was

depressed in the same manner as the flame of a stearine candle, and at different elevations assumed the corresponding forms. The combustion was increased, and the original dark violet colour became of a beautiful *yellow*.

24. The *increase of the flame* which has always hitherto been found to occur, evidently depends upon the flame of the source from which it is supplied being brought nearer by the magnet. The alcohol, the stearine and the sulphur are more rapidly consumed on account of the increased heat. To ascertain specially the cause of the production of the yellow colour of the flame by the magnet, which did not occur in the combustion of the sulphur, alcohol *without a wick* was placed under the apices of the poles in a thimble-shaped vessel of copper and ignited, thus replacing the common spirit lamp. The flame was then changed in form as before, but without becoming coloured yellow. Hence the yellow colour in the former experiment appears to have arisen from carbonaceous particles having become detached from the wick, and this again is produced by the flame being pressed down upon the wick by the excitation of the magnetism, and the wick thus partly carbonized.

25. It was still a point of interest to examine the *flame of hydrogen*, which, as is well known, merely consists of aqueous vapour at a red heat. The hydrogen gas was evolved in the ordinary way from pieces of zinc and dilute sulphuric acid, and, on turning a cock, issued from a glass tube drawn out to a fine point, the aperture of which was placed beneath the middle of the space between the apices of the poles, under a perpendicular pressure of water, which at the commencement was 340 millims. in height. After igniting it, the magnetism was excited. At first, from the great force with which the gas escaped, no action upon the flame was perceptible; but as the pressure of the water continued to diminish, the flame at the same time becoming smaller, the effect soon became apparent, and the flame was driven laterally and depressed exactly as in the former analogous cases.

26. When we reflect upon the phænomena which the different flames have exhibited (11–25), all the various forms are explicable on the assumption, that the mass of the flame is repelled by the magnet, and that this repulsion principally takes place from the axial line in all directions. The form of the flame becomes changed in consequence of external influences, in exactly the same manner as in the case of a volume of gas enclosed in a

thin envelope. The original form is determined by the ascent of the gases evolved from the wick. In the first and second figure, the lateral repulsion, by expanding the flame laterally, produces depression of it. If this action increases, the two outermost apices become inclined upwards towards the equatorial plane, just as the original flame does (Pl. IV. figs. 3–5), whence in the first two instances a depression is produced in the centre. But when the poles are situated lower down, the flame, which derives the gas by which it is fed from the wick, cannot leave the wick, an elevation then occurs in the centre, as seen in the fifth figure. The elevated portion is here less luminous, for the same reason that the lower part is so in the case of the flame of the common wax candle.

27. In most cases the flame is depressed, and consequently increased, by the magnet. Both these effects are the results of the repulsion produced by the latter This repulsion however, under altered circumstances, must be capable of producing a diminution, as also an elongation of the flame. The former was accidentally noticed in a small stearine flame, which, when the apices of the poles were depressed considerably below the wick, was in fact extinguished on the excitation of the magnetism; evidently because the gas which fed the flame, before arriving at it, was repelled laterally The flame was moreover diminished by the refrigeration arising from the apices of the poles.

28. When, on the other hand, the flame is completely above the surface of the poles, and the direction in which it is repelled by the magnet is directed perpendicularly upwards, the flame, if our view be correct, should be elongated instead of being shortened. To confirm this by direct experiment, the parallelopipedal halves of the keeper (A) were laid on flat, and retained at a distance of 3·5 millims A common lamp wick was led close above the middle of the upper angles of the two corresponding rectangular surfaces of the poles, and then with both ends between them into a vessel situated beneath, and filled with alcohol. When the wick had become soaked, it was set light to above, so that the flame was not diffused over that portion of the wick which had been conducted to the upper angles. The flame, on closing the circuit, became *higher*.

In this experiment the burning portion of the wick must not be advanced from the centre of the upper angles to their extremities, because the flame is then simultaneously repelled laterally.

29. The experiment described above proves that the different mes which were examined act diamagnetically, and more er that they are all diamagnetic to a greater extent than the rounding air. Granting on the one hand that finely divided rbon at a red heat exists in the flame of tallow, stearine and ix candles, as also in that of turpentine, and on the other nd solid phosphoric acid at a red heat in that of the phos orus, and that the repulsion observed might be attributed to se two bodies, by supposing that they carried off the red t gases with them,—still in the other flames which were amined no finely divided solid bodies exist which could pro ce the repulsion. *Thus the watery vapour of the hydrogen me, the carbonic acid mixed with watery vapour in the alcohol me, and the sulphurous acid in that of burning sulphur, are all re powerfully diamagnetic when at a red heat than air.* This remarkable result surprises us more when we consider w rarified these gases are at a red heat, and we are involuntarily to inquire whether the elevated temperature is not favourable the appearance of diamagnetism, just as on the other hand it minishes magnetism.

30. At the very commencement of my magnetic researches, I leavoured, by means of the magnet, to detect the iron in an holic flame, the wick of which had been rubbed with finely ided protosulphate of iron, and which consequently burned h a beautiful yellow light. I then obtained a negative result, ich in subsequent repetitions I found confirmed by the flame experiencing any less repulsion from the iron in admixture n without it: in fact even isolated particles of iron at a red it which ascended with the flame, were not interrupted in ir motion by the magnet.

31. The phænomena which occurred with flame did not for an tant leave room for the idea that the repulsion observed could e from currents of air. Such currents could only be pro ed by the action of the magnet upon it, not by the flame: it because its action ceases simultaneously with the magnetism might be imagined, as we regard the air as diamagnetic, that en we apply the parallelopipedal halves of the keeper (A) as paragraph 28, the space between the surfaces of the poles of halves of the keeper acted as a chimney, in such a manner t the air at those spots where the diamagnetic action was ongest (the magnetism being strongest at the surfaces of the

poles) would be expelled and constantly replaced by more, which entered at those spots where the action was more feeble. The flame, when brought laterally into proximity with the surfaces of the poles, was repelled; and when I inserted a lamina of mica between the flame and the halves of the keeper, the flame was *less powerfully* repelled. But as the proximity of the lamina of mica exerts a disturbing influence, I shall not at present venture to decide whether the diminution of the repulsion should be ascribed to a current of air or not, although it would have been very interesting to have directly proved the diamagnetism of air in this manner.

32. If air is diamagnetic, which we can hardly doubt, it is repelled by the poles of the magnet, and hence is *rarefied* when in proximity to them. This rarefaction however *cannot* be detected by means of the barometer, because the air to a certain extent simultaneously acquires a greater state of tension. For this purpose I therefore adopted another method, and took a glass cylinder about 90 millims in length and 30 millims. in breadth, which was so depressed in the middle as to allow of the conical apices of the halves of the keeper (B) being placed in the depressions, and which were then only a few millimetres apart. A narrow tube was fused to the cylinder, and the air contained in it confined by placing a drop of alcohol in the tube, so as to allow of its being decided as to the magnetism or diamagnetism of the enclosed air by the motion of the drop on closing the circuit. Although no definite result ensued, I think of repeating the experiment under different circumstances and with greater intensity of force.

§ 2 *On the Magnetic or Diamagnetic Deportment of Liquids.*

33. When masses of fine iron filings are placed upon the approximated poles of a magnet, in consequence of their being attracted by these poles they form heaps, which assume different configurations according to the form of the poles and their distance apart Hence magnetic fluids must be affected in a corresponding manner. Starting from this point of view, I first placed different liquids in a watch-glass above the poles of the electro-magnet, and the phænomena which were observed corresponded perfectly to my expectation. The liquids were more or less strongly attracted towards those points at which the magnetic action is strongest, hence principally towards the angles of

the surfaces of the poles, and in consequence their surface assumes remarkable forms, the perfect explanation of which we find in the analogous appearance from which we started in this paragraph. I shall next describe these phenomena as they occur in a powerfully magnetic liquid, a tolerably concentrated *aqueous solution of perchloride of iron*.

34. I chose the halves of the keeper (C), mentioned in the second paragraph, on account of their greater size, and applied them, with the polished grooves downwards, to the surfaces of the poles of the electro-magnet in such a manner that the rounded ends were turned towards each other, and retained at a definite distance apart. A watch-glass cut from a sphere, the radius of which was 36 millims., was so placed upon the halves of the keeper as to be in contact with them at those points which were nearest each other, and the liquid was then put into it. When the least distance apart of the two halves of the keeper amounted to 2·5 millims., and when the quantity of the liquid was such that its circumference formed a circle of 35 millims. in diameter, on closing the circuit the liquid assumed such a form as, when seen from above, to appear bounded by an almost true geometrical ellipse, the large axis of which was in the equatorial plane and the small axis in the perpendicular meridional plane of the magnet*. The former was 10 millims. and the latter 25 millims. in length. The height of the liquid before and after the closing of the circuit, was measured by the spherometer, and it was thus proved that the magnetic force had raised the fluid in the centre 1·12 millim. In fig. 8 the larger circle represents the original boundary of the liquid viewed from above, and this circle is changed by the magnetic action into the outer ellipse. The liquid is forced into the equatorial plane, forming in it an elevated ridge, the crest of which is constituted by a curve, which in the middle almost slopes off to a straight line, and at the ends, its convexity being altered, rapidly sinks down to the glass. The section in the meridional plane is bounded above by a curve elevated in the middle.

* By this term, in conformity with Faraday's system of notation, I denote all planes passing through the axial (that uniting the poles) right line. When any two symmetrical halves of the keeper are applied to the surfaces of the poles of a horseshoe magnet, there are always two principal planes, one of which passes through the axes of the two arms, the second, which is perpendicular to the first, through the central line of the magnet. Hereafter we shall denominate the former the *meridional* and the second the *equatorial* plane.

35. The magnetic action exerted upon the liquid is more apparent when the quantity of the latter is diminished When its circumference originally formed a circle of 25 millims., this, viewed from above, extended itself, becoming narrower in the axial and broader in the equatorial direction, into a more excentric ellipse, the axes of which were 30·5 millims. and 13 millims. (Pl. IV. fig 8). The axial section of the liquid, in the direction A B, is represented in fig 8 *a*; the equatorial, in the direction C D, in fig. 8 *b*.

The upper angles of the two halves of the keeper upon which the watch-glass stands are indicated in figs. 8 to 12 by the two arcs of larger radii

36. When the shortest distance apart of these two halves of the keeper was increased to 8 millims., the form of the fluid, the amount last used being retained, was very essentially altered. When viewed from above, it formed (Pl. IV. fig 9) an oval figure, which deviated considerably from an ellipse. Its dimensions in the equatorial plane remained the same, but in the meridional plane the fluid had contracted to 14·5 millims. Fig. 9 *a* and fig. 9 *b* represent the sections of the fluid in the directions of A B and C D respectively; both are bounded above by almost straight lines.

37. The quantity of fluid remaining the same, the poles were separated 15 millims. from each other; the form given in fig. 10 then resulted, by the contraction of the original circle in the axial as also in the equatorial direction; in the former the convexity of the circle was diminished, in the latter it was changed in the centre into a concavity. The new boundary curve came into contact with the above circle, by which it was completely enclosed, in those four points through the perpendicular projection of which the upper angles of the two halves of the keeper passed. Viewed from above, two elevated ridges were seen; these coincided with two right lines, the projections of which were in contact with the angles of the halves of the keeper at those points where they were least separated, and in the centre between the two a depression running parallel with them and situated in the equatorial plane. The two sections of the liquid in the directions of A B and C D, in the instance described in the preceding paragraph, were here so changed that the former (Pl. IV. fig. 10 *a*) acquired a depression in the centre, and the latter (fig. 10 *b*)

remaining rectilinear in the centre, formed a curve with its concavity turned towards the watch glass.

38. The two halves of the keeper were next placed at a distance of 23 millims.; the fluid then projected even beyond their upper angles. On closing the circuit the fluid became extended into an elongated oval, the greatest dimension of which coincided with the *axial* direction. In the centre the surface of the fluid was depressed almost to the glass, and became accumulated perpendicularly above the angles of the two halves of the keeper. The view from above is represented in fig. 11; the section in the direction of A B in Pl. IV. fig. 11 *a*. In the equatorial plane the oval was somewhat compressed in the centre; the section in this plane corresponded with the upper surface of the watch glass.

39. Lastly, the poles were placed at a distance of 31 millims. apart, so that the original circle formed by the liquid was entirely contained between the halves of the keeper. On exciting the magnetism, it was converted into a slightly excentric ellipse, the long axis of which was in the meridional plane (Pl. IV. fig. 12). In the equatorial plane the surface of the liquid was slightly depressed.

If, with the greatly diminished magnetic tension consequent upon the separation of the halves of the keeper, a current of greater intensity had been used and the liquid had reached the halves of the keeper, it would of course have assumed a form somewhat approaching that seen in the preceding case. The gradual transition of the form in fig. 8 to that of 12 may be readily followed, and considered as necessarily dependent upon the attractive forces and the cohesion of the liquid.

40. *Protochloride of iron* when dissolved in water was found to be somewhat less strongly magnetic than the perchloride, and protosulphate of iron still less so. When the arrangement was the same as in paragraph 34, a concentrated solution of the latter in the watch glass formed a circle 26 millims. in diameter, which on the excitation of the magnetism was converted into an ellipse, the axes of which were 27.5 millims. and 23.5 millims. in length.

41. A saturated solution of *pernitrate of nickel* was more powerfully magnetic than the solution of the protosulphate of iron; at first it formed a circle 30.5 millims. in diameter, and

after the excitation of the magnetism, an ellipse, the axes of which were 33 millims. and 26 millims. in length.

42. In all the preceding experiments ten of Grove's cells were set in action; but *a single cell* only was sufficient to render the effect perceptible. In the experiments upon flame, described in the earlier paragraphs of this memoir, *two* cells at least were required for this purpose. To observe the effect with less powerfully magnetic liquids, the halves of the keeper were usually placed at a distance of 2 millims. to 4 millims. apart. The reflection of the window in the liquid much facilitated the observation of the magnetic action, especially in those cases where the expansion and contraction of the fluid was only slight and difficult to be perceived. A solution of commercial sulphate of copper afforded an instance of a *slightly* magnetic fluid, probably in consequence of its containing iron, as did likewise a solution of protosulphate of iron in water, 1 part to 50 by weight.

43. It was an important point to subject *diamagnetic* as well as magnetic liquids to the same experiment; the results corresponded to my anticipations. When the two halves of the keeper were 2·5 millims. apart, the section in fig. 8 *a* corresponding to the magnetic fluids became that represented in fig. 13. [In this and the following figure the diamagnetic effect is represented as of greater intensity than really occurs with the above intensity of the current.] The fluid was expanded in the axial, whilst it was contracted in the longitudinal direction. Above the centre between the two halves of the keeper, only a *depression* contracted in the equatorial direction was formed, instead of the former elevated ridge. When the two halves of the keeper were 15 millims. apart, instead of the section of the magnetic liquid, represented in Pl. IV. fig. 10 *a*, a section of the form of fig. 14 was produced. The fluid *expanded* both in the axial and equatorial direction, forming in the equatorial plane an elevated ridge and two depressions parallel with it, the perpendicular projections of which were in contact with the upper angles of the halves of the keeper.

44. It was always found, when the two halves of the keeper were retained at a distance of from 2 to 4 millims. apart on account of the greater magnetic tension, that *no one* of the various fluids which I examined was found to be indifferent. In this way, amongst others, the following bodies were found to be diamagnetic:—water, alcohol, sulphuric ether, sulphuric acid, nitric

acid, hydrochloric acid, solution of ammonia, sulphuret of carbon, fatty and volatile oils, wax in a state of fusion, saturated solutions of nitrate of bismuth, chloride of sodium, nitre, sulphate of soda, and especially ferrocyanide of potassium, milk and blood

45 *Mercury* placed in the watch glass was found to be indifferent to the magnet which at once gave rise to the supposition that this depended upon its slight amount of mobility, and that this again might be explained by the fact, that it did not moisten the surface of the glass But when the mercury was placed in a small brass cup which had been recently amalgamated, it exhibited its diamagnetic reaction distinctly

46 The diamagnetic reaction of ferrocyanide of potassium which Dr Faraday also observed on allowing the crystals of this salt to oscillate, was least to be anticipated A saturated solution of the ferrocyanide in water was more powerfully diamagnetic than pure water Hence it was at least to be anticipated, that together with the large amount of iron it contained, unusually powerful diamagnetic matters must be present in the double salt, and that the same would be found to exist in the cyanide of potassium, which, when combined with cyanide of iron, would overpower the magnetism of the latter and have rendered it decidedly diamagnetic But cyanide of potassium, when dissolved in water, *did not appear to render it more diamagnetic*

47 The preceding method of observation appeared to me worthy of further application for two reasons —on the one hand, for enabling us to detect the presence of even *the slightest trace of magnetism or diamagnetism* in any liquid, and on the other, for *measuring* the intensity of both With regard to the first point, it might be expected that the action of the magnet upon the liquid would increase considerably, when the latter, instead of being put into a watch glass, was placed upon a thin lamina of mica lying upon the two halves of the keeper (C), a short distance only apart A strongly diamagnetic liquid under these circumstances formed two double elevated ridges, several millimetres in height, which following exactly the two semicircular edges, ascended highest at that part where the distance between the two halves of the keeper was least

The action of the magnet was strongest when the same two halves of the keeper were applied in such a manner that as before they were about 3 millims apart, with their planed

grooves uppermost, and the latter, one of which formed the continuation of the other, were covered with a thin lamina of mica at the spot where they are most approximated, forming a bridge from one half of the keeper to the other. When water, which is by no means one of the most strongly diamagnetic liquids, was then placed upon the lamina of mica, and walled in on both sides at some distance from the centre by wax, on looking between the two keepers, the water existing there was seen depressed from 5 to 6 millims. towards the lamina of mica, so as to enter the two grooves on opposite sides. Magnetic liquids, on the contrary, moved towards the centre, and there became raised up upon the lamina of mica

48. On the other hand, the two parallelopipedal halves of the keeper (A) were laid flat upon the magnet, and placed at a distance of 8 millims. apart, in which position they were fixed. A parallelopipedal box, made of thin sheet brass open above, the longest sides of which were of about the same dimensions as the surfaces of the poles, was placed between the latter, and being about 7 millims. wide, a small space was left for its play. A long glass tube of about 1·5 millim. internal diameter, which slowly ascended in the equatorial plane, was cemented water-tight into the under part of one of the two narrow sides. When a diamagnetic fluid was then placed in the box in such a quantity that about a third of it was filled and at the same time the liquid in the tube rose to about its middle, on closing the circuit of a battery of from six to eight Grove's cells, the fluid column, in the case of water, solution of ferrocyanide of potassium, and alcohol, rose from 1 to 3 millims The reverse occurred on using a magnetic fluid; a saturated aqueous solution of protosulphate of iron receded in the glass tube more than 80 millims.

I shall reserve for a future communication the investigation of the question, how far the idea of an accurate comparative method of determining the intensity of the magnetism and diamagnetism of liquids may be derived from the preceding experiments, at present confining myself to the phænomenon alone.

49. In the consideration of the motion of the liquids, the idea of observing also *the motion of the finely divided solid matters mixed with them* was of importance, and especially occurred in the examination of the blood. Faraday has already ascertained

that this liquid is diamagnetic, which does not surprise us, considering that even the ferrocyanide of potassium proved to be so. I found this equally confirmed whether the blood of a recently killed frog, human blood or the beaten blood of an ox was put into the watch glass (14), and also when the corpuscles were separated by filtration, dried, and suspended in the form of a solid mass between the apices of the poles by means of a silk worm thread. Another point of interest was the observation of the corpuscles of the blood separately, as swimming in the serum.

50. To render the *microscope* applicable, which for this purpose was essential, I at first used the halves of the keeper (B), and instead of the conical points, screwed on two others, the lower half of which was filed off into them so that, even when the points were approximated as much as possible, the glass upon which the object was placed could be brought into almost absolute contact with them, whence both the points and the object could be simultaneously brought into the field of the microscope.

In most cases however, on account of the greater magnetic tension and more ready adjustment, I preferred applying the two parallelopipedal halves of the keeper (A) flat, and fixing them at a distance of 3.5 millims. apart, then placing a lamina of mica or thin glass above two of the upper and opposite angles of the two halves of the keeper, and so adjusting the microscope that the mirror reflected the light to the object through the space between the surfaces of the poles.

51. In whatever manner the observation was made, indifferently whether the blood of the frog or of any other animal was placed upon the glass or plate of mica, or whether it was used in its pure state or mixed with water, a repulsion of the whole mass of the liquid was always observed, as also a distinct repulsion of the corpuscles of the blood themselves. Hence the latter, in which chemical analysis has proved the iron present to be contained, appear more powerfully diamagnetic than the serum in which they are originally suspended, and also more so than the water in which they were placed.

52. Milk, with its minute fatty globules, exhibited the same reaction under the microscope as the blood.

53. For the purpose of ascertaining by a direct experiment whether very minute corpuscles contained in a liquid really acquire independent movements by the action of a magnet, I placed

some globules of starch from a recently-cut potato, in a little water on mica upon one of the angles of the parallelopipedal halves of the keeper; simultaneously with the water, they were repelled and driven outwards between the angles of the poles. But when placed in a dilute solution of the protosulphate of iron, instead of water, they were at first forcibly attracted towards the middle of the upper angles simultaneously with the liquid; but when the liquid had acquired a state of rest, they resumed their independent motion, and were repelled outwards by the poles.

* * * * *

54. The first paragraph in the preceding memoir had already been despatched, before I had obtained the slightest knowledge of Faraday's last paper upon the diamagnetism of gases. The mere communication of the statement that No. 732 of the Journal *l'Institut* contained a report of this paper induced me to hasten in despatching this first paragraph, which, although it had long been completed, had been intentionally kept back, to await the result of two experiments, one of which merely consists in a repetition of one of the experiments discussed in the preceding paragraphs; and the other, which had been designed at the very commencement of my experiments upon the diamagnetism of gases, was obliged to be postponed for want of sunshine to the time when I should be able to institute my experiments. The last experiment certainly now loses the interest of novelty in consequence of Faraday's communication; however I shall detail it in these supplemental investigations, although still in an imperfect state, because it serves to complete the series of experiments which I have described in the earlier paragraphs.

55. To begin with the first of the two experiments mentioned in the preceding paragraph, it is a matter of daily experience that when the sun shines upon a hot stove, the ascending air, and upon an adjacent wall its shadow, are very distinctly seen. This then was a means of rendering ascending heated air visible, and thus of deciding whether heated air is more diamagnetic than cold air, which, judging from the experiment upon flame, I considered as probable (14), although I could not form any opinion as to the demonstrability of the point by experiment. I therefore placed a spiral of thin platinum wire, somewhat curved toward the top, and with its long axis equatorial beneath the approximated apices of the poles of the electro-magnet. When

the current of six elements was transmitted through it, the spiral became white hot, a heated current of air quietly ascended between the apices of the poles under the case of the torsion balance, and threw a well defined shadow upon a sheet of white paper which was held behind it. Unfortunately the sun completely disappeared just at the moment when the magnetism was about to be excited by closing the circuit. But as Faraday has made the corresponding experiment by showing the equatorial deflection of an ascending current of heated air, by thermometers placed above and at the sides, there is no longer any doubt that even in my arrangement the shadow which directly ascended would have separated into two parabolic branches.

56. Every one, who has but once seen the smoke from a flame of turpentine or the vapour of iodine ascending between the apices of the poles, must also be convinced how by Faraday's beautiful process,—in which he placed vessels above (or beneath), and on the side of the gaseous current whilst directly ascending, to receive it and subsequently found that those gases, which are more diamagnetic than the air, did not enter the upper (or under) vessel, but that placed at the side,—we may obtain general results. But when the examination of the various gases becomes the question, my series of experiments, in which I confined myself to the action of the magnet upon *visible* gases,—the experiment of causing colourless gases to ascend in coloured gases, and thus rendering them visible, was neither carried out by myself nor was its practicability at all tested,—become inferior to those of Faraday.

57. However I shall finally again direct attention to the experiment described in the earlier paragraphs, viz. that *of directly proving the diamagnetism of the air by its rarefaction in exactly the same manner as its expansion by heat is shown by an ordinary air thermometer.* Although I was then compelled to report the failure of the experiment, which was twice performed in an unsatisfactory manner, still I never lost sight of it, nor doubted for an instant of its ultimate success. I am now able to assert that it has succeeded most beautifully.

I applied the two halves of the keeper (C), with their semicircular ends turned towards each other and fixed at the very short distance of 5 millims. apart, upon the poles of the horseshoe electro-magnet, and had a vessel made of thin sheet brass, which fitted as accurately as possible between the two keepers, but

leaving a small space for its play. The vessel was 41 millims. in height and 93 millims. in length in the equatorial direction, in the axial direction, 4·5 millims in breadth in the middle, and 40 millims. in breadth at the two ends; it was everywhere closed air-tight, except that a glass tube, 1 millim. in diameter, which was placed horizontally in the equatorial plane, was let into the middle of one of the two almost square walls. After the temperature was found to remain constant at 53° F., the vessel was slightly warmed by the contact of the hand; and then, to confine the air inside, a drop of alcohol was placed at the orifice of the tube. On withdrawing the hand, the drop ran into the tube and settled near the vessel. The magnetism was then excited by closing a circuit of twelve Grove's elements, and at the same instant the drop of alcohol was driven 3 millims. nearer to the orifice of the glass tube; and after some time, when it had become stationary, the circuit was again opened; it then returned to exactly the original position, the latter movement being more rapid than the former. *Thus, in consequence of the diamagnetic repulsion, the air became expanded by the electro-magnet, and on the disappearance of the magnetism it returned to its original volume.*

58. The vessel described in the preceding paragraph can be opened and closed in the middle of the other lateral wall, that which is opposite to the one in which the glass tube is let in, and thus may be filled with *any kind of gas*. Hence these can be tested as to their diamagnetism in the same manner as the air.

Lastly, on the same principle, we can determine the influence of *heat* upon the diamagnetism of gases; and we are not only able to observe the diamagnetism of any gases at a given temperature, *but we can also measure it.*

59. Since, according to the other methods of determination, any gas which is not enclosed can only be experimented upon in the air or some other gas by this means, independently of admeasurements being then out of the question, we can only arrive at *relative determinations.* We can, in that case, only suppose that air, with all other gases, is diamagnetic. The phænomena would remain the same in all experiments of the kind, if the air and all gases, instead of being diamagnetic, were magnetic, supposing however that those which in reality are most powerfully diamagnetic were least magnetic.

60. The results obtained in par. 57 appeared to me remarkable in many respects, and especially because they directly prove

the existence of *an analogy between heat and the power of the magnet: both expand air and gaseous bodies.* It has been shown, that around a magnet, even the air which is not enclosed becomes rarefied by it; and this rarefaction must necessarily increase with the approximation of the poles. Hence Mariotte's law, *strictly considered*, ceases to be correct as ordinarily admitted. The barometer or manometer does not indicate the density of the air when we take into consideration the temperature only without regard to its diamagnetic condition.

Bonn, 22nd and 31st of January, 1848.

ARTICLE XVII.

On a simple Method of increasing the Diamagnetism of Oscillating Bodies: Diamagnetic Polarity. By Prof. PLÜCKER.

[From Poggendorff's *Annalen* for March 1848.]

1. FARADAY'S fundamental experiment may even be repeated with a feeble steel magnet, and it can thus be shown that a bar of bismuth, when suspended by means of a silkworm thread between its two poles, is repelled by them, and settles in the equatorial position. But the diamagnetic repulsion of bismuth is very slight in comparison with the magnetic attraction of iron; hence every means of increasing the diamagnetic force is desirable. A means of effecting this is obtained by merely placing close beneath the oscillating bar of bismuth a bar of iron, arranged equatorially in the middle of the space between the two poles of the horseshoe iron magnet, and fixing it in this position. It is at once evident that the bar of bismuth then tends to assume the equatorial position with greater force; and it is easy, by the oscillating method, to determine the proportion in which the diamagnetic directive force has become increased by the application of the iron bar.

2. Among other experiments I made the following:—I placed the two halves of the keeper C (see par. 2 of the preceding memoir) in such a manner upon the large electro-magnet, with their grooves downwards, that their rectangular terminal surfaces were turned towards each other and formed the surfaces of the poles. They were 40 millims. in height, 59 millims. broad in the equatorial direction, and were kept at a constant distance of 16·5 millims. by placing between them two pieces of brass, each of which was 6 millims. in thickness, and in the middle enclosed an iron bar 4·5 millims. in thickness and as long as the breadth of the surfaces of the poles. A small cylinder of bismuth, 2 millims thick and 25 millims. long, when suspended by a silkworm thread so as to oscillate between the surfaces of the poles at a level somewhat below that of their upper angles and close

above the enclosed iron bar on exciting the magnetism by four feebly charged Grove's cells, assumed a very decided equatorial position, and when moved from the equatorial position made 90 oscillations during the space of half a minute. After opening the circuit, the pieces of brass and the iron bar were removed without disturbing the two halves of the keeper. The bar of bismuth then also assumed a decided equatorial position, making, however only 36 oscillations during the space of half a minute. Hence the directive force of the bar of bismuth had increased by the addition of the equatorial bar of iron in the proportion of 36^2 to 90^2, or of

1 to 6.25,

thus more than *sixfold*.

3. When a hardened bar of steel a little thicker was used instead of the bar of soft iron, the corresponding numbers of the oscillations were 78 and 34. Hence the directive force had become increased by the steel bar in the proportion of

1 to 5.26.

4. I shall limit myself here to the communication of a single observation, and merely add a few words upon what gave rise to it.

Faraday's explanation of diamagnetic phenomena is so evident, that it would have occurred to every philosopher. In my lectures last summer I expressed it in the following words:—"In bismuth every north pole of a magnet induces a north pole, each south pole a south pole." Diamagnetic polarity is a necessary consequence of this explanation. I then tried in vain to detect this polarity. Among other things, I caused a small bar of bismuth to oscillate between the ends of the poles of the cylinders, inserting however in the two perforated appendages of the poles, instead of the cylinder of iron pointed in front, a similar one made of bismuth, but on superficial examination I did not observe that any change was produced by the cylinder of bismuth. The results obtained by Reich and Weber induced me again to take up my former experiments, and this was the origin of the observation described above. It appears to confirm (at least it was instituted with a view to this point) what the experiments of Poggendorff have already directly proved, viz. that a bar of bismuth in the equatorial position is a real transversal magnet, which turns the line of its north pole to the north

pole and the line of its south pole to the south pole of the magnet. When this was communicated to me in a friendly manner, I immediately repeated the experiment with a single pole of the magnet only, and showed the polarity of the bar of bismuth alluded to in Poggendorff's *Annalen* for March, p. 475, and this not only on the side turned towards the exciting pole, but also on that turned from it, by the attraction and repulsion in the ordinary manner. Except that I principally used for this purpose a soft iron bar, and that pole of the magnet which acted diamagnetically upon the bismuth; I also gave it, according to the position in which it was kept, the polarity required in each instance.

5. Although the above experiments show indisputably that diamagnetism consists in a polar excitation, still there are great difficulties to be overcome before this theory can be considered as generally established. These long appeared to me so great, that I completely laid them aside, and did not again resume them until the polarity had been so decidedly proved. In fact, I think, after a superficial mathematical consideration, that the repulsion of the optic axes of crystals by the magnet, which I was the first to observe, and the preponderance of this repulsion, when the poles are a great distance apart, over the magnetic or diamagnetic action upon their substance, might be very easily explained by a supplemental assumption; but the facts described in my *second* memoir appear to me inexplicable even at the present time. When a piece of wood-charcoal (merely in consequence of its form, and as the longitudinal direction may be taken as either in or against the direction of the fibres, quite independently of its structure) in close proximity to the poles assumes an equatorial position and at a great distance an axial position, this is expressed in the language of theory as follows:—The poles of the magnet excite in the charcoal Ampère's molecular currents, which, when the poles are approximated, perhaps run from east to west; when the poles are separated, run from west to east.

6. According to the experiment last detailed, it cannot be doubted that *as the distance of the two poles apart is increased* the molecular currents in the charcoal (at least the resulting ones) *reverse their direction*, because they first produce diamagnetism and then magnetism: we are therefore compelled to admit that the same pole of the magnet, according to the dif-

ferent distance excites* diamagnetic or magnetic molecular currents in the same mass or, what I consider more probable that in phænomena of this kind magnetic and diamagnetic substances are always mixed, in which the different molecular currents simultaneously exist, and that then, as the distance between the poles increases the diamagnetic currents diminish in intensity more rapidly than the magnetic The following question appears important as regards theory —*Does the different distance of the poles of the electro magnet come under consideration directly as such, or indirectly only inasmuch as a diminution of the force corresponds to a greater distance?* In the first of these two alternatives, I should unconditionally lay aside the theory, for if magnetic and diamagnetic currents *when the magnetic force is the same diminish with the distance according to a different law,* I could not possibly regard the two as *identical* in their nature I can however far more easily imagine, that when two forces produce opposite rotations these rotations, when a different resistance is to be overcome are not proportional at the same distance to the forces which produce them If, *e g*, by way of comparison, we take two wheels, which two forces A and B strive to rotate in opposite directions and premise that the resistance to the first direction of rotation is much greater than that to the second, the rotation of the first wheel may then be slower than that of the second, although the force B is considerably less than the force A, which however ceases to be the case when, on the increase of the forces, the total resistance to the magnitude of the force A ceases to be appreciable In this point of view the force A would correspond to the diamagnetic, and the force B to the magnetic forces

7 For the purpose of determining which of these two alternatives occurs in nature, I made the following decisive experiment —

I took a small piece of box wood charcoal, which when at a considerable distance from the poles of the electro magnet acted magnetically whilst at a less distance it was diamagnetic I excited the current by means of *a single* Grove's cell and placed the poles at such a distance from each other that the pieces of charcoal assumed a decidedly axial position I next used *two*

* The magnetic currents must be considered as pre existing as in iron and merely turned by the magnet into the same direction as its own —POGGENDORFF

elements; it was then difficult to determine whether it acted magnetically or diamagnetically. With *three* elements however it assumed a decidedly equatorial position, and the diamagnetic force which brought it decidedly into this position continued to increase on using a larger number of elements. I went on with this experiment until I had used six elements.

A second experiment with another piece of charcoal gave the same result, except that with *three* elements it was still very slightly magnetic, but with *four* elements it acted decidedly diamagnetically.

Hence, the distance remaining the same, augmentation of the force of the poles of the magnet converts the magnetism of wood-charcoal into diamagnetism.

This result, which I had not before anticipated, but which was to be expected in a purely theoretical point of view, appears to me to form a remarkable confirmation of the theory of diamagnetism adopted by Faraday, Reich, Weber and Poggendorff, in which I now entirely coincide.

Bonn, February 21, 1848

Article XVIII

Experimental and Theoretical Researches on the Figures of Equilibrium of a Liquid Mass withdrawn from the Action of Gravity By J Plateau, *Professor at the University of Ghent, Member of the Royal Academy of Belgium, &c*

Second Series*

Preface

At the period when attacked by the disease which has entirely deprived me of sight I had terminated the greater part of the experiments relating to this series, as well as the following M Duprez, correspondent of the Brussels Academy, and M Donny had the kindness to undertake those which were still wanting I constantly directed their execution, nearly all were made in my presence, and I followed all the details I have therefore considered myself justified, in order to simplify the description in expressing myself in the course of this investigation as if I had made the experiments

With respect to the theoretical portions, I am indebted to the able assistance of one of my colleagues, M Lamarle, who has most kindly devoted many long hours to listening to the details of my investigations, and to aiding me in the explanation of several difficult points I am also indebted to another of my colleagues, M Manderlier, for the execution of a part of the calculations

May I be permitted to express in this place my gratitude to

* The memoir published in vol xvi of the Memoirs of the Royal Academy of Belgium (and translated in vol iv of the Scientific Memoirs) under the title of Memoir on the Phænomena of a free Liquid Mass withdrawn from the Action of Gravity First Part constitutes the first series of these researches A different title has been adopted for the other series because the preceding was not applicable to the entire work

these devoted friends? Thanks to their generous help, science is still an open field for me: notwithstanding the infirmity with which I am afflicted, I am able to put in order the materials I have collected, and even to undertake fresh researches.

Preliminary Considerations and Theoretical Principles. General Condition to be satisfied by the free Surface of a Liquid Mass withdrawn from the Action of Gravity, and in a state of equilibrium. Liquid Sphere.

1. THE process described in the previous memoir enabled us to destroy the action of gravity upon a liquid mass of considerable volume, leaving the mass completely at liberty to assume the figure assigned to it by the other forces to which it is subject. This process consists essentially in introducing a mass of oil into a mixture of water and alcohol, the density of which is exactly equal to that of the oil employed. The mass then remains suspended in the surrounding liquid, and behaves as if withdrawn from gravity. By this means we have studied a series of phænomena of configuration, dependent either simply upon the proper molecular attraction of the mass, or upon the combination of this force with the centrifugal force. We shall now abandon the latter force, and introduce another of a different kind, the molecular attraction exerted between liquids and solids: in other words, we shall cause the liquid mass to adhere to solid systems, and study the various forms assumed under these circumstances by those portions of the surface which remain free In this way we shall have the curious spectacle presented by the figures of equilibrium appertaining to a liquid mass, absolutely devoid of gravity and adherent to a given solid system.

But the figures which we shall obtain present another kind of interest. The free portions of their surface belong, as we shall show, to more extended figures, which may be conceived by the imagination, and which, in the same condition of total absence

of gravity, would belong to a perfectly free liquid mass; thus our processes will partially realize the figures of equilibrium of a mass of this kind. The latter are far from being confined to the sphere; but among them the sphere alone is capable of being completely formed, the others presenting either infinite dimensions in certain directions, or other peculiarities which we shall point out, and which equally render their realization in the complete state impossible.

Moreover, the results at which we shall arrive will constitute so many new and unexpected confirmations of the theory of the pressures exerted by liquids upon themselves in virtue of the mutual attraction of their molecules, a theory, upon which the explanation of the phænomena of capillarity is based.

Lastly, in our liquid figures we shall discover remarkable properties, which will lead us to some important applications.

2. In order to guide us in our experiments, and also to enable us to comprehend their bearing, we shall first consider the question in a purely theoretic point of view. The action of gravity being eliminated and the liquid mass being at rest, the only forces upon which the figure of equilibrium will depend will be the molecular attraction of the liquid for itself, and that exerted between the liquid and the solid system to which we cause it to adhere. The action of the latter force ceases at an excessively minute distance from the solid; hence in regard to any point of the surface of the liquid situated at a sensible distance from the solid, we have only to consider the first of the two above forces, *i. e.* the molecular attraction of the liquid for itself.

The general effect of the adhesive force exerted between the liquid and the solid, is to oblige the surface of the former to pass certain lines; for instance, if a liquid mass of suitable volume be caused to adhere to an elliptic plate, the surface of the mass will pass the elliptic outline of the plate. At every point of this surface, situated at a sensible distance from this margin, the molecular attraction of the liquid for itself alone is in action.

Let us now examine into the fundamental condition which all points of the free surface of the mass must satisfy, in virtue of the latter force.

The determination of this condition and its analytical expression, are comprised in the beautiful theories upon which the explanation of the phænomena of capillarity are based, although

geometricians have not specially studied the problem of the figure of a liquid mass void of gravity adherent to a given solid system We shall therefore now resume the principles and the results of the theories in question, at least those which relate directly to our subject

3 Within the interior of a liquid mass, at any notable distance from its surface, each molecule is equally attracted in every direction, but this is not the case at or very near the surface In fact, let us consider a molecule situated at a distance from the surface less than the radius of the sphere of sensible activity of the molecular attraction, and let us imagine this molecule to be the centre of a small sphere having this same radius It is evident that one portion of this sphere being outside the liquid, the central molecule is no longer equally attracted in every direction, and that a preponderating attraction is directed towards the interior of the mass If we now imagine a rectilinear canal, the diameter of which is very minute, to exist in the liquid, commencing at some point of the surface in a direction perpendicular to the latter, and extending to a depth equal to the above radius of activity, the molecules contained in this minute canal, in accordance with what we have stated, will be attracted towards the interior of the mass, and the sum of all these actions will constitute a pressure in the same direction Now the intensity of this pressure depends upon the curves of the surface at that point at which the minute canal commences In fact, let us first suppose the surface to be concave, and let us pass a tangent plane through the point in question All the molecules situated externally to this plane, and which are sufficiently near the minute canal for the latter to penetrate within their sphere of activity, will evidently attract the line of molecules which it contains from the interior towards the exterior of the mass If therefore we suppressed that portion of the liquid situated externally to the plane, the pressure excited by the line would be augmented Hence it follows that the pressure corresponding to a concave surface is less than that which corresponds to a plane surface, and we may conceive that it will be less in proportion as the concavity is more marked,

If the surface is convex, the pressure is, on the contrary, greater than when the surface is plane To render this evident, let us again draw a tangent plane at that point at which the line of molecules commences, and let us imagine for a moment

that the space included between the convex surface and this plane is filled with liquid. Let us then consider a molecule, m, of this space sufficiently near, and from this point let fall a perpendicular upon the minute canal. The action of the molecule m upon the portion of the line comprised between the base of the perpendicular and the surface, will attract this portion towards the interior of the mass. If afterwards we take a portion of the line equal to the former from the other side of the perpendicular, and commencing at the base of the latter, the action of the molecule m upon this second portion will be equal and opposite to that which it exerted upon the first, so that these two portions conjointly would neither be attracted towards the interior nor the exterior of the mass; if beyond these two same portions another part of the line is comprised within the sphere of activity of m, this part will evidently be attracted towards the exterior. The definitive action of m upon the line will then be in the latter direction. Hence it follows that all the molecules of the space comprised between the surface and the tangent plane which are sufficiently near the line to exert an effective action upon it, will attract it towards the exterior of the mass. If then we suppress this portion of the liquid so as to reproduce the convex surface, the result will be an augmentation of the pressure on the part of the line. Thus the pressure corresponding to a convex surface is greater than that corresponding to a plane surface, and its amount will evidently be greater in proportion as the convexity is more marked.

4. If the surface has a spherical curvature, it may be demonstrated that, representing the pressure corresponding to a plane surface by P, the radius of the sphere to which the surface belongs by r, and by A a constant, the pressure exerted by a line of molecules, and reduced to unity of the surface, will have the following value,

$$P+\frac{A}{r} \qquad (1.)$$

r being positive in the case of a convex, and negative in that of a concave surface.

Whatever be the form of the surface, let us imagine two spheres, the radii of which are those of greatest and least curvature at the point under consideration. It is evident that the pressure exerted by the line will be intermediate between those corresponding to these two spheres, and calculation shows

that it is exactly their mean. Denoting the two radii in question by R and R′, the pressure exerted by the line, referred to the unity of surface, would be

$$P+\frac{A}{2}\left(\frac{1}{R}+\frac{1}{R'}\right) \qquad (2.)$$

The radii R and R′ are positive when they belong to convex curves, or, in other terms, when they are directed to the interior of the mass, whilst they are negative when they belong to concave curves, *i.e.* when they are directed towards the exterior.

5. From the preceding details we can now easily deduce the condition of equilibrium relative to the free surface of the mass.

The pressures exerted by the lines of molecules which commence at the different points of the surface are transmitted to the whole mass; consequently, for the existence of equilibrium in the latter, all the pressures must be equal to each other. In fact, let us imagine a minute canal running perpendicularly from some point of the surface, and subsequently becoming recurved so as to terminate perpendicularly at a second point of this same surface; it is evident that equilibrium can only exist in this minute canal when the pressures exerted by the lines which occupy its two extremities are equal; and if this equality exists, equilibrium will necessarily exist also. Now the pressures exerted by the different lines depend upon the curves of the surface at the point at which they commence; these curves must therefore be such, at the various points of the free surface of the mass, as to determine everywhere the same pressure.

Such is the condition which it was our object to arrive at, and to which in each case the free surface of the mass must be subject.

The analytical expression of this condition is directly deducible from the general value of the pressure given in the preceding paragraph; we only require to equalize this value to a constant, and as the quantities P and A are themselves constant, it is in fact sufficient to make

$$\frac{1}{R}+\frac{1}{R'}=C, \qquad (3.)$$

the quantity C being constant for the same figure of equilibrium.

This equation is the same as those which are given by geometricians for capillary surfaces, when, in the latter equations, the quantity representing gravity is supposed to be 0.

R and R′ may be replaced by their analytical values; we are

thus led to a complicated differential equation, which only appears susceptible of integration in particular cases. Yet the equation (3.) will be useful to us in the above simple form. Now we know that the normal plane sections which correspond to the greatest and the least curvature at the same point of any surface form a right angle with each other. Geometricians have shown, moreover, that if any two other rectangular planes be made to pass through the same normal, the radii of curvature, ρ and ρ' corresponding to the two sections thus determined, will be such that the quantity $\frac{1}{\rho}+\frac{1}{\rho'}$ will be equal to the quantity $\frac{1}{R}+\frac{1}{R'}$. Hence the first of these two quantities may be substituted for the second, and consequently, the equation of equilibrium, in its most general expression, will be

$$\frac{1}{\rho}+\frac{1}{\rho'}=C, \qquad (4.)$$

in which equation ρ and ρ' denote the radii of curvature of any two rectangular sections passing through the same normal.

6. These geometric properties lead to another signification of the equation (4.). We know that unity divided by the radius of curvature corresponding to any point of a curve is the measure of the curvature at this point. The quantity $\frac{1}{\rho}+\frac{1}{\rho'}$ represents then the sum of the curvatures of two normal rectangular sections at the point of the surface under consideration. This being admitted, if we imagine that the system of the two planes occupies successively different positions in turning round the same normal, a sum of curvatures $\frac{1}{\rho}+\frac{1}{\rho'}$, $\frac{1}{\rho''}+\frac{1}{\rho'''}$, $\frac{1}{\rho^{\text{iv}}}+\frac{1}{\rho^{\text{v}}}$, &c. will correspond to each of these positions, and, according to the property noticed in the preceding paragraph, all these sums will have the same value. Consequently, if we add them together, and let n denote the number of positions of the system of the two planes, the total sum will be equal to n times the value of one of the partial sums, or to $n\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)$. Now this total sum is that of all the curvatures $\frac{1}{\rho}$, $\frac{1}{\rho'}$, $\frac{1}{\rho''}$, $\frac{1}{\rho'''}$, &c. in number $2n$, corresponding to all the sections determined by the two

planes If then we divide the above equivalent quantity by $2n$, the result $\frac{1}{2}\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)$ will represent the mean of all these curvatures Now as this result is independent of the value of n, or of the number of positions occupied by the system of the two planes, it will be equally true if we suppose this number to be infinitely great, or, in other words, if the successive positions of the system of the two planes are infinitely approximated, and consequently if this same system turns around the normal in such a manner as to determine all the curvatures which belong to the surface around the point in question The quantity $\frac{1}{2}\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)$ represents then the mean of all the curvatures of the surface at the same point, or the mean curvature at this point Now if, in passing from one point of the surface to another, the quantity $\frac{1}{\rho}+\frac{1}{\rho'}$ retains the same value, *i e* if for the whole surface we have $\frac{1}{\rho}+\frac{1}{\rho'}=C$, this surface is such that its mean curvature is constant

Considered in this purely mathematical point of view, the equation (1) has formed the object of the researches of several geometricians, and we shall profit by these researches in the subsequent parts of this memoir

Thus our liquid surfaces should satisfy this condition, that the mean curve must be the same everywhere We can understand that if this occurs, the mean effect of the curvatures at each point upon the pressure corresponding to this point also remains the same, and that this gives rise to equilibrium Hence we now see more clearly the nature of the surfaces we shall have to consider, and why they constitute surfaces of equilibrium

64 We must now call attention to an immediate consequence of the theoretical principles which have led us to the general condition of equilibrium According to these principles, each of the lines of molecules exerting upon the mass the pressures upon which its form depends, commences at the surface and terminates at a depth equal to the radius of the sensible activity of the molecular attraction, so that these lines collectively constitute a superficial layer the thickness of which is equal to the radius itself, and we know that this is of extreme minuteness It results from this that the formative forces exerted by the

liquid upon itself emanate solely from an excessively thin superficial layer. We shall denominate this consequence *the principle of the superficial layer*.

7. A spherical surface evidently satisfies the condition of equilibrium, because all the curvatures in it are the same at each point: also when our mass is perfectly free, *i. e.* when it is not adherent to any solid which obliges its surface to assume some other curve, it in fact takes the form of the sphere, as shown in the preceding memoir.

8. Before proceeding further we ought to elucidate one point of great importance in regard to the experimental part of our investigations. The liquid mass in our experiments being immersed in another liquid, the question may be asked whether the molecular actions exerted by the latter exert no influence upon the figure produced: or in other words, whether the figure of equilibrium of a liquid mass adherent to a solid system, and withdrawn from the action of gravity by its immersion in another liquid of the same density as itself, is exactly the same as if the mass adherent to the solid system were really deprived of gravity and were placed *in vacuo*. Now we shall show that this really is the case. The molecular actions resulting from the presence of the surrounding liquid are of two kinds, viz. those resulting from the attraction of this liquid for itself, and those resulting from the mutual attraction of the two liquids. Let us first consider the former, imagining for an instant that the others do not exist. The surrounding liquid being applied to the free surface of the immersed mass, the former presents *in intaglio* the same figure as the latter mass presents in relief. Those molecules of this same liquid which are near the common surface of the two media must then exert pressures of the same nature as those which we have considered throughout the preceding details, towards the interior of the liquid to which they belong, and these pressures must consequently also impart a figure of equilibrium to the surface *in intaglio*; so that if the immersed mass of itself had no tendency to assume any one figure rather than another, the surrounding liquid would give it a determinate one, by compelling it to mould itself in the above hollow figure. This is why a bubble of air in a liquid assumes the globular form, solely in consequence of the pressures exerted by the liquid upon it. Now let us suppose that the immersed mass has assumed that figure which it would acquire *in vacuo* if really deprived of

gravity; the analytical condition of paragraph 5 would then be satisfied as regards this mass. Now at each point of the common surface of the two media, the radii of curvature ρ and ρ' have the same absolute values, both in the case of the immersed mass and of the hollow figure of the surrounding liquid, except that their signs are contrary, according as they are considered as referring to one or the other of the two liquids. To pass from one of the two figures to the other, we need therefore only change the signs ρ and ρ', or, what comes to the same thing, change the sign of the constant C. Changing the sign does not destroy the condition of equilibrium; and consequently, if the immersed mass is in equilibrium as regards its own molecular attractions, the same will hold good in the case of the hollow figure of the surrounding liquid. The pressures of the latter liquid cannot therefore by themselves produce any modification in the figure of equilibrium of the immersed mass.

Let us now introduce the second kind of molecular actions, *i. e.* the mutual attraction of the two liquids, and see what will be its effects. Let us imagine, for an instant, that the immersed mass, or, for the sake of fixing the ideas, the mass of oil in our experiments is replaced by the same kind of liquid as that which surrounds it, *i. e.* by the alcoholic mixture. In other words, supposing the vessel to contain only the alcoholic mixture and the solid system, let us limit, in the imagination, a portion within the liquid, of the same figure and dimensions, and situated in the same manner as the preceding mass of oil. It is then clear that the molecules of the mass near its surface being, like those of the interior, completely surrounded by the same kind of liquid beyond their sphere of activity, these molecules will no longer exert any pressure upon the mass; consequently the pressures which would exist if this mass could be isolated, must be considered as destroyed by the attractions emanating from the surrounding liquid. The latter forces are therefore all equal and opposite to the pressures in question. Now as these are all equal to each other in accordance with the figure which we have attributed to the imaginary surface of the mass, the attractions emanating from the surrounding liquid will also all be equal to each other. If we now replace the mass of oil, the attractions emanating from the surrounding liquid may certainly alter in absolute value, but it is evident that they will retain their directions, and that they will remain equal to each other; we there-

fore see that they will only diminish, by the same quantity, all the pressures exerted by the mass of oil upon itself; consequently, as all the differences remain equal to each other, the condition of equilibrium will still be satisfied as regards that mass. It is evident that the same mode of reasoning may be applied to the pressures exerted by the surrounding liquid upon itself; pressures which will retain their directions, all of which will only be diminished to the same extent by the attractions emanating from the oil; so that the condition of equilibrium will still be satisfied as regards the hollow figure of the surrounding liquid. Thus the whole of the molecular actions due to the presence of the surrounding liquid will not tend in any way to modify the figure of equilibrium of the immersed mass, which figure will consequently be identically the same as if that mass were really void of gravity and were placed *in vacuo*. We can therefore leave the surrounding liquid completely out of the question, its sole function being to neutralize the action of gravity upon the mass forming the object of the experiments.

9. We shall now pass to the experimental part. And first, to avoid useless repetition, we shall say a few words relative to the apparatus to be used. As the liquid always consists of a mass of oil immersed in an alcoholic mixture of the same density as itself, our solid systems will all consist of iron, and this for the following reasons. In ordinary circumstances oil contracts, I believe, perfect adhesion with all solids, but this is not exactly the case when the same oil is plunged into a mixture of water and alcohol, for then, in the case of certain solids, as *e. g.* glass, the phænomena of adhesion sometimes undergo modifications which give rise to trouble in the experiments. We shall meet with an instance of this in the subsequent parts of this memoir. Now the metals do not present this inconvenience; moreover, the form which we have given to most of our solid systems would render their construction of any other substance besides a metal difficult. Now among metals we prefer iron, not copper, because oil removes nothing from iron, whilst by prolonged contact with copper it slightly attacks it, acquires a green colour, and increases in density, which is a great inconvenience*.

* In a letter which Dr. Faraday did me the honour of sending to me regarding the preceding memoir, he *informed me* that when about to repeat my experiments before a numerous audience, wishing to produce a still greater

When we wish to use one of these solid systems of iron, before introducing it into the vessel, it must be completely moistened with oil, and for this purpose it is not sufficient simply to immerse it in this liquid, but it must be carefully rubbed with the finger. The presence of this coating facilitates the adherence of the liquid mass.

We shall continue to make use of the vessel with plane walls, described in the preceding memoir, § 8*, a common-shaped bottle, and the flask previously mentioned (§ 5 and 8) in the same memoir, are not well adapted, because they do not exhibit the true figure of the mass.

When the solid system is composed of a single piece, it is supported by a vertical iron wire, which is screwed to the lower end of the axis traversing the metallic stopper; but for certain experiments the solid system is formed of two isolated parts, and then only one of them is attached to the axis, as I have stated, the other is supported by small feet which rest upon the bottom of the vessel. It need not be mentioned, that those liquids only which are prepared in such a manner as to be incapable of exciting any chemical action upon each other, can be employed (§ 6 and 24 of the preceding memoir).

In addition to the little funnel for introducing the mass of oil into the vessel, the iron wire which serves for uniting the isolated spheres, &c., of which I have spoken in the preceding memoir, the experiments require some other accessory instruments, as, in the first place, a small glass syringe, the point of which is elongated and slightly bent. It is used as a sucking-

difference in the aspect of the two liquids, he dissolved intentionally a little oxide of copper in the oil, so as to render the latter of a green colour. The compound having thus been made beforehand, and rendered perfectly homogeneous, and the alcoholic mixture having been regulated according to the density of the modified oil, the presence of the copper in solution could not produce any inconvenience; but in this case also the solid systems should unquestionably be made of iron.

* In making the experiments relating to the present memoir, I found that it was requisite slightly to modify the apparatus in question. The second perforation in the plate forming the lid of the vessel should be but little smaller than the central aperture, its neck should be less elevated, and lastly, it should be placed near the other; if left as previously described and figured, the employment of the accessory instruments which we shall describe would be impossible. Moreover, the neck of the central aperture should be furnished with a slight rim, so that it may be easily taken hold of when we wish to remove the lid, as e. g. when it is required to attach a solid system which is too large to pass through this same aperture to the axis which traverses the stopper. Lastly, the vessel should be furnished with a stop-cock at its lower part, so that it may be easily emptied.

pump, to remove for instance, a portion of the oil composing the liquid mass, when it is required to diminish the volume of the latter, or to withdraw the entire mass of oil from the vessel, an operation which is sometimes required &c In the second place, two wooden spatulas, one being slightly bent, the other straight, covered with fine linen or cotton stuff When these spatulas are introduced into the vessel, and the cloth with which they are furnished is thoroughly impregnated with the alcoholic liquid, the mass of oil does not adhere to them Hence, by means of one or the other of these spatulas, the mass can be moved in the surrounding liquid, and conducted to the place which it is required to occupy in the interior of the vessel without any of it remaining upon the spatula This is the purpose for which these instruments are intended After they have been used care must always be taken to agitate them in pure alcohol before allowing them to dry If this precaution be omitted, the alcoholic mixture with which they are impregnated, on evaporating would leave the small quantity of oil which it held in solution upon their surface and when the same instruments are used again, the mass of oil would adhere to it In the third place, an iron spatula, the uses of which we shall point out in the proper place Lastly, as it is necessary, in all the experiments which we shall relate that the alcoholic liquid should be homogeneous, the process indicated in the preceding memoir (§ 25) cannot be used to prevent the mass of oil from becoming occasionally adherent to the bottom of the vessel, but the same result is obtained by covering the bottom with a square piece of linen

New experiments in support of the theoretical principles brought forward in the preceding observations Figures of equilibrium terminated by surfaces of spherical curvature New principle relating to layers of liquids

10 The facts which we shall first describe may be considered as constituting the experimental demonstration of the principle of the superficial layer (§6 *bis*) Let us imagine any solid system to be immersed in the liquid mass and let us give to this mass such a volume that it may constitute a sphere which completely envelopes the solid system without the latter reaching the surface at any point Then, if the above principle be true, the presence of the solid system will exert no influence upon the

figure of equilibrium, because, under these circumstances, the superficial layer, from which the configuring actions emanate, remains perfectly free; whilst if these actions emanated from all points of the mass, any unsymmetrical modification occurring in the internal parts of the latter would necessarily produce one in the external form. This is confirmed by experiment The condition of a solid system completely enveloped by the mass of oil would be somewhat difficult to realize; but it must be remembered, that in the experiments relating to the preceding memoir, the system of the disc, by means of which the mass was made to revolve, was very nearly in this condition, because it did not reach the external surface of the mass excepting at the two very small spaces which gave passage to its axis. But we then saw (§ 9 of the preceding memoir), that when the mass was at rest, its sphericity was only very slightly altered by the presence of this system. The theoretical condition may be more nearly approached by taking a very fine metallic wire for the axis of this same system; in this case the alteration in form is quite imperceptible. The axis being supposed to be vertical, the disc may moreover be placed so that its centre coincides with that of the mass of oil, or is situated above or below the latter without producing any difference. I shall relate another fact of an analogous nature. In the course of the experiments, it sometimes happens that portions of the alcoholic liquid become imprisoned in the interior of the mass of oil, forming so many isolated spheres Now, however these spheres may be situated in the interior of the mass, not the least alteration is produced in the figure of the latter.

11. Again, let us cause some kind of solid system to penetrate the liquid mass; but now let the mass be of too small a volume to be capable of completely enveloping this system. The latter will then necessarily reach the superficial layer; and, if the principle in question be true, the figure of the liquid mass will be modified, or, in other words, will cease to remain spherical. This does really occur, as we might have expected; the liquid mass becomes extended at those portions of the solid system which project externally from its surface; it finally either occupies the whole of these portions, or only a part of their extent, according to the form and the dimensions of the solid system, and thus assumes a new figure of equilibrium. We shall meet with examples of this hereafter (§§ 14, 15, 17).

12. Instead of causing the solid system to penetrate the interior of the liquid mass, let it simply be placed in contact with the external surface of the latter. An action being then established at a point of the superficial layer, equilibrium must be destroyed, and the figure of the liquid mass ought again to be modified. This really occurs: the mass becomes extended upon the surface presented to it, and consequently acquires a different shape. This result might also have been anticipated from what occurs under ordinary circumstances, when a drop of water is placed upon a previously moistened solid surface. One might be induced to believe that, as regards the actual result, this case is referable to that of the preceding paragraph or that in paragraph 10; for it appears that the liquid mass becoming extended upon the solid sytem so as to obtain the new figure of equilibrium should ultimately occupy or envelope this system in the same manner as if the latter had been made to penetrate its interior directly. Under certain circumstances this must occur; but the experiments which are about to be related will show that under other circumstances the result is totally different.

13. Let us take for the solid system a thin circular plate*, attached by its centre to the iron wire which supports it (Pl. V. fig. 1), and let us produce the adhesion of its lower surface to the upper part of the mass of oil†. Directly contact is completely established the oil extends rapidly over the surface presented to it; but, what is remarkable, although the precaution has been taken of rubbing the whole of the system (§ 9), that is the two faces of the plate as well as its rim, with oil, the oil terminates

* The diameter of that which I have used is 4 centimetres. I mention this diameter for the sake of being definite. It is evident that in our experiments the dimensions of the apparatus are completely arbitrary, except that if these dimensions exceed certain limits the operations will be more embarrassing in consequence of the large quantities of liquid which would be required.

† In order that this operation may be effected with facility, the sphere of oil must first remain in the surrounding liquid beneath the central aperture in the lid; the plate being then introduced into the vessel, we have merely to lower it by means of the axis traversing the stopper, to bring it towards the liquid mass. If the latter does not occupy the position in question, it must be previously placed there by means of a spatula covered with linen (§ 9). It must be remarked here, that true contact between the plate and the sphere of oil does not usually ensue immediately; a certain resistance has to be overcome analogous to that treated of in the note to paragraph 4 of the preceding memoir; but to overcome this, the liquid sphere need only be gently moved by means of the plate. The slight resulting pressure soon causes the rupture of the obstacle and the production of adhesion.

abruptly at this rim without passing to the other side of the plate, and thus presents a sudden interruption in the curvature of its surface. In the case in question, the new figure acquired by the mass is a portion of a sphere. This portion will be as much larger in proportion to the complete sphere as the volume of oil is greater; but the curvature will always terminate abruptly at the margin of the plate (see fig. 2, which represents a section of the solid system and the adherent mass in the case of three different volumes of the latter).

The cause of this singular interruption of continuity is readily understood. The rim of the plate reaching to the superficial layer, it is natural that something peculiar should occur along this margin, and that the continuity of form should cease at that point where a foreign attractive action is exerted without transition on the superficial layer.

11. Let us again make use of the above plate; but instead of presenting one of its faces to the exterior of the sphere of oil, let us insert the plate edgewise into the interior of this sphere*. The liquid will necessarily extend over both faces of the solid; and if the diameter of the primitive sphere were less than that of the plate, the oil will be seen to form two spherical segments upon the two faces in question, the curvatures of which will still terminate abruptly at the margin of the plate. These two segments may be either equal or unequal, according as the edge of the plate has been introduced into the liquid sphere in such a manner that the plane of the plate passes through the centre of the sphere or not. The upper segment will be slightly deformed by the action of the suspending wire; but this effect will

* This operation is performed as follows. The stopper to which the system of the plate is attached is kept at some distance above the neck of the central aperture, in such a manner however that the latter is immersed to a sufficient depth in the alcoholic mixture. The plate can then be moved with tolerable freedom, and it is conducted towards the liquid mass. For this purpose, the latter ought previously to occupy a suitable position. Immediately the liquid mass is cut, the plate is kept still until the action is terminated, after which the stopper is carefully placed in the neck. A process the reverse to the preceding may also be made use of. The liquid mass is first made to occupy a position near the second aperture, and a sufficient distance from the axis which passes through the centre of the central aperture; then, having fixed the solid system firmly in the position which it is to occupy, move the liquid mass towards it, and when this has been cut, allow the action to continue uninterruptedly. These processes are also employed in other experiments, and it is enough to have pointed them out once. In some cases the second is the only practicable one. This may be easily decided upon in making the experiments.

be less sensible in proportion to the thinness of the wire in question Fig 3 represents the result of the experiment with two unequal segments The discontinuity of the curvatures is a very general fact, which we shall frequently find to recur in the course of our experiments it will hereafter lead us to very important consequences

15 I have repeated the same experiment, substituting a plate of an elliptic form for the circular plate In this, as in the preceding case, the oil extends over both faces of the solid, so as entirely to cover them and, if the volume of the liquid mass is not too great the curvatures again terminate abruptly along the rim of the plate By gradually augmenting the volume of the primitive sphere of oil without however rendering it sufficiently large to allow of the mass completely enveloping the plate so as to retain the spherical form, a limit is attained at which the edge of the plate ceases to reach the superficial layer of the new figure of equilibrium except at the two summits of the ellipse The discontinuity in the curvatures then only occurs at these two places Figs 4 and 5 exhibit the result of the experiment in this case In fig 4 the long axis of the ellipse is presented to view, in fig 5 its short axis

16 All the facts which we have hitherto detailed show, that so long as the interior of the mass is modified, its external shape undergoes no alteration but that directly the superficial layer is acted upon the mass acquires a different form To complete the proof by experiment alone, that the configuring actions exerted by the liquid upon itself emanate solely from the superficial layer, the only point would then be the possibility of reducing a liquid mass to its superficial layer, or at least to a thin pellicle, and to see if in this state it would assume the same figure of equilibrium as a complete mass Now this is completely realized in soap bubbles, for these bubbles, when detached from the tube in which they have been made, assume, as is well known, a spherical form, i e the same figure as that which we find a complete mass acquires in our apparatus, when withdrawn from the action of gravity and perfectly free When the mass adheres to a solid system, which modifies its figure, it is clear that the entire configurative action is composed of two parts, one of which belongs to the solid system, and we find that this system only exerts it when acting upon the superficial layer the other belongs to the liquid, and emanates

directly from the free portion of this same superficial layer. The facts which we have related show clearly what is the seat of this second part of the whole configurative action; but they do not make us acquainted with the nature of the forces of which it consists. On referring to theory, we find that these forces consist in pressures exerted upon the mass by all the elements of the superficial layer, pressures the intensity of which depend upon the curvatures of the surface at the points to which they correspond. Hence it follows that the mass is pressed upon by every part of its superficial layer, with an intensity depending in the same manner upon the curvatures of the surface. For instance, a mass the free surface of which presents a convex spherical curvature, will be pressed upon by the whole of the superficial layer belonging to this free surface, with a greater intensity than if this surface had been plane, and this intensity will be more considerable in proportion as the curvature is greater, or as the radius of the sphere to which the surface belongs is less. Let us see whether experiment will lead us to the same conclusions

17. The solid system which we shall employ is a circular perforated plate (fig. 6). It is placed vertically, and attached by a point of its circumference to the iron wire which supports it Let the diameter of the sphere of oil be less than that of the plate, and let the latter be made to penetrate the mass by its edge in a direction which does not pass through the centre of the sphere. At first, as in the experiment at paragraph 14, the oil will form two unequal spherical segments, but matters do not remain in this state. The most convex segment is seen to diminish gradually in volume, consequently in curvature, whilst the other increases, until they have both become exactly equal One part of the oil then passes through the aperture in the plate, so as to be transferred from one of the segments towards the other, until the above equality is attained.

Let us now examine into the consequences deducible from this experiment, judging from the preceding ones, and independently of all theoretical considerations. When the oil has once become extended over both surfaces of the plate, in such a manner that the superficial layer is applied to every part of the margin of the latter, the action of the solid system is completed; and the movements which subsequently ensue in the liquid mass, to attain the figure of equilibrium, can only then be due to an action emanating from the free part of the superficial layer. It is therefore the latter which

compels the liquid to pass through the aperture in the plate, and the phænomenon must necessarily result either from a pressure exerted by that portion of the superficial layer which belongs to the most convex segment, or by a traction produced by the portion of this same layer belonging to the other segment. Our experiment not being alone capable of determining our choice between these two methods of explaining the effect in question, let us provisionally adopt the first, i. e. that which attributes it to pressure. In our experiment, this pressure emanates from the superficial layer of the most curved segment; but it is easy to see that the superficial layer of the other segment also exerts a pressure which, alone, is less than the preceding. In fact, if for the most curved segment a segment less curved than the other were substituted, the oil would then be driven in the opposite direction. Hence it follows that the entire superficial layer of the mass exerts a pressure upon the liquid which it encloses, and that the intensity of this pressure depends upon the curvatures of the free surface. Moreover, as the liquid proceeds from the most curved segment to that which is least so, it is evident that in the case of a convex surface the curvature of which is spherical, the pressure is greater in proportion as the curvature is more marked, or as the radius of the sphere to which the surface belongs is smaller. Lastly, since a plane surface may be considered as belonging to a sphere, the radius of which is infinitely great, it is evident that the pressure corresponding to a convex surface, the curvature of which is spherical, is superior to that which would correspond to a plane surface. All these results were announced by theory. They perfectly verify then that part of the latter to which they refer; and this concordance ought now to decide in favour of the hypothesis of pressure. This same part of the theory was already verified, in its application to liquids submitted to the action of gravity, by the phænomenon of the depression presented by liquids in capillary tubes, the walls of which they do not moisten; but the series of our experiments setting out with the elements of the theory, and following it step by step, yields far more direct and complete verification. Our last experiment leads us to still further consequences. The liquid passing from one of its segments to the other so long as their curvatures have not become identical, and the pressures corresponding to the two portions of the superficial layer becoming equal to each other simultaneously

with the two curvatures, it follows that the mass only attains its figure of equilibrium when this equality of pressure is established We thus have a primary verification of the general theory of equilibrium which governs our liquid figures, a condition in virtue of which the pressures exerted by the superficial layer ought to be everywhere the same. Moreover, it is evident that if a superficial layer, having a spherical curvature, exerts by itself a pressure, this principle must be true however small the extent of this layer may be supposed to be. It follows, therefore, that an extremely minute portion of the superficial layer of our mass, taken from any part of either of the two segments, ought itself to be the seat of a slight pressure; consequently, that the total pressure exerted by the superficial layer is the result of individual pressures emanating from all the elements of this layer. This was also shown by theory. Further, following the same train of reasoning, we see that the intensity of each of the minute individual pressures ought to depend upon the curvature of the corresponding element of the layer, which is also in conformity with theory. Lastly, as in a state of equilibrium the two segments belong to spheres of equal radii, the curvature is the same in all points of the surface of the mass; whence it follows, that all the minute elementary pressures are equal to each other The general condition of equilibrium (§ 5) is therefore perfectly verified in the instance of our experiment

18. The principle of the superficial layer, applied to the preceding experiment, allows of the latter being modified in such a manner as to obtain a very remarkable result. When the figure of equilibrium is once attained, the perforated plate acts upon the superficial layer by its external border only. The whole of the remainder of this plate then exerts no influence upon the figure in question. Hence it follows that this figure would still be the same if the aperture were enlarged, only the greater the diameter of the latter the less time is required for the establishment of the equality between the two curvatures. Lastly, we ought to be able to enlarge the aperture nearly to the margin of the plate without changing the figure of equilibrium; or, in other words, to reduce the solid system to a simple ring of thin iron wire. Now this is confirmed by experiment; but, to put it in execution, we cannot confine ourselves, as before, to making the solid system penetrate a sphere of oil of less diameter than that of this same system, and subsequently

to allow the molecular forces to act because the metallic wire, on account of its small extent of surface would not exert a sufficient action upon the superficial layer to cause the liquid to extend so as to adhere to the entire surface of the ring. The mass would then remain traversed by part of the latter, and its spherical form would not be sensibly altered if the metallic wire were small: the liquid surface would merely be slightly raised upon the wire in the two small spaces at which it issued from the mass. To speak more exactly, under the circumstances in question two figures of equilibrium are possible. One of these differs but very slightly from the sphere; it is not symmetrical with regard to the ring, one part of which traverses it whilst the other part remains free. The second figure is perfectly symmetrical as regards the ring, and completely embraces its margin; its surface is composed of two equal spherical curves, the margins of which rest upon the ring; in other words, it constitutes a true doubly convex lens of equal curvatures. This is the figure which it is our object to obtain. For this purpose, we first give the sphere of oil a diameter slightly greater than that of the metallic ring; we then introduce the latter into the mass so that it is completely enveloped; lastly, by means of the small glass syringe (§ 9) some of the liquid is gradually removed from the mass*. As this diminishes in volume its surface is soon applied to every part of the margin of the ring, and the volume continuing to diminish, the lenticular form becomes manifest. Afterwards by withdrawing more of the liquid, the curvatures of the two surfaces may be reduced to that degree which is considered suitable. In this way a beautiful double convex lens is obtained, which is entirely liquid except at its circumference. Moreover, in consequence of the index of refraction of the olive oil being much greater than that of the alcoholic mixture, the lens in question possesses all the properties of converging lenses: thus, it magnifies objects seen through it, and this magnifying power may be varied at pleasure by removing some of the liquid from, or adding more to, the mass. Our figure therefore realizes that which could not be obtained with glass lenses, *i. e.* it forms a lens the curvature and magnifying power of which are variable. The diameter of that which I formed was 7 centimetres, and the thickness of the metallic wire was

* The point of the instrument is introduced into the vessel through the second aperture in the lid.

about ½ a millimetre. A much finer wire might have been used with the same success; but the apparatus would then become inconvenient, on account of the facility with which it would be put out of shape. By operating with care, the curvatures of the lens may be diminished so as almost to make them vanish; thus I have been enabled to reduce the lens which I formed, and the diameter of which, as I have stated, was 7 centimetres, to such an extent that it was only 2 or 3 millimetres in thickness. Hence we might presume that it would be possible to obtain, by a proper mode of proceeding, a layer of oil with plane faces. This is, in fact, confirmed by experience, as we shall see further on.

19. To render the curvatures of the liquid lens very slight, the point of the syringe must naturally be applied to the middle of the lens, because the maximum of thickness exists there. Now when a certain limit has been attained, the mass suddenly becomes divided at that point, and a curious phænomenon is produced. The liquid rapidly retires in every direction towards the metallic circumference, and forms a beautiful liquid ring along the latter; but this ring does not last for more than one or two seconds, after which it spontaneously resolves itself into several small, almost spherical masses, adhering to various parts of the ring of iron wire, which passes through them like the beads of a necklace.

20. The reasoning which led us, at the commencement of paragraph 18, to reduce the primitive solid system to a simple metallic wire representing the line in the direction of which this system is met by the superficial layer belonging to the new figure of equilibrium, may be generalized. We may conclude, that whenever a solid system introduced into the mass is not met by the superficial layer of the figure produced, excepting in the direction of small lines only, simple iron wires, representing the lines in question, may be substituted for the solid system employed. But if the volume of the primitive solid system were considerable, it would evidently be requisite to add to the mass of oil an equivalent volume of this liquid, to occupy the place of the solid parts suppressed.

There is however an exception to this principle; it occurs when the solid system separates the entire mass into isolated portions, as in the experiment of paragraph 14, for then these portions assume figures independent of each other, and which may correspond to different pressures. In this case the sup-

pression of one portion of the solid system would place the figures primitively isolated in communication and the inequality of the pressures would necessarily induce a change in the whole figure Excluding this exception, the principle is general, and the result of it is that well developed effects of configuration may be obtained on employing simple iron wires instead of solid systems The experiment of the biconvex lens furnishes one instance of this and we shall meet with a great many others hereafter Nevertheless, to be enabled to comprehend the influence of a simple metallic wire upon the configuration of the liquid mass, it is not requisite to consider this wire as substituted for a complete solid system, it may also be considered by itself It is, in fact, clear that the solid wire acting by attraction upon the superficial layer of the mass the curvatures of the two portions of the surface resting upon it ought not to have any further relation of continuity with each other The metallic wire may therefore determine a sudden transition between these two portions of the surface the curvatures of which will terminate abruptly at the limit which it places to them The principles which we have established ought undoubtedly to be considered as among the most remarkable and curious consequences of the principle of the superficial layer, and one cannot avoid being astonished when we see the liquid maintained in such different forms by an action exerted upon the extremely minute parts of the superficial layer of the mass

21 We have experimentally studied the influence of convex surfaces of spherical curvature let us now ascertain what experiment is able to teach us in regard to plane surfaces and concave surfaces of spherical curvature Let us take for the solid system a large strip of iron curved circularly so as to form a hollow cylinder and attached to the suspending iron wire by some point on its outer surface (fig 7) To prevent the production of accessory phænomena in the experiment, we shall suppose that the breadth of the metallic band is less than the diameter of the cylinder formed by the same band, or that it is at least equal to it Make the mass of oil adhere to the internal surface of this system, and let us suppose that the liquid is in sufficient quantity then to project outside the cylinder In this case the mass will present on each side a convex surface of spherical curvature, and the curvatures of these two surfaces will be equal This figure is a consequence of what we have previously seen, and we

must not stop here, for it will serve us as a starting-point in obtaining other figures which we require. Apply the point of the syringe to one of the above convex surfaces, and gradually withdraw some of the liquid. The curvatures of the two surfaces will then gradually diminish, and with care they may be rendered perfectly plane. It follows from this first result, that a plane surface is also a surface of equilibrium, which is evidently in conformity with theory. Let us now apply the end of the syringe to one of these plane surfaces, and again remove a small quantity of liquid. The two surfaces will then become simultaneously hollow, and will form two concave surfaces of spherical curvature, the margins of which rest upon the metallic band, and the curvatures of which are the same. Finally, by the further removal of the liquid, the curvatures of the two surfaces become greater and greater, always remaining equal to each other.

Hence it results, first, that concave surfaces of spherical curvature are still surfaces of equilibrium, which is also in accordance with theory. Moreover, as the plane surface left free sinks spontaneously as soon as that to which the instrument is applied becomes concave, it must be concluded that the superficial layer belonging to the former exerts a pressure which is counterbalanced by an equal force emanating from the opposite superficial plane layer, but which ceases to be so, and which drives away the liquid as soon as this opposite layer commences to become concave. Again, as further abstraction of the liquid determines a new rupture of equilibrium, so that the concave surface opposite to that upon which we directly act exhibits a new spontaneous depression when the curvature of the other surface increases, it follows that the concave superficial layer belonging to the former still exerted a pressure, which at first was neutralized by an equal pressure arising from the other concave layer, but which becomes preponderant, and again drives away the liquid when the curvature of this other layer is increased.

Hence it follows,—1st, that a plane surface produces a pressure upon the liquid; 2nd, that a concave surface of spherical curvature also produces a pressure; 3rd, that the latter is inferior to that corresponding to a plane surface; 4th, that it is less in proportion as the concavity is greater, or that the radius of the sphere to which the surface belongs is smaller. These results were also pointed out by theory, and had already been verified in the application of the latter to liquids submitted to the action

of gravity by the phænomenon of the elevation of a liquid column in a capillary tube, the walls of which are moistened by it.

Reasoning upon these facts, as we have done at the end of paragraph 17 in regard to convex surfaces of spherical curvature, we shall arrive at the conclusion that the entire pressure exerted by a concave superficial layer of spherical curvature is the result of minute individual pressures arising from all the elements of this layer, and that the intensity of each of these minute pressures depends upon the curvature of that element of the layer from which it emanates. Our last experiment therefore perfectly verifies that part of the theory which relates to plane and convex surfaces of spherical curvature. Lastly, in the state of equilibrium of our liquid figure, the curvature being the same at all points of each of the two concave surfaces, it is again evident that all the minute elementary pressures are equal to each other, which gives a new complete verification of the general condition of equilibrium.

22. The figure we have just obtained constitutes a biconcave lens of equal curvatures, and possesses all the properties of diverging lenses, *i. e.* it diminishes objects seen through it, &c. Moreover, as the curvature of the two surfaces may be increased or diminished by as small degrees as is wished, it follows that we thus obtain a diverging lens, the curvature and action of which are variable.

23. Now let us suppose that we have increased the curvatures of the lens until the two surfaces nearly touch each other by their summits.* We might presume, that if the removal of the liquid were continued, the mass would become disunited at that point at which this contact took place, and that the oil would recede in every direction towards the metallic band. This is however not the case; we then observe in the centre of the figure the formation of a small sharply defined circular space, through which objects no longer appear diminished, and we easily recognize that this minute space is occupied by a layer of oil with plane faces. If the removal of the liquid be gradually continued, this layer increases more and

* To effect this operation the point of the syringe must not be placed in the middle of the figure, as in the case of the doubly convex lens, but on the contrary near the metallic band, as this is now the point where the greatest thickness of the liquid exists.

more in diameter, and may thus be extended to within a tolerably short distance of the solid surface. In my experiment, the diameter of the metallic cylinder was 7 centimetres, and I have been enabled to increase the size of the layer until its circumference was not more than about 5 millimetres from the solid surface; but at this instant it broke, and the liquid of which it consisted rapidly receded towards that which still adhered to the metallic band. The fact which we have just described is very remarkable, both in itself and in the singular theoretical consequences to which it leads. In fact, that part of the mass to which the layer adheres by its margin presents concave surfaces, whilst those of the layer are plane; now the existence of such a system of surfaces in a continuous liquid mass seems in opposition to theory, since it appears evident that the pressures cannot be equal in this case. But let us investigate the question more minutely.

24. According to theory, the pressure corresponding to any point of the surface of a liquid mass, as we have seen (§ 3), is the integral of the pressures exerted by each of the molecules composing a rectilinear line perpendicular to the surface at that point, and equal in length to the radius of the sphere of activity of the molecular attraction. The analytical expression of this integral contains no other variables than the radii of the greatest and of the least curvature at the point under consideration (§ 4), consequently the pressure in question varies only with the curvatures of the surface at the same point. This is rigorously true when the liquid is of any notable thickness, but we shall show, that in the case of an extremely thin layer of liquid, there is another element which exerts an influence upon the pressure. Let us conceive a liquid layer, the thickness of which is less than twice the radius of the sphere of sensible activity of the molecular attraction. Let each molecule be conceived to be the centre of a small sphere with this same radius (§ 3), and let us first consider a molecule situated in the middle of the thickness of the layer. The little sphere, the centre of which is occupied by this molecule, will be intersected by the two surfaces of the layer, consequently it will not be entirely full of liquid, but the segments suppressed on the outside of the two surfaces being equal, the molecule will not be more attracted perpendicularly in one direction than in the other. Now let a small right line, normal to and terminating at the two surfaces, pass through

this same molecule, and let us consider a second molecule situated at some other point of this right line. The little sphere which belongs to the second molecule in question may again be intersected by the two surfaces of the layer, but then the two suppressed segments will be unequal; the molecule will consequently be subjected to a preponderating attraction, evidently directed towards the thickness of the layer. The molecule will then exert a pressure in this direction, and it must be remarked that this pressure will be less than if the liquid had any notable thickness, the molecule being situated at the same distance from the surface; for in the latter case the little sphere would only be cut on one side, and its opposite part would be perfectly full of liquid. It might also happen that the little sphere belonging to the molecule in question in the thin layer is only cut on one side; the molecule will then still exert a pressure in the same direction, but its intensity will then be as great as in the case of a thick mass. It is easy to see that if the thickness of the layer is less than the simple length of the radius of the molecular attraction, the little spheres will all be cut on both sides; whilst if the thickness in question is comprised between the length of the above radius and twice this same length, a portion of the minute spheres will be cut on one side only. In both cases, the pressure exerted by any molecule being always directed towards the middle of the thickness of the layer, it is evident that the integral pressure corresponding to any point of either of the two surfaces will be the result of the pressures individually exerted by each of those molecules, which, commencing at the point in question, are arranged upon half the length of the small perpendicular. Now each of the two halves of the small perpendicular being less than the radius of the sphere of activity of the molecular attraction, it follows that the number of molecules composing the line which exerts the integral pressure is less than in the case of a thick mass. Thus, on the one hand, the intensities of part or the whole of the elementary pressures composing the integral pressure will be less than in the case of a thick mass, and, on the other hand, the number of these elementary pressures will be less; from this it evidently follows that the integral pressure will be inferior to that which would occur in the case of a thick mass. P always denoting the pressure corresponding to any point of a plane surface belonging to a thick mass (§ 4), the pressure corresponding to any point of

either of the surfaces of an extremely thin plane layer will therefore be less than P. Moreover, this pressure will be less in proportion as the layer is thinner, and it may thus diminish indefinitely; for it is clear that it would be reduced to zero if we supposed that the thickness of the layer was equal to no more than that of a simple molecule.

We can obtain liquid layers with curved surfaces, soap-bubbles furnish an example of these, and we shall meet with others in the progress of this investigation. Now by supposing the thickness of such a layer to be less than twice the radius of the molecular attraction, we should thus evidently arrive at the conclusion, that the corresponding pressures at either of its two surfaces would be inferior in intensity to those given by paragraph 4, and that moreover these intensities are less in proportion as the layer is smaller. We thus arrive at the following new principle:—

In the case of every liquid layer, the thickness of which is less than twice the radius of the sphere of activity of the molecular attraction, the pressure will not depend solely upon the curvatures of the surfaces, but will vary with the thickness of the layer.

25. We thus see that an extremely thin plane liquid layer, adhering by its edge to a thick mass the surfaces of which are concave, may form with this mass a system in a state of equilibrium; for we may always suppose the thickness of the layer to be of such value, that the pressure corresponding to the plane surfaces of this layer is equal to that corresponding to the concave surfaces of the thick mass. Such a system is also very remarkable in respect to its form, inasmuch as surfaces of different nature, as concave and plane surfaces, succeed each other. This heterogeneity of form is moreover a natural consequence of the change which the law of pressures undergoes in passing from the thick to the thin part.

26. As we have already seen, theory demonstrates the possibility of the existence of such a system in a state of equilibrium. As regards the experiment which has led us to these considerations, although the result presented by it tends to realize in an absolute manner the theoretical result, there is one circumstance which is unfavourable to the completion of this realization. We can understand that the relative mobility of the molecules of oil is not sufficiently great to occasion the immediate formation of the liquid layer with that excessive tenuity which is

requisite for equilibrium: the thickness of this layer, although very minute absolutely speaking, is undoubtedly, during the first moments, a considerable multiple of the theoretical thickness. If then we produce the layer without extending it to that limit to which it is capable of increasing during the operation, and afterwards leave it to itself, the pressure corresponding to its plane surfaces will still exceed that corresponding to the concave surfaces of the remainder of the liquid system. Hence it follows that the oil within the layer will be driven towards this other part of the system, and that the thickness of the layer will progressively diminish. The equilibrium of the figure will then be apparent only, and the layer will in reality be the seat of continual movements. The diminution in thickness, however, will be effected slowly, because in so confined a space the movements of the liquid are necessarily restrained; this is why, as in the experiment in paragraph 17, the mass only acquires its figure of equilibrium slowly, because there is a cause which impedes the movements of the liquid. The thickness of the layer gradually approximates to the theoretical value, from which the equilibrium of the system would result; but unfortunately it always happens that before attaining this point, the layer breaks spontaneously. This effect depends, without doubt, upon the internal movements of which I have spoken above; we can imagine in fact that when the layer has become of extreme thinness, the slightest cause is sufficient to determine its rupture. The exact figure which corresponds to the equilibrium is therefore a limit towards which the figure produced tends; this limit the latter approaches very nearly, and would attain if it were not itself previously destroyed by an extraneous cause.

Our experiment has led us to modify the results of theory in one particular instance; but we now see, that, far from weakening the principles of this theory, it furnishes, on the contrary, incomplete as it is, a new and striking verification of it. The conversion of the doubly concave lens into a system comprising a thin layer, is connected with an order of general facts; we shall see that a large number of our liquid figures become transformed by the gradually produced diminution of the mass of which they are composed into systems consisting of layers, or into the composition of which layers enter.

27. If by some modification of our last experiment, we could succeed in obtaining the equilibrium of the liquid system, we

might be able to deduce from it a result of great interest—an indication of the value of the radius of the sphere of activity of the molecular attraction. In fact, we might perhaps find out some method of determining the thickness of the layer; these might, for instance, then exhibit colours, the tint of which would lead us to this determination. Now we have seen that in the state of equilibrium of the figures, half the thickness of the layer would be less than the radius in question, hence we should then have a limit above which the value of this same radius would exist. In other words, we should know that the molecular attraction produces sensible effects, even at a distance from its centre of action beyond this limit. Our experiment, although insufficient, may thus be considered as the first step towards the determination of the distance of sensible activity of the molecular attraction, of which distance at present we know nothing, except that it is of extreme minuteness.

28. Let us now return to the consideration of thick masses. It follows from the experiments related in paragraphs 13, 14, 17, 18 and 21, that when a continuous portion of the surface of such a mass rests upon a circular periphery, this surface is always either of spherical curvature or plane. But to admit this principle in all its generality, we must be able to deduce it from theory. We shall do this in the following series, at least on the supposition that the portion of the surface in question is a surface of revolution. We shall then see that this same principle is of great importance. We may remark here, that in the experiment in paragraph 23, the layer commences to appear as soon as the surfaces can no longer constitute spherical segments. Now we shall again find, that in the other cases, when a full figure is converted, by the gradual withdrawal of the liquid, into a system composed of layers, or into the composition of which layers enter, the latter begin to be formed when the figure of equilibrium, which the ordinary law of pressures would determine, ceases to be possible. The mass then assumes, or tends to assume, another figure, compatible with a modification of this law. Such is the general principle of the formation of layers under the circumstances in question.

29. After having formed a converging and a diverging liquid lens, it appeared to me curious to combine these two kinds of lens so as to form a liquid telescope. For this purpose, I first substituted for the ring of iron wire in paragraph 18 a circular

plate of the same diameter perforated by a large aperture (fig. 8). This plate having been turned in a lathe, I was certain of its being perfectly circular, which would be a very difficult condition to fulfill in the case of a simple curved iron wire. In the second place, I took for the solid part of the doubly concave lens, a band of about 2 centimetres in breadth, and curved into a cylinder $3\frac{1}{2}$ centimetres in diameter. These two systems were arranged as in fig. 9, in such a manner that the entire apparatus being suspended vertically in the alcoholic mixture by the iron wire a, and the two liquid lenses being formed, their two centres were at the same height, and 10 centimetres distant from each other. In this arrangement the telescope cannot be adjusted by altering the distance between the objective and the eye piece, but this end is attained by varying the curvatures of these two lenses. With the aid of a few preliminary experiments I easily managed to obtain an excellent Galilean telescope, magnifying distant objects about twice, like a common opera glass, and giving perfectly distinct images with very little irisation. Fig. 10, which represents a section of the system, shows the two lenses combined.

Figures of equilibrium terminated by plane surfaces. Liquid polyhedra. Laminar figures of equilibrium.

30. In the experiment detailed at paragraph 21, we obtained a figure presenting plane surfaces. These were two in number, parallel, and bounded by circular peripheries; but it is evident that these conditions are not necessary in order to allow plane surfaces to belong to a liquid mass in equilibrium. We can understand that the forms of the solid contours might be indifferent provided they constitute plane figures. We can moreover understand that the number and the relative directions of the plane surfaces may be a matter of indifference, because these circumstances exert no influence upon the pressures which correspond to these surfaces, pressures which will always remain equal to each other. Lastly, it follows from the principle at which we arrived at the end of paragraph 20, relative to the influence of solid wires, that for the establishment of the transition between a plane and any other surface a metallic thread representing the edge of the angle of intersection of these two surfaces will be sufficient. We are thus led to the curious result, that we ought to be able to form polyhedra which are entirely liquid excepting at their edges

Now this is completely verified by experiment. If for the solid system we take a frame work of iron wire representing all the edges of any polyhedron, and we cause a mass of oil of the proper volume to adhere to this frame-work, we obtain, in fact, in a perfect manner, the polyhedron in question, and the curious spectacle is thus obtained of parallelopipedons, prisms, &c., composed of oil, and the only solid part of which is their edges.

To produce the adhesion of the liquid mass to the entire frame-work, a volume is first given to the mass slightly larger than that of the polyhedron which it is to form; it is then placed in the frame work; and lastly, by means of the iron spatula (§ 9), which must be introduced by the second aperture of the lid of the vessel, and which is made to penetrate the mass, the* latter is readily made to attach itself successively to the entire length of each of the solid edges. The excess of oil is then gradually removed with the syringe, and all the surfaces thus become simultaneously exactly plane. But that this end may be attained in a complete manner, it is clearly requisite that the equilibrium of density between the oil and the alcoholic mixture should be perfectly established; and the slightest difference in this respect is sufficient to alter the surfaces sensibly. It should also be borne in mind, that the manipulation with the spatula sometimes occasions the introduction of alcoholic bubbles into the interior of the mass of oil; these are, however, easily removed by means of the syringe.

31. Now, having formed a polyhedron, let us see what will happen if we gradually remove some of the liquid. Let us take, for instance, the cube, the solid frame-work of which, with its suspending wire, is represented at fig. 11 b. Let the point of the syringe be applied near the middle of one of the faces, and let a small quantity of the oil be drawn up. All the faces will immediately become depressed simultaneously and to the same extent, so that the superficial square contours will form the bases of six similar hollow figures. We should have imagined this to have been the case for the maintenance of equality between the pressures.

If fresh portions of the liquid are removed, the faces will become more and more hollowed; but to understand what happens when this manipulation is continued, we must here enunciate a preliminary proposition. Suppose that a square

* The edges of all the frames which I used were 7 centims in length

plate of iron, the sides of which are of the same length as the edges of the metallic frame, is introduced into the vessel and that a mass of oil equal in volume to that which is lost by one of the faces of the cube is placed in contact with one of the faces of this plate, I say that the liquid after having become extended upon the plate will present in relief the same figure as the face of the modified cube presents in intaglio. Then in fact, in passing from the hollow surface to that in relief the radii of curvature corresponding to each point will only change their signs without changing in absolute value, consequently (§ 8) since the condition of equilibrium is satisfied as regards the first of these surfaces, it will be equally so with regard to the second.

Now let us imagine a plane passing through one side of the plate and tangentially to the surface of the liquid which adheres to it at that point. As long as this liquid is in small quantity we should imagine, and experiment bears us out, that the plane in question will be strongly inclined towards the plate, but if we gradually increase the quantity of liquid, the angle comprised between the plane and the plate will also continue to increase and instead of being acute as before, will become obtuse. Now so long as this angle is less than 45° the convex surface of the liquid adhering to the plate will remain identical with the concave surfaces of the mass attached to the metallic frame, and suitably diminished but beyond this limit, the coexistence in the frame of the six hollow identical surfaces with the surface in relief becomes evidently impossible for these surfaces must mutually intersect each other. Thus when the withdrawal of the liquid from the mass forming the cube is continued, a point is attained at which the figure of equilibrium ceases to be realizable in accordance with the ordinary law of pressures. We then meet with a new verification of the principle enunciated in § 28 *i e* that the formation of layers commences. These layers are plane they commence at each of the wires of the frame, and connect the remainder of the mass to the latter, which continues to present six concave surfaces. In fact we can imagine, that by this modification of the liquid figure the existence of the whole of this in the metallic frame again becomes possible, as also the equilibrium of the system, for there is then no further impediment to the concave surfaces assuming that form which accords with the ordinary law of pressures, and on the other hand, in supposing the layers to be sufficiently thin, the

pressure belonging to them might be equal to that which corresponds to these same concave surfaces (§ 25).

On removing still further portions of the liquid, the layer will continue to enlarge, whilst the full mass which occupies the middle of the figure will diminish in volume, and this mass can thus be reduced to very minute dimensions: fig. 12 represents the entire system in this latter state. It is even possible to make the little central mass disappear entirely, and thus to obtain a complete laminar system, but for this purpose certain precautions must be taken, which I shall now point out. When the central mass has become sufficiently small, the point of the syringe must first be thoroughly wiped, otherwise the oil adheres to its exterior to a certain height, and this attraction keeps a certain quantity of oil around it, which the instrument cannot absorb into its interior. In the second place, the point of the syringe must be depressed to such an extent, that it nearly touches the inferior surface of the little mass. During the suction, this surface is then seen to become raised, so as to touch the orifice of the instrument, and the latter then absorbs as much of the alcoholic mixture as of the oil, but this is of no consequence, and the minute mass is seen to diminish by degrees, so as at last completely to disappear. The system then consists of twelve triangular layers, each of which commences at one of the wires of the frame, and all the summits of which unite at the centre of the figure; it is represented in fig. 13. But this system is only formed during the action of the syringe. If, when this is complete, the point of the instrument is slowly withdrawn, an additional lamina of a square form is seen to be developed in the centre of the figure (fig. 14). This then is the definitive laminar system to which the liquid cube is reduced by the gradual diminution of its mass.

32. In the preceding experiment, as in that of paragraph 23, the thickness of the layers is at first greater than that which would correspond to equilibrium. If then the system were left to itself whilst it still contains a central mass, we should imagine that one portion of the liquid of the layers would be slowly driven towards this mass, and that the layers would gradually become thinner. Moreover it always happens that one or the other of the latter increases after some time, undoubtedly for the reason which we have already pointed out (§ 26). Hence, for the perfect success of the transformation of the cube into the

laminar system, one precaution, which has not yet been spoken of must be attended to It consists in the circumstance, that from the instant at which the layers arise, the exhaustion of the liquid must be continued as quickly as possible until the central mass has attained a certain degree of minuteness In fact, as soon as the formation of the layers commences, their tendency to become thinner also begins to be developed, and if the operation is effected too slowly, the system might break before it was completed When the central mass is sufficiently reduced, and experience soon teaches us to judge of the suitable point, the action of the syringe must be gradually slackened, and at last the other precautions which we have mentioned must be taken

We are able then to explain the rupture of the layers so long as there is a large or small central mass, but when the laminar system is complete we do not at the first glance see the reason why the thickness of the layers diminishes, and consequently why destruction of the system takes place Nevertheless the rupture ultimately takes place in this as in the other case, and the time during which the system persists rarely extends to half an hour In ascertaining the cause of this phænomenon, it must be remarked that the intersections of the different layers cannot occur suddenly, or be reduced to simple lines it is evident that the free transition between two liquid surfaces could not be thus established in a discontinuous manner These transitions must therefore be effected through the intermedium of minute concave surfaces, and with a little attention we can recognise that in fact this really takes place We can then understand that the oil of the layers ought also to be driven towards the places of junction of the latter and consequently the absence of the little central mass does not prevent the gradual attenuation of the layers, and the final destruction of the system

33 If during the action of the syringe, when the system shown in fig 13 has been attained, instead of slowly withdrawing the instrument it is suddenly detached by a slight shake in a vertical direction, the additional layer is not developed, but the little mass in fig 12 is seen to be reproduced very rapidly This fact confirms in a remarkable manner the explanation which we have given in the preceding paragraph In fact, at the moment at which the point of the instrument is separated from the system the latter may be considered as composed of hollow pyramids now it also follows, from causes relating to their conti-

nuity, that the summits of these pyramids should not constitute simple points, but little concave surfaces. But as the curvatures of these minute surfaces are very great in every direction, they would give rise to still far less pressure than those which establish the transitions between each pair of surfaces of the layers, for in the latter there is no curvature in one direction. The oil of the layers will therefore be driven with much greater force towards the centre of the figure than towards the other parts of the junctions of these layers. Again, the twelve layers terminating in this same centre, the oil flows there simultaneously from a large number of sources. These two concurrent causes ought then, in conformity with experiment, to produce the rapid reappearance of the small central mass; and we can understand why it is impossible to obtain the complete system of the pyramids otherwise than during the action of the syringe.

34 All the other polyhedric liquids become transformed, like the cube, into laminar systems when the mass of which they are composed is gradually diminished. Among these systems, some are complete; the others still contain very small masses, which cannot be made to disappear entirely. Analogous considerations to those which we applied with regard to the cube would show, in each case, that the formation of layers commences as soon as the hollow surfaces which would correspond to the ordinary law of pressures cease to be able to coexist in the solid frame. Figs. 15, 16, 17 and 18, represent the laminar systems resulting from the triangular prism, the hexahedral prism, the tetrahedron and the pyramid with a square base, these systems being supposed to be complete. They are all formed of plane layers, commencing at each of the metallic wires; and that of the hexahedral prism, as is shown, contains an additional layer in the centre of the figure.

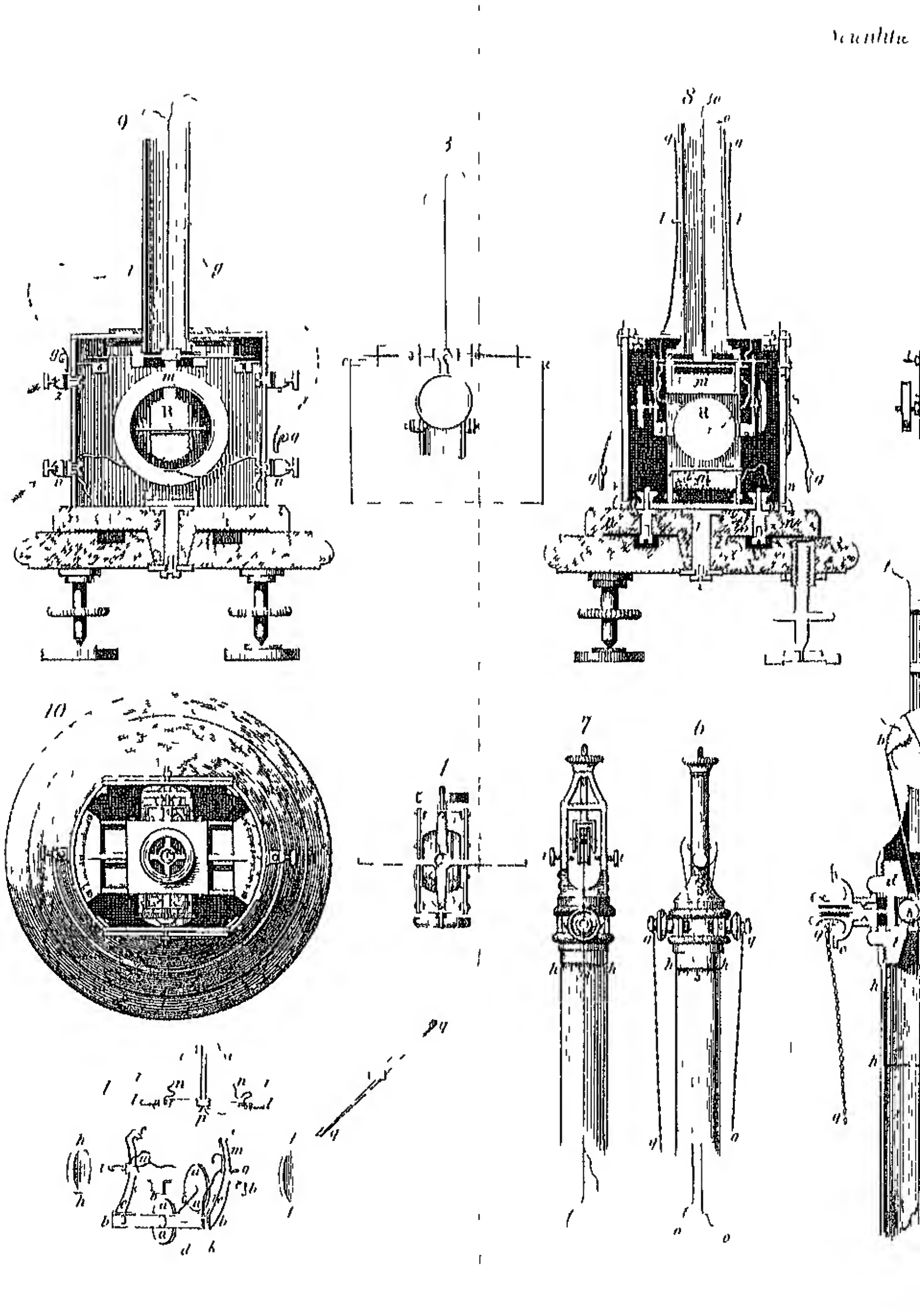
35. The system arising from the regular octohedron presents a singular exception, which I have not been able to explain. The layers of which this system is composed are curved, and form a fantastical group, of which it is difficult to give an exact idea by graphic representations. Fig. 19 exhibits them projected upon two rectangular vertical planes; and it is seen that the aspects of the system observed upon two adjacent sides are inverse as regards each other. The formation of this system presents a curious peculiarity. At the commencement of the operation, all the faces of the octohedron become simultaneously

hollow the layers in progress of formation are plane, and arranged symmetrically, so that the system tends towards the form represented at fig 20 But when a certain limit is attained a sudden change occurs the layers become curved, and the system tends to assume the singular form which we have mentioned I have several times repeated the experiment varying the circumstances as much as possible, and the same effects are always produced

In the course of this memoir, I shall point out another process for obtaining laminar systems, it is an extremely simple one and has moreover the advantage of producing all the systems in a complete state

36 In concluding our observations upon polyhedric liquids, I shall remark that the triangular prism may be employed to produce the phænomena of dispersion In this way a beautiful solar spectrum may be obtained by means of a prism with liquid faces But as the effect only depends upon the excess of the refracting action of the oil above that of the alcoholic liquid, to obtain a considerably extended spectrum the angle of refraction of the prism must be obtuse, an angle of 110 gives a very good result Moreover it is evidently requisite that the faces of the prism should be perfectly plane which is obtained by using a carefully made frame by establishing exact equilibrium between the density of the liquids, and lastly, by arresting the action of the syringe exactly at the proper point

[To be continued]



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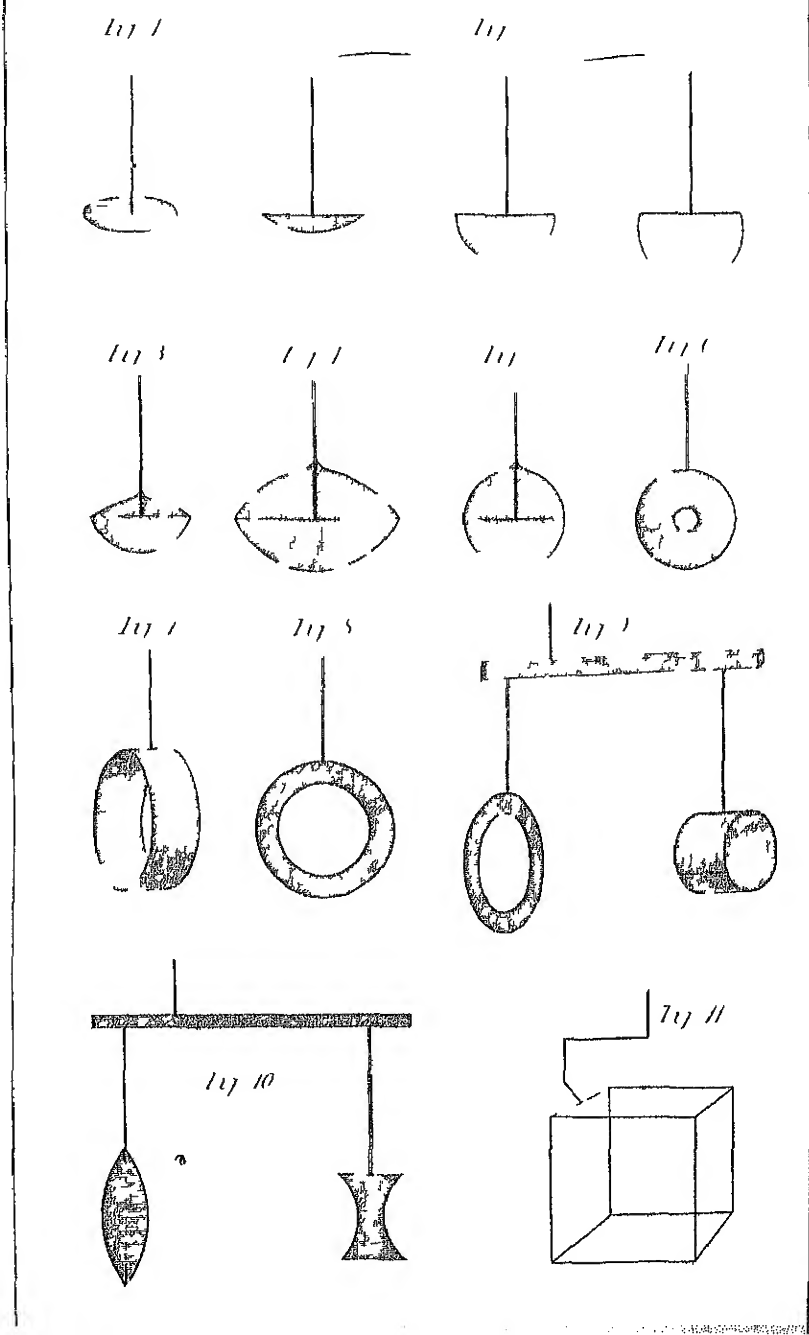

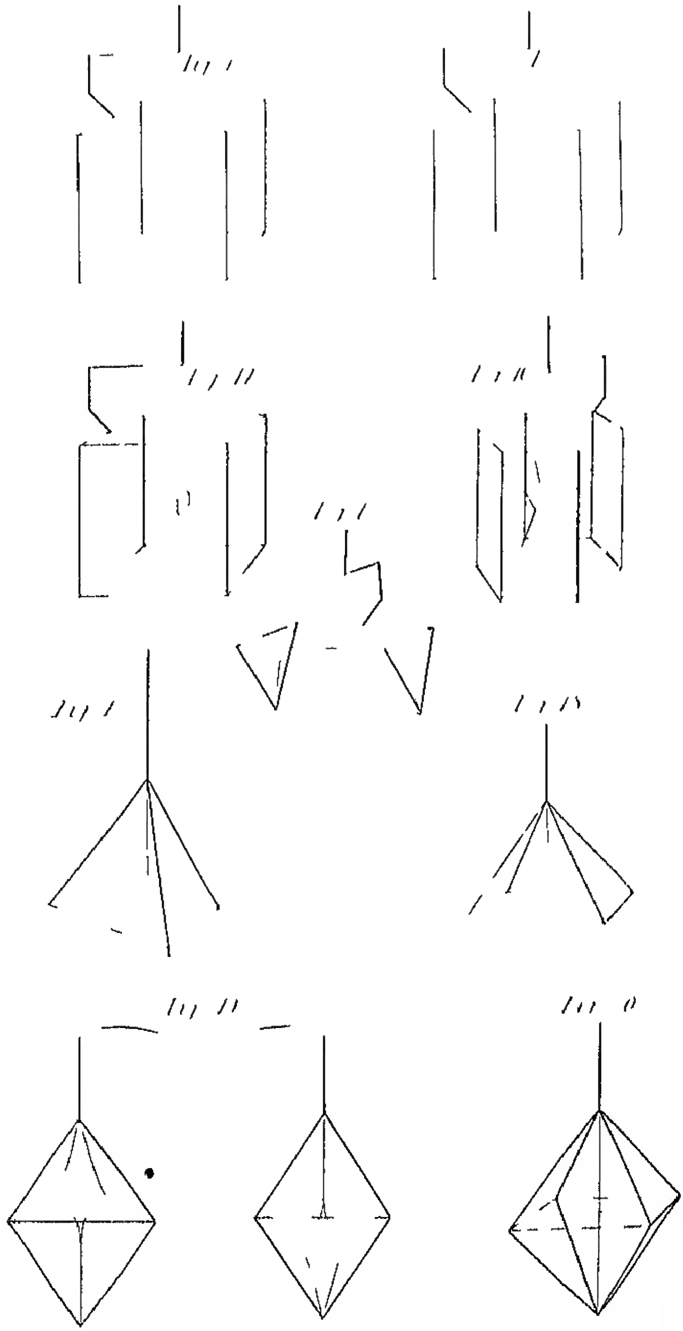

SCIENTIFIC MEMOIRS.

VOL. V.—PART XXI.

ARTICLE XVIII. *continued.*

Experimental and Theoretical Researches on the Figures of Equilibrium of a Liquid Mass withdrawn from the Action of Gravity. By J. PLATEAU, *Professor at the University of Ghent, Member of the Royal Academy of Belgium, &c.*

SECOND SERIES.

Other figures of Revolution besides the Sphere. Liquid Cylinder.

37. LET us now endeavour to form some new liquid figures. Those best adapted to theoretical considerations would be figures terminated by surfaces of revolution other than the sphere and lenticular figures which we have already studied. Surfaces of revolution enjoy simple properties in regard to the radii of the greatest and least curvature at every point; we know that one of these two radii is the radius of curvature of the meridional line, and that the other is that portion of the normal to this line which is included between the point under consideration and the axis of revolution. We shall now endeavour to obtain figures of this nature.

38. Let our solid system be composed of two rings of iron wire, equal, parallel, and placed opposite to each other. One of these rings rests upon the base of the vessel by three feet composed of iron wire; the other is attached, by means of an intermediate piece, to the axis traversing the central stopper, so that it may be approximated to or removed from the former by depressing or elevating this axis*. The system formed by these

* In the experiments which we are now about to describe, the short axis represented in fig. 2 of the preceding memoir, and which has hitherto answered our purpose, must be replaced by another of about 15 centims. in length.

two rings is represented in Plate VII fig 20 *bis*, the diameter of those which I employed was 7 centims

After having raised the upper ring as much as possible, let a sphere of oil, of a slightly larger diameter than that of the rings, be formed, and conducted towards the lower ring, in such a manner as to make it adhere to the entire circumference of the latter, then depress the upper ring until it comes into contact with the liquid mass and the latter is uniformly attached to it When the mass has thus become adherent to the system of the two rings, let the upper ring be slowly raised, when the two rings are at a proper distance apart, the liquid will then assume the form the vertical projection of which is represented in fig 21, in which the lines *a b* and *c d* are the projections of the rings The two portions of the surface which are respectively applied to each of the rings are convex spherical segments, and the portion included between the two rings constitutes a figure of revolution, the meridional curve of which, as is shown, is convex externally We shall recur, in the following series, to this part of the liquid figure If we now continue gradually to raise the upper ring, the curvature of the two extremities and the meridional curvature of the intermediate portion will be diminished, and if there is exact equilibrium between the density of the oil and the surrounding liquid, the surface included between the two rings will be seen to assume a perfectly cylindrical form (fig 22) The two bases of the liquid figure are still convex spherical segments, but their curvature is less than in the preceding figure If the interval between the rings be still further increased, it is evident that the surface included between them would lose the cylindrical form, and that a new figure would result This is what occurs, but the consideration of the figure thus produced must be deferred

Instead then of immediately increasing the distance between the rings, let us commence by adding a certain quantity of oil to the mass, which will again render the surface included between the rings convex Let us then gradually elevate the upper ring, and we shall produce a cylinder of greater height than the first If we repeat the same manipulation a suitable number of times, we shall ultimately obtain the cylinder of the greatest height which our apparatus permits* I have in this manner obtained a perfectly cylindrical mass 7 centims in diameter and about 14 centims in height (fig 23) To allow

the cylinder of this considerable height being perfect, it is requisite that perfect equality be established between the densities of the oil and the alcoholic liquid As a very slight difference in either direction tends to make the mass ascend or descend, the latter assumes, to a more or less marked extent one of the two forms represented in fig 24 Even when the cylindric form has been obtained by the proper addition of alcohol of 16 or absolute alcohol, as occasion may require (§ 21 of the preceding memoir), slight changes in temperature are sufficient to alter and reproduce one of the above two forms

39 Let us now examine the results of these experiments in a theoretical point of view First, it is evident that a cylindrical surface satisfies the general condition of equilibrium of liquid figures, because the curvatures in it are the same at every point Moreover, such a surface being convex in every direction except in that of the meridional line, where there is no curvature, the pressure corresponding to it ought to be greater than that corresponding to a plane surface The same conclusions are deducible from the general formulæ (2) and (3) of paragraphs 4 and 5 In fact, as we have already stated in paragraph 37, one of the quantities R and R′ is the radius of curvature of the meridional line and the other is the portion of the normal to this line included between the point under consideration and the axis of revolution Now in the case of the cylinder, the meridional line being a right line, its radius of curvature is everywhere infinitely great and, on the other hand, this same right line being parallel to the axis of revolution, that portion of the normal which constitutes the second radius of curvature is nothing more than the radius itself of the cylinder Hence it follows, that one of the terms of the quantity $\frac{1}{R}+\frac{1}{R'}$ disappears and that the other is constant, this same quantity is therefore constant, and consequently the condition of equilibrium is satisfied Now if we denote by λ the radius of the cylinder, the general value of the pressure for this surface would become

$$P+\frac{A}{2}\,\frac{1}{\lambda}$$

Now λ being positive because it is directed towards the interior of the liquid (§ 1), the above value is greater than P, *i e* than that which would correspond to a plane surface It is therefore evident that the bases of our liquid cylinder must necessarily be

convex, as is shown to be the case by experiment; for as equilibrium requires that the pressure should be the same throughout the whole extent of the figure, these bases must produce a greater pressure than that which corresponds to a plane surface.

Our plane figure then fully satisfies theory; but verification may be urged still further. Theory allows us to determine with facility the radius of those spheres of which the bases form a part. In fact, if we represent this radius by r, the formula (1.) of paragraph 4 will give, for the pressure corresponding to the spheres in question,

$$P+A\ \frac{1}{r}.$$

Now as this pressure must be equal to that corresponding to the cylindrical surface, we shall have

$$P+\frac{A}{2}\ \frac{1}{\lambda}=P+A\ \frac{1}{r},$$

from which we may deduce

$$r=2\lambda.$$

Thus the radius of the curvature of the spherical segments constituting the bases is equal to the diameter of the cylinder.

Hence, as we know the diameter, which is the same as that of the solid rings, we may calculate the height of the spherical segments; and if by any process we afterwards measure this height in the liquid figure, we shall thus have a verification of theory even as regards the numbers. We shall now investigate this subject.

40. If we imagine the liquid figure to be intersected by a meridional plane, the section of each of the segments will be an arc belonging to a circle the radius of which will be equal to 2λ, according to what we have already stated, and the versed sine of half this arc will be the height of the segment. If we suppose the metallic filaments forming the rings to be infinitely small, so that each of the segments rests upon the exact circumference of the cylinder, the chord of the above arc will also be equal to 2λ; and if we denote the height of the segments by h, we shall have

$$h=\lambda(2-\sqrt{3})=0.268\ \lambda.$$

Now the exact external diameter of my rings, or the value of 2λ corresponding with my experiments, was 71.1 millims., which gives $h=9.57$ millims. But as the metallic wires have a certain thickness, and the segments do not rest upon the external circum-

ference of the rings it follows that the chord of the meridional arc is a little less than 2λ, and that consequently the true theoretical height of the segments is a little less than that given by the preceding formula. To determine it exactly, let us denote the chord by $2c$, which will give

$$h=2\lambda-\sqrt{4\lambda^2-c^2}$$

Now let us remark, that the meridional plane intersects each of the rings in two small circles to which the meridional arc of the spherical segment is tangential, and upon each of which the chord of this arc intercepts a small circular segment. The meridional arc being tangential to the sections of the wire, it follows that the above small circular segments are similar to that of the spherical segment, and as the chord of the latter differs but very slightly from the radius of the circle to which the arc belongs, the chords of the small circular segments may be considered as equal to the radius of the small sections, which radius we shall denote by r. It is moreover evident that the excess of the external radius of the ring over half the chord c is nothing more than the excess of the radius r over half the chord of the small circular segments, which half chord, in accordance with what we have stated, is equal to $\frac{1}{2}r$. Thence we get $\lambda-c=\frac{1}{2}r$, whence $c=\lambda-\frac{1}{2}r$, and we have only to substitute this value in the preceding formula to obtain the true theoretical value of h. The thickness of the wire forming my rings is 0·74 millim., hence $\frac{1}{2}r=0{\cdot}18$ millim., which gives as the true theoretical height of the segments under these circumstances,

$$h=9{\cdot}46 \text{ millims.}$$

I may remark, that it is difficult to distinguish in the liquid figure the precise limit of the segments, *i. e.* the circumferences of contact of their surfaces with those of the rings. To get rid of this inconvenience, I measured the height of the segments, commencing only at the external planes of the rings, *i. e.* in the case of each segment, commencing at a plane perpendicular to the axis of revolution and resting upon the surface of the ring on that side which is opposite the summit of the segment. The quantity thus measured is evidently equal to the total height minus the

versed sine of the small circular segments which we have considered above; consequently these small circular segments being similar to that of the spherical segment, we obtain for the determination of this versed sine, which we shall denote by f, the proportion $\frac{h}{c}=\frac{f}{\frac{1}{2}}$, which in the case of our liquid figure gives

$f=0\cdot05$ millim., whence

$$h-f=9\cdot41 \text{ millims.}$$

This then is definitively the theoretical value of the quantity which was required to be measured.

41. Before pointing out the process which I employed for this purpose, and communicating the result of the operation, I must preface a few important remarks If the densities of the alcoholic mixture and of the oil are not rigorously equal, the mass has a slight tendency to rise or descend, and the height of one of the segments is then a little too great, whilst that of the other is a little too small, but we can understand that if their difference is very small, an exact result may still be obtained by taking the mean of these two heights. We thus avoid part of those preliminary experiments, which the establishment of perfect equality between the two densities requires. But one circumstance which requires the greatest attention, is the perfect homogeneity of each of the two liquids. If this condition be not fulfilled with regard to the alcoholic mixture, *i. e.* if the upper part of this mixture be left containing a slightly greater proportion of alcohol than the lower portion, the liquid figure may appear regular and present equal segments; all that is required for this is, that the mean density of that part of the mixture which is at the same level as the mass, must be equal to the density of the oil, but under these circumstances the level of the two segments is too low. In fact, the oil forming the upper segment is then in contact with a less dense liquid than itself, and consequently has a tendency to descend, whilst the opposite applies to the oil forming the inferior segment*. Heterogeneity of the liquid produces an opposite effect, *i. e.* it renders the height of the segments too great. In fact, the least dense portions rising to the upper part of the mass, tend to lift it up, whilst the most

* By intentionally producing very great heterogeneity in the alcoholic mixture (§ 9 of the preceding memoir), and employing suitable precautions, a perfectly regular cylinder may be formed, the bases of which are absolutely plane.

dense portions descend to the lower part and tend to depress it. Now the quantities of pure alcohol, and that at 16° added to the alcoholic mixture to balance the mass, necessarily produce an alteration in the homogeneity of the oil; for, in the first place, the oil during these operations being in contact with mixtures which are sometimes more, sometimes less charged with alcohol must absorb or lose some of this by its surface; in the second place these same additions of alcohol to the mixture diminish the saturation of the latter with the oil, so that it removes some of it from the mass, and this action is undoubtedly not equally exerted upon the two principles of which the oil is composed. Hence before taking the measures, the different parts of the oil must be intimately mixed together, which may be effected by introducing an iron spatula into the mass, moving it about in it in all directions, and this for a long time, because the mixture of the oil can only be perfectly effected with great difficulty on account of its viscidity.

To avoid the influence of the reactions which render the oil heterogeneous, the operations must be conducted in the following manner:—The mass being introduced into the vessel and attached to the two rings, and the equality of the densities being perfectly established, allow the mass to remain in the alcoholic liquid for two or three days, re-establishing from time to time the equilibrium of the densities altered by the chemical reactions and the variations of temperature. Afterwards remove the two rings from the vessel, so that the mass remains free; remove almost the whole of this, by means of a siphon, into a bottle, which is to be carefully corked; withdraw with the syringe the small portion of oil which is left in the vessel, and reject this portion. Next replace the two rings, and mix the alcoholic liquid perfectly; then again introduce the oil into the vessel, taking the precaution of enveloping the bottle containing it with a cloth several times folded, so that the temperature may not be sensibly altered by the heat of the hand.* Then attach

* The following is the reason why the oil must be removed from the vessel before employing it for the experiment. After having remained a considerable time in the alcoholic liquid the oil becomes enveloped by a kind of thin pellicle, or more strictly speaking the superficial layer of the mass has lost part of its liquidity, an effect which undoubtedly arises from the unequal action of the alcohol upon the principles of which the oil is composed. The necessary result of this is that the mass loses at the same time part of its tendency to assume a determinate figure of equilibrium, which tendency must therefore be

the mass to the lower ring only, the upper ring being raised as much as possible; mix the oil intimately, as we have said above; then depress the upper ring, cause the mass to adhere to it, elevate it so as to form an exact cylinder, and proceed immediately to the measurement.

42. The instrument best suited for effecting the latter operations in an exact manner is undoubtedly that which has received the name of *cathetometer*, and which, as is well known, consists of a horizontal telescope moving along a vertical divided rule. The distance comprised between the summits of the two segments is first measured by the aid of this instrument; the distance included between the external planes of the two rings (§ 40) is then measured by the same means. The difference between the first and the second result evidently gives the sum of the two heights, the mean of which must be taken; and consequently this mean, or the quantity sought, $h-f$, is equal to half the difference in question.

The determination of the distance between the external planes of the rings requires peculiar precautions. First, as the points of the rings at which we must look are not exactly at the external surface of the figure, the oil interposed between these

completely restored to it. This is why the oil is withdrawn by the siphon. In fact, the pellicle does not penetrate the interior of the latter, and during its contraction continues to envelope the small portion remaining; so that after the latter has been removed by the syringe, which ultimately absorbs the pellicle itself, we get completely rid of the latter.

Before using the siphon, the thickness and consistence of the pellicle are too slight to enable us distinctly to perceive its presence, but when the operation of the siphon is nearly terminated, and the mass is thus considerably reduced, we find that the surface of the latter forms folds, hence implying the existence of an envelope. Moreover, when the siphon is removed, the small residuary mass, which then remains freely suspended in the alcoholic liquid, no longer assumes a spherical form, but retains an irregular aspect, appearing to have no tendency to assume any regular form.

This indifference to assume figures of equilibrium, arising from a diminution in the liquidity of the superficial layer, constitutes a new and curious proof of the fundamental principle relating to this layer (§§ 6 *bis* and 10 to 16). M. Hagen (*Mémoire sur la Surface des Liquides*, in the Memoirs of the Academy of Berlin, 1845) has observed a remarkable fact, to which the preceding appears to be related. It consists in this, that the surface of water, left to itself for some time, undergoes a peculiar modification, in consequence of which the water then rises in capillary spaces to elevations which are very distinctly less than is the case when its surface is exempt or freed from this alteration. This fact might perhaps be explained by admitting that the water dissolves a small proportion of the substance of the solid with which it is in contact, and that the external air acts chemically at the surface of the liquid upon the substance dissolved, thus giving rise to the formation of a slight pellicle which modifies the effects of the molecular forces.

points and the eye must produce some effects of refraction, which would introduce a slight error into the value obtained. To avoid this inconvenience, we need only expose the rings by allowing the liquids to escape from the vessel by the stop cock (note 2 to § 9), then remove the minute portions of the liquid which remain adherent to the rings by passing lightly over their surface a small strip of paper which must be introduced into the vessel through the second aperture. The drops of alcoholic liquid remaining attached to the inner surface of the anterior side of the vessel must also be absorbed in the same manner. In the second place, as it would be difficult for the rings to be rigorously parallel, their distance must be measured from two opposite sides of the system, and the mean of the two values thus found taken. The following are the results which I obtained. The mensuration of the distance between the summits gave first, in four successive operations, the values 76.77, 76.80, 76.85 and 76.75 millims., the mean of which is 76.79 millims. But after the alcoholic liquid had been again agitated for some time to render its homogeneity more certain, two new measurements taken immediately afterwards gave 77.05 and and 77.00 millims., or a mean of 77.02 millims. The distance between the external planes of the rings was found, on the one hand, by two observations, which agreed exactly, to be 57.73 millims.; on the other hand, two observations furnished the values 57.87 and 57.85 millims., or as the mean 57.86 millims. Taking then the mean of these two results, we get 57.79 millims. as the value of the distance between the centres of the external planes. Hence, if we assume the first of the two values obtained for the distance of the summits, 76.79 millims., we find

$$h-f=\frac{76.79-57.79}{2}=9.50 \text{ millims.},$$

and if from the second result, 77.02 millims., we find

$$h-f=\frac{77.02-57.79}{2}=9.61 \text{ millims.}$$

These two elevations evidently differ but little from 9.11 millims., the altitude deduced from theory (§ 40); in the first case the difference does not amount to the $\frac{1}{100}$dth part of its theoretical value, and in the second it hardly exceeds $\frac{3}{50}$dths. These differences undoubtedly arise from slight re-

mains of heterogeneity in the liquids; it is probable that in the first case neither of the two liquids was absolutely homogeneous, and that the two contrary effects which thence resulted (§ 41) partly neutralized each other, whilst in the second case, the alcoholic liquid being rendered perfectly homogeneous, the effect of the slight heterogeneity of the oil exerted its full influence. However this may be, these differences in each case are so small, that we may consider experiment as in accordance with theory, of which it evidently presents a very remarkable confirmation.

43. Mathematically considered, a cylindrical surface extends indefinitely in the direction of the axis of revolution. Hence it follows that the cylinder included between the two rings constitutes one portion only of the complete figure of equilibrium. Hence also if the liquid mass were free, it could not assume the cylindrical form as the figure of equilibrium; for the volume of this mass being limited, it would be necessary that the cylinder should be terminated on both sides by portions of the surface presenting other curvatures, which would not admit of the law of continuity But this heterogeneity of curvature, which is impossible when the mass is free, becomes realizable, as our experiments show, through the medium of solid rings. As each of these renders the curvatures of the portions of the surface resting upon it (§ 20) independent of each other, the surface comprised between the two rings may then be of cylindrical curvature, whilst the two bases of the figure may present spherical curvatures We therefore arrive at the very remarkable result, that with a liquid mass of a limited volume we may obtain isolated portions of figures of equilibrium, which in their complete state would be extended indefinitely.

44 With the view of obtaining a cylinder in which the proportion between the height and the diameter was still greater than that in fig 23, I replaced the rings previously employed by two others, the diameter of which was only 2 centims. I first tried to make a cylinder 6 centims. in height, *i. e* the height of which was thrice the diameter, and in this operation I adopted a slightly different process from that of paragraph 38. The uniformity in the density of the two liquids being accurately established, I first gave the mass of oil a somewhat larger volume than that which the cylinder would contain, having then attached the mass to the two rings, I elevated the upper ring until it was at a distance of 6 centims. from the other; this

distance was measured by a scale introduced into the vessel and kept in a vertical position by the side of the liquid figure. In consequence of the excess of oil, the meridional line of the figure was convex externally, and as there was still a slight difference between the densities, this convexity was not symmetrical in regard to the two rings. I corrected this irregularity by successive additions of pure alcohol and alcohol of 16°, an operation which requires great circumspection, and towards the end of which these liquids could only be added in single drops. The figure being at last perfectly symmetrical, I carefully removed the excess of oil by applying the point of the syringe to a point at the equator of the mass, and in this manner I obtained a perfect cylinder. Subsequently, after having added some oil to the mass, I increased the distance between the rings until it was equal to 8 centims., *i.e.* to four times their diameter. The oil was in sufficient quantity to allow of the meridional line of the figure being convex externally, but the curvature was not perfectly symmetrical, and I encountered still greater difficulties in regulating it than in the preceding case. The defect in the symmetry being ultimately corrected, the meridional convexity presented a versed sine of about 3 millims. (fig. 25). I then proceeded to the removal of the excess of oil; but before the versed sine was reduced to 2 millims., the figure appeared to have a tendency to become thin at its lower part and to swell out at the upper part, as if the oil had suddenly become slightly increased in density. At this moment I withdrew the syringe, so as to be enabled to observe the effect in question better: the change in form then became more and more pronounced; the lower part of the figure soon presented a true strangulation, the neck of which was situated nearly at a fourth part of the distance between the rings (fig. 26); the constricted portion continued to narrow gradually, whilst the upper part of the figure became swollen; finally, the liquid separated into two unequal masses, which remained respectively adherent to the two rings; the upper mass formed a complete sphere, and the lower mass a doubly convex lens. The whole of these phænomena lasted a very short time only.

With a view to determine whether any particular cause had in reality produced the alteration of the densities, I approximated the rings, then, after having reunited the two liquid masses, I again carefully raised the upper ring, ceasing at the height of

$7\frac{1}{2}$ centims., so that the versed sine of the meridional convexity was slightly greater than when this was 8 centims. The figure was then found to be perfectly symmetrical, and it did not exhibit any tendency to deformity; whence it follows that the uniformity in the densities had not experienced any appreciable alteration. I recommenced, with still more care, the experiment with that figure which was 8 centims. in height, and I was enabled to approach the cylindrical form still more nearly; but before it was attained, the same phænomena again presented themselves, except that the alteration in form was effected in an inverted manner, i. e. the figure became narrow at the upper part and dilated at the base; so that after the separation into two masses, the perfect sphere existed in the lower ring and the lens in the upper ring. On subsequently uniting, as before, the two masses, and placing the rings at a distance of $7\frac{1}{2}$ centims. apart, the figure was again obtained in a regular and permanent form. Thus when we try to obtain between two solid rings a liquid cylinder the height of which is four times the diameter, the figure always breaks up spontaneously, without any apparent cause, even before it has attained the exactly cylindrical form. Now as the cylinder is necessarily a figure of equilibrium, whatever may be the proportion of the height to the diameter, we must conclude that the equilibrium of a cylinder the height of which is four times the diameter is unstable. As the shorter cylinders which I had obtained did not present analogous effects, I was anxious so satisfy myself whether the cylinders were really stable. I therefore again formed a cylinder 6 centims. in height with the same rings; but this, when left to itself for a full half hour, presented a trace only of alteration in form, and this trace appeared about a quarter of an hour after the formation of the cylinder, and did not subsequently increase, which shows that it was due to some slight accidental cause.

The above facts lead us then to the following conclusions,—1st, that the cylinder constitutes a figure the equilibrium of which is stable when the proportion between its height and its diameter is equal to 3, and with still greater reason when this proportion is less than 3; 2nd, the cylinder constitutes a figure the equilibrium of which is unstable when the proportion of its height to its diameter is equal to 4, and with still greater reason when it exceeds 4; 3rd, consequently there exists an intermediate relation, which corresponds to the passage from stability to

instability. We shall denominate this latter proportion *the limit of the stability of the cylinder*.

15. These conclusions however are liable to a well founded objection. Our liquid figure is complex, because its entire surface is composed of a cylindrical portion and of two portions which present a spherical curvature. Now we cannot affirm that these latter portions exert no influence upon the stability or the instability of the intermediate portion, and consequently upon the value of the proportion which constitutes the limit between these two states. To allow of the preceding conclusions being rigorously applicable to the cylinder, it would be requisite that the figure should present no other free surface than the cylindrical surface, which is easily managed by replacing the rings by entire discs. I effected this substitution by employing discs of the same diameter as the preceding rings, but the results were not changed; the cylinder, 6 centims. in height, was well formed, and was found to be stable; whilst the figure 8 centims. in height began to change before becoming perfectly cylindrical and was rapidly destroyed. The final result of this destruction did not however consist, as in the case of the rings, of a perfect sphere and a double convex lens, but, as evidently ought to have been the case, of two unequal portions of spheres, respectively adherent to the two opposite solid surfaces. The limit of the stability of the cylinder therefore really lies between 3 and 4.

The experiments which we have just related are very delicate, and require some skill. In this, as in all other cases of measurements, the oil must be allowed to remain in the alcoholic mixture for two or three days, then the pellicle must be removed from it (note to p. 627); afterwards, when the mass after having been again introduced into the vessel, has been attached to the two solid discs, some time must be allowed to elapse in order that the two liquids may be exactly at the same temperature; moreover, it must be understood that the experiments should be made in an apartment the temperature of which remains as constant as possible. Lastly, it is scarcely necessary to add, that when the alcoholic liquid is mixed, after having added small quantities of pure alcohol or alcohol at 16°, the movements of the spatula should be very slow, so as to avoid the communication of too much agitation to the mass of oil; we are even sometimes compelled momentarily to depress the upper disc, so as to

give greater stability to the mass, and thus to prevent the movements in question from producing the disunion.

46. It might be asked, whether the want of symmetry, which is constantly seen in the spontaneous modification of the above unstable figures, is the result of a law which governs these figures, or whether it simply arises, as we should be led to believe at first sight, from imperceptible differences still existing between the densities of the two liquids, which differences acting upon unstable figures might produce this want of symmetry, notwithstanding their extreme minuteness.

After having concluded the preceding experiments, I imagined that to solve the question in point, all that would be requisite would be to arrange matters so that the axis of the figure, instead of being vertical, as in the above experiments, should have a horizontal direction. In fact, in the latter case, the slightest difference between the densities ought to have the effect of slightly curving the figure, but evidently cannot give the liquid any tendency to move in greater quantity towards one extremity of the figure than the other, whence it follows, that, if the spontaneous alteration of the figure still occurs unsymmetrically, this can only be owing to a peculiar law.

On the other hand, if the figure really tends of itself to change its form unsymmetrically, it is clear, that, in the case of the vertical position of the axis, the effect of a trace of difference between the densities ought to concur with that of the instability, and thus to accelerate the moment at which the figure commences to alter spontaneously. Consequently, on avoiding this extraneous cause by the horizontal direction of the axis of the figure, we may hope to approximate more nearly to the cylindrical form, or even to attain it exactly; we can moreover understand, that the difficulty in the operations will be found to be considerably diminished.

I therefore constructed a solid system, presenting two vertical discs of the same diameter, placed parallel with each other, at the same height, and opposite each other. Each of these discs is supported by an iron wire fixed normally to its centre, then bent vertically downwards, and the lower extremities of these two wires are attached to a horizontal axis furnished with four small feet. This system is represented in perspective in fig. 27. The diameter of the discs is 30 millims., but the distance which separates them is not four times this diameter. I thought that

by approximating the figure more to the limit of stability, the operations would require still less trouble the distance in question is only 108 millims, so that the relation between the length and the diameter of the liquid cylinder which would extend between the two discs, would be equal to 3 6

We shall now detail the results obtained by the employment of this system In the first place the operations were much more easily performed[†] In the second place, the figure still had a tendency to deformity before it had been rendered perfectly cylindrical but this tendency always exhibited itself unsymmetrically, as in the vertical figures from which circumstance alone we might conclude that the unsymmetrical nature of the phænomenon is not occasioned by a difference between the densities of the two liquids In the third place, by a little management, I have pursued the experiment further, and succeeded in forming an exact cylinder | This lasted for a moment, it then began to be narrowed at one part of its length, becoming dilated at the other, like the vertical figures, and the phænomenon of disunion was completed in the same manner, giving rise ultimately to two masses of different volumes

I repeated the experiment several times, and always with the same results, except that the separation occurred sometimes on one, sometimes on the other side of the middle of the length of the figure However, although the phænomenon is produced in an unsymmetrical manner with regard to the middle of the length of the figure, whether horizontal or vertical, on the contrary there is always symmetry with regard to the axis, in other

* The two discs in this solid system being placed at an invariable distance from each other it is necessary in making a mass of oil the volume of which is not too great adhere to them to employ an extra piece consisting of a ring of iron wire of the same diameter as the discs supported by a straight wire of the same metal the free extremity of which is held in the hand by means of this ring the mass which has been previously attached to one of the discs is drawn out until it is equally attached to the other the ring is then removed The latter removes a small portion of the mass at the same time but on leaving the vessel it leaves this portion in the alcoholic liquid it is then removed by means of the syringe

† To effect this the following proceeding must be adopted for the removal of the excess of oil The operation is at first carried on with a suitable rapidity until the figure begins to alter in form the end of the point of the syringe is then drawn gently along the upper part of the mass proceeding from the thickest to the other portion This slight action is sufficient to move a minute quantity of oil towards the latter and thus to re establish the symmetry of the figure a new absorption is then made the figure again regulated and these proceedings are continued until the exactly cylindrical form is attained

words, throughout the duration of the phænomenon the figure remains constantly a figure of revolution. We may add here, that in the horizontal figure the respective lengths of the constricted and dilated portions appear to be equal; we shall show, in the following series, that this equality is rigorously exact, at least at the commencement of the phænomenon

It is now evident that the alteration in the form of these cylinders is really the result of a property which is inherent in them. We shall hereafter deduce this property as a necessary consequence of the laws which govern a more general phænomenon.

It moreover results from the above experiment, that the proportion 3 6 is still greater than the limit of stability, so that the exact value of the latter must lie between the numbers 3 and 3 6. It is obvious that this method of experiment might be employed to obtain a closely approximative determination of the value in question, I propose doing this hereafter, and I shall give an account of the result in the following series, when I shall have to return to the question of the limit of stability of the cylinder

47 In the unstable cylinders which we have just formed, the proportion of the length to the diameter was inconsiderable; but what would be the case if we were to obtain cylinders of great length relatively to their diameter? Now under certain circumstances, figures of this kind, more or less exactly cylindrical, may be realized, and we shall proceed to see what the results of the spontaneous rupture of equilibrium are.

A fact which I described in paragraph 20 of the preceding memoir, and which I shall now describe more in detail, affords us the means of obtaining a cylinder of this kind, and of observing its spontaneous destruction. When some oil is introduced by means of a small funnel into an alcoholic mixture containing a slight excess of alcohol, and the oil is poured in sufficiently quick to keep the funnel full, the liquid forms, between the point of the funnel and the bottom of the vessel where the mass collects, a long train, the diameter of which continues to increase slightly from the upper to the lower part, so as to form a kind of very elongated cone, which does not differ much from a cylinder* This nearly cylindrical figure, the height of which

* This slight increase in diameter depends upon the retardation which the resistance of the surrounding liquid occasions in the movement of the oil.

is considerable in proportion to the diameter remains without undergoing any perceptible alteration so long as the oil of which it consists has sufficient rapidity of transference but when the oil is no longer poured into the funnel and consequently the motion of transference is retarded, the cylinder is soon seen to resolve itself rapidly into a series of spheres which are perfectly equal in diameter, equally distributed, and with their centres arranged upon the right line forming the axis of the cylinder

To obtain perfect success the elements of the experiment should be in certain proportions The orifice of the funnel which I used was about 3 millims in diameter and 11 centims in height It rested upon the neck of a large bottle containing the alcoholic mixture, and its orifice was plunged a few millimetres only beneath the surface of the liquid Lastly, the length of the cylinder of oil, or the distance between the orifice and the lower mass, was nearly 20 centims Under these circumstances three spheres were constantly formed, the upper of which remained adherent to the point of the funnel the latter was therefore incomplete We may add, that the excess of alcohol contained in the mixture should neither be too great nor too small, the proper quantity is found by means of a few preliminary trials

18 The constancy and regularity of the result of this experiment complete then the proof that the phenomena to which the spontaneous rupture of equilibrium of an unstable liquid cylinder gives rise, are governed by determinate laws

In this same experiment, the transformation ensues too rapidly to allow of its phases being well observed, but the phenomena presented to us by larger and less elongated cylinders, *i e* the formation of a dilatation and constriction in juxtaposition, and equal or nearly so in length the gradual increase in thickness of the dilated portion and the simultaneous narrowing of the constricted portion, &c , authorize us to conclude that in the case of a cylinder the length of which is considerable in proportion to the diameter, the following order of things takes place —The figure becomes at first so modified as to present a regular and uniform succession of dilated portions, separated by constricted portions of the same length as the former, or nearly so This alteration, the indications of which are very slight, gradually becomes more and more marked, the constricted portions

gradually becoming narrower, whilst the dilated portions increase in thickness, the figure remaining a figure of revolution; at last the constrictions break, and each of the various parts of the figure, which are thus completely isolated from each other, acquire the spherical form. We must add, that the termination of the phænomenon is accompanied by a remarkable peculiarity, of which we have not yet spoken, but as it only constitutes, so to speak, an accessory portion of the general phænomenon, we shall transfer the description of it to a subsequent part of this memoir (see § 62).

49. It might be asked, why, in the experiment which we have last described, the cylinder is only resolved into spheres when the rapidity of the transference of liquid of which it is composed is diminished. In fact, we cannot understand how a motion of transference could give stability to a liquid figure which in a state of repose was unstable. In explaining this apparent peculiarity, we must remark, that, as the spontaneous transformation of an unstable cylinder is effected under the action of continued forces, the rapidity with which the phænomenon occurs ought to be accelerated, this may be, moreover, easily verified in experiments relating to larger and less elongated cylinders; this same rapidity ought therefore always to be very minute at the commencement of the phænomenon. Now, in the case in question, as the changes in figure occur in the liquid of the cylinder whilst this liquid is animated by a movement of transference, it is evident, from what we have stated, that if this movement of transference is sufficiently rapid, the changes of form could only acquire a very slightly-marked development during the passage of the point of the funnel to the mass accumulated at the bottom of the vessel; so that, the liquid being continually renewed, there will be no time for any alteration in form to become very perceptible to the eye Hence, so long as the rapidity of the flow is sufficiently great, the liquid figure will appear to retain its almost cylindrical form, although its length is considerable in comparison with its diameter. On the other hand, when the velocity of the transference is sufficiently small, there will be time for the alterations in form to take place in a perfect manner, and we shall be able to see the cylinder resolve itself into spheres throughout the whole of its length.

50. We shall now describe another method of experimenting, which allows us to observe the result of the transformation under

less restrained and more regular conditions in some respects than those of the preceding experiment, and which will moreover lead us to new consequences as regards the laws of the phenomenon. We shall first succinctly describe the apparatus and the operations, and afterwards add the necessary details.

The principal parts of which the apparatus consists are:—1st, a rectangular plate of plate glass 2[illegible] centims. in length and 20 in breadth; 2nd, two strips of the same glass, 13 centims. in length and 5–6 millims. in thickness, perfectly prepared and polished at the edges; 3rd, two ends of copper wire, about 1 millim. in thickness and 5 centims. in length; these wires should be perfectly straight, and one extremity of each of them should be cut very accurately, then carefully amalgamated. The plate being placed horizontally, the two strips are laid flat upon its surface and parallel with its long sides, so as to leave an interval of about a centimetre between them; the two copper wires are then introduced into this, placing them in a right line in the direction of the length of the strips, and in such a manner that the amalgamated extremities are opposite to, and a few centimetres distant from each other. A globule of very pure mercury, from 5 to 6 centims. in diameter, is next placed between the same extremities; the two strips of glass are then approximated until they touch the wires, so as only to leave between them an interval equal in width to the diameter of these wires. The little mass of mercury, being thus compressed laterally, necessarily becomes elongated, and extends on both sides towards the amalgamated surfaces. If it does not reach them, the wires are made to slide towards them until contact and adhesion are established. The wires are then moved in opposite directions, so as to separate them from each other, which again produces elongation of the little liquid mass and diminution of its vertical dimensions. By proceeding carefully, and accompanying the operation with slight blows given with the finger upon the apparatus to facilitate the movements of the mercury, we succeed in extending the little mass until its vertical thickness is everywhere equal to its horizontal thickness, *i.e.* to that of the copper wires. Thus the mercury forms a liquid wire of the same diameter as the solid wires to which it is attached, and from 8 to 10 centims. in length. This wire, considering the small size of its diameter, which renders the action of gravitation insensible in comparison with that of molecular attraction, may

be considered as exactly cylindrical; so that in this manner we obtain a liquid cylinder, the length of which is from 80 to 100 times its diameter, and attached by its extremities to solid parts, which cylinder preserves its form so long as it remains imprisoned between the strips of glass. Weights are next placed upon the parts of the two copper wires which project beyond the extremities of the bands, so as to maintain these wires in firm positions; lastly, by means which we shall point out presently, the two strips of glass are raised vertically. At the same instant, the liquid cylinder, being liberated from its shackles, becomes transformed into a numerous series of isolated spheres, arranged in a straight line in the direction of the cylinder from which they originated*. Ordinarily the regularity of the system of spheres thus obtained is not perfect; the spheres present differences in their respective diameters and in the distances which separate them; this undoubtedly arises from slight accidental causes, dependent upon the method of operation; but the differences are sometimes so small, that the regularity may be considered as perfect. As regards the number of spheres corresponding to a cylinder of determinate length, it varies in different experiments, but these variations, which are also due to slight accidental causes, are comprised within very small limits.

51. Let us now complete the description of the apparatus, and add some details regarding the operations. As the plate of glass requires to be placed in a perfectly horizontal position, it is supported for this purpose upon four feet with screws. A small transverse strip of thin paper is glued to each of the extremities of the lower surface of the strips of glass, in such a manner that the strips of glass resting upon the plate through the medium of these small pieces of paper, their lower surface is not in contact with the surface of the plate. Without this precaution, the strips of glass might contract a certain adhesion to the plate, which would introduce an obstacle when the strips are raised vertically. Moreover, the latter are furnished, on their upper surface and at a distance of 6 millims. from each of their extremities, with a small screw placed vertically in the glass with the point upwards, firmly fixed to it with mastic, and rising 8 millims. above its surface. These four screws are for the pur-

* We may remark, that the conversion of a metallic wire into globules by the electric discharge, must undoubtedly be referred to the same order of phænomena.

pose of receiving the nuts which fix the strips to the system by means of which they are elevated. This system is made of iron: it consists, in the first place, of two rectangular plates 55 millims. in length, 12 in breadth, and 3 in thickness. Each of them is pierced, perpendicularly to its large surfaces, by two holes, so situated, that on placing each of these plates transversely upon the extremities of the two strips of glass, the screws with which the latter are furnished fit into these four holes. The screws being long enough to project above the holes, nuts may then be adapted to them, so that on screwing them the strips of glass become fixed in an invariable position with regard to each other. The holes are of an elongated form in the direction of the length of the iron plates; hence, after having loosened the nuts, the distance between the two strips of glass may be increased or diminished without the necessity of removing the plates. A vertical axis, 5 centims. in height, is implanted upon the middle of the upper surface of each of the plates, and the upper extremities of these two axes are connected by a horizontal axis, at the middle of which a third vertical axis commences; this is directed upwards, and is 15 centims. in length. The section of the latter axis is square, and it is 5 millims. in thickness. When the nuts are screwed up, it is evident that the strips of glass, the iron plates, and the kind of fork which connects them, constitute an invariable system. The long vertical axis serves to direct the movement of this system; with this view, it passes with very slight friction through an aperture of the same section as itself, and 5 centims. in length, pierced in a piece which is fixed very firmly by a suitable support 10 centims. above the plate of glass. Lastly, the perforated piece is provided laterally with a thumb-screw, which allows the axis to be screwed into the tube. By this arrangement, if all parts of the apparatus have been carefully finished, when once the little nuts have been screwed up, the two strips of glass can only move simultaneously in a parallel direction to each other, and always identically in the same direction perpendicular to the plate of glass. When the liquid cylinder is well formed, and the weights are placed upon the free portions of the copper wires, the finger is passed under the horizontal branch of the fork, and the moveable system is raised to a suitable distance above the plate of glass; it is then maintained at this height by means of the thumb-screw, so as to allow the result of the transformation of the cylinder

to be observed. As the amalgamation of the copper wires always extends slightly upon their convex surface, the latter is coated with varnish, so that the amalgamation only occurs upon the small plane section. It would be almost impossible to judge by simple inspection of the exact point at which the separation of the copper wires from each other, to allow of the liquid attaining a cylindrical form, should be discontinued To avoid this difficulty, the length of the cylinder is given beforehand, and this length is marked by two faint scratches upon the lateral surface of one of the strips of glass; the weight of the globule of mercury, which is to form a cylinder of this diameter and of the length required, is then determined by calculation from the known diameter of the wire; lastly, by means of a delicate balance, the globule to be used in the experiment is made exactly of this weight All that then remains to be done, is to extend the little mass until the extremities of the copper wires between which it is included have reached the marks traced upon the glass Lastly, in making a series of experiments, the same mercury may be used several times if the isolated spheres are united into a single mass at the end of each observation However, after a certain number of experiments, the mercury appears to lose its fluidity, and the mass always becomes disunited at some point, in spite of all possible precautions, before it has become extended to the desired length, which phænomena arise from the solid wires imparting a small quantity of copper to the mercury. The latter must then be removed, the plates of glass and the strips cleaned, and a new globule taken The amalgamation of the wires also sometimes requires to be renewed

52 By means of the above apparatus and methods, I have made a series of experiments upon the transformation of the cylinders; but before relating the results, it is requisite for their interpretation that we should examine the phænomenon a little more closely

Let us imagine a liquid cylinder of considerable length in proportion to its diameter, and attached by its extremities to two solid bases, let us suppose that it is effecting its transformation, and let us consider the figure at a period of the phænomenon anterior to the separation of the masses, *i. e.* when this figure is still composed of dilatations alternating with constrictions. As the surfaces of the dilatations project externally from the primitive

cylindrical surface and those of the constrictions on the contrary are internal to this same surface. We can imagine in the figure a series of plane sections perpendicular to the axis and all having a diameter equal to that of the cylinder; these sections will evidently constitute the limits which separate the dilated from the constricted portions, so that each portion whether constricted or dilated will be terminated by two of them; moreover, as the two solid bases are necessarily part of the sections in question, each of these bases should occupy the very extremity of a constricted or dilated portion. This being granted, three hypotheses present themselves in regard to these two portions of the figure, *i.e.* to those which rest respectively upon each of the solid bases. In the first place we may suppose that both of the portions are expanded. In this case, each of the constrictions will transfer the liquid which it loses to the two dilatations immediately adjacent to it; the movements of transport of the liquid will take place in the same manner throughout the whole extent of the figure, and the transformation will take place with perfect regularity, giving rise to isolated spheres exactly equal in diameter, and at equal distances apart. This regularity will not however extend to the two extreme dilatations, for as each of these is terminated on one side by a solid surface, it will only receive liquid from the constriction which is situated on the other side, and will therefore acquire less development than the intermediate dilatations. Under these circumstances, then, after the termination of the phenomenon we ought to find two portions of spheres respectively adherent to two solid bases, each presenting a slightly less diameter than that of the isolated spheres arranged between them.

In the second place, we may admit that the terminal portions of the figure are, one a constriction and the other a dilatation. The liquid lost by the first, not being then able to traverse the solid base, will necessarily all be driven into the adjacent dilatation; so that, as the latter receives all the liquid necessary to its development on one side only, it will receive none from the opposite side; consequently all the liquid lost by the second constriction will flow in the same manner into the second dilatation, and so on up to the last dilatation. The distribution of the movements of transport will therefore still be regular throughout the figure, and the transformation will ensue in a perfectly regular manner. This regularity will evidently extend even to

the two terminal portions, at least so long as the constrictions have not attained their greatest depth, but beyond that point this will not exactly be the case, for independence being then established between the masses, each of the dilatations, excepting that which rests upon the solid base, will enlarge simultaneously on both sides, so as to pass into the condition of the isolated sphere, by appropriating to itself the two adjacent semi-constrictions, whilst the extreme dilatation can only enlarge on one side. Consequently, after the termination of the phænomenon, we should find, at one of the solid bases, a portion of a sphere of but little less diameter than that of the isolated spheres, and at the other base a much smaller portion of a sphere, arising from the semi-constriction which has remained attached to it.

Lastly, in the third place, let us suppose that the terminal portions of the figure were both constrictions, in which case, after the termination of the phænomenon, a portion of a sphere equal to the smallest of the two above would be left to each of the solid bases. In this case, to be more definite, let us start from one of these terminal constrictions, for instance that of the left. All the liquid lost by this first constriction being driven into the contiguous dilatation, and being sufficient for its development, let us admit that all the liquid lost by the second constriction also passes into the second dilatation, and so on, then all the dilatations, excepting the last on the right, will simply acquire their normal development; but the right dilatation, which, like each of the others, receives from that part of the constriction which precedes it the quantity of liquid necessary for its development, receives in addition the same quantity of liquid from that part of the constriction which is applied to the adjacent solid, so that it will be more voluminous than the others. Hence it is evident, in the case in point, that the opposed actions of the two terminal constrictions introduce an excess of liquid into the rest of the figure. Now, whatever other hypothesis may be made respecting the distribution of the movements of transport, it must always happen, either that the excess of volume is simultaneously distributed over all the dilatations, or that it only augments the dimensions of one or two of them; but the former of these suppositions is evidently inadmissible, on account of the complication which it would require in the movements of transport; hence we must admit the second, and then the isolated spheres will not all be equal. Thus this third mode of transformation

ould necessarily of itself induce a cause of irregularity and
oreover it would not allow of a uniform distribution of the
ovements of transport, because there would be opposition in
egard to these movements at least in the terminal constrictions

It may therefore be regarded as very probable that the transfor
ation takes place according to one or the other of the two first
ethods and never according to the third *i e* that things will
e so arranged that the figure which is transformed may have
n its terminal portions either two dilatations or one constric
on and one dilatation, but not two constrictions In the former
ase as we have seen the movement of the liquid of all the con
rictions would ensue on both sides simultaneously and in the
econd this movement would occur in all in one and the same
irection If this is really the natural arrangement of the phe
omenon, we can also understand how it will be preserved
ven when it is disturbed in its regularity by slight extraneous
auses Now this, as we shall see, is confirmed by the experi
ments relating to the mercurial cylinder although the trans
ormation of this cylinder has rarely yielded a perfectly regular
ystem of spheres, I have found in the great majority of the re
ults, either that each of the solid bases was occupied by a mass
ttle less in diameter than the isolated spheres, or that one of
he bases was occupied by a mass of this kind and the other by
much smaller mass

53 For the sake of brevity, let us denominate *divisions* of the
ylinder those portions of the figure each of which furnishes a
phere, whether we consider these portions in the imagination
s in the cylinder itself, before the commencement of the trans
ormation, or whether we take them during the accomplishment
f the phænomenon, *i e* during the modification which they
ndergo in arriving at the spherical form The length of a divi
ion is evidently that distance which, during the transformation,
s comprised between the necks of two adjacent constrictions,
onsequently it is equal to the sum of the lengths of a dilatation
nd two semi constrictions Let us therefore see how the length
n question, *i e* that of a division, may be deduced from the
esult of an experiment Let us suppose the transformation to be
erfectly regular, and let λ be the length of a division, l that of
he cylinder, and n the number of isolated spheres found after
he termination of the phænomenon Each of these spheres
eing furnished by a complete division, and each of the two ter

minal masses by part of a division, the length l will consist of n times λ, plus two fractions of λ. To estimate the values of these fractions, we must recollect that the length of a constriction is exactly or apparently equal to that of a dilatation (§ 46); now, in the first of the two normal cases (§ 52), *i. e.* when the masses remaining adherent to the bases after the termination of the phænomenon are both of the large kind, each of them evidently arises from a dilatation plus half a constriction, therefore three-fourths of a division, the sum of the lengths of the two portions of the cylinder which have furnished these masses is therefore equal to once and a half λ, and we shall have in this case $l=(n+1.5)\lambda$, whence $\lambda=\frac{l}{n+1.5}$. In the second case, *i. e.* when the terminal masses consist of one of the large and the other of the small kind, the latter arises from a semi-constriction, or a fourth of a division, so that the sum of the lengths of the portions of the cylinder corresponding to these two masses is equal to λ, consequently we shall have $\lambda=\frac{l}{n+1}$.

As the respective denominators of these two expressions represent the number of divisions contained in the total length of the cylinder, it follows that this number will always be either simply a whole number, or a whole number and a half. On the other hand, as the phænomenon is governed by determinate laws, we can understand, that for a cylinder of given diameter composed of a given liquid, and placed under given circumstances, there exists a normal length which the divisions tend to assume, and which they would rigorously assume if the total length of the cylinder were infinite. If then it happens that the total length of the cylinder, although limited, is equal to the product of the normal length of the divisions by a whole number, or rather a whole number plus a half, nothing will prevent the divisions from exactly assuming this normal length. If, on the other hand, which is generally the case, the total length of the cylinder fulfills neither of the preceding conditions, we should think that the divisions would assume the nearest possible to the normal length, and then, all other things being equal, the difference will evidently be as much less as the divisions are more numerous, or, in other words, as the cylinder is longer. We should also believe that the transformation would adopt that of the two methods which is best adapted to diminish the difference in

question, and this is also confirmed by experiment, as we shall see presently. Hence although, as I have already stated, the transformation of the cylinder of mercury almost always ensues in one of the two normal methods, the result is rarely very regular: we must therefore admit, that slight accidental disturbing causes in general render the divisions formed in any one experiment unequal in length; but then the expressions of λ obtained above evidently give in each experiment the mean length of these divisions, or in other words, the common length which the divisions would have taken if the transformation had occurred in a perfectly regular manner, giving rise to the same number of isolated spheres and to the same state of the terminal masses.

Lastly, since the third method of transformation presents itself, i. e. since it sometimes happens that each of the bases is occupied by a mass of the small kind, if we would leave out of consideration the particular cause of irregularity inherent in this method (the preceding par.), and find the corresponding expression of λ, it need only be remarked that each of the terminal masses then proceeds from a semi-constriction or the fourth of a division, which will evidently give $\lambda = \frac{l}{n+0.5}$.

51. I shall now relate the results of the experiments. The diameter of the copper wires, consequently of the cylinder, was 1.05 millim. I first gave the cylinder a length of 90 millims., and repeated the experiment ten times, noting after each the number of isolated spheres produced and the state of the masses adherent to the bases; I then calculated for each result the corresponding value of the length of a division, by means of that of the three formulæ of the preceding paragraph which refers to this same result. I afterwards made ten more experiments, giving the cylinder a length of 100 millims., and also calculated the corresponding values of the length of a division. The table contains the results furnished by these cylinders, and the values deduced for the length of a division. I only obtained a perfectly regular result in one case in each series; I have placed an † opposite the corresponding number of isolated spheres.

Length of the cylinder 90 millims			Length of the cylinder 100 millims		
Number of isolated spheres	Masses adherent to the bases	Length of a division	Number of isolated spheres	Masses adherent to the bases	Length of a division
		millims			millims
10	Two large	7·83	11	One large and one small	8·33
*12	Two large	6·67	14	Two large	6·45
12	Two small	7·20	14	Two large	6·45
15	Two large	5·45	14	Two large	6·45
14	Two large	5·81	*14	One large and one small	6·67
11	Two large	7·20	13	One large and one small	7·14
11	Two large	7·20	11	Two large	8·00
12	One large and one small	6·92	14	One large and one small	6·67
13	Two large	6·21	13	Two large	6·90
11	Two large	7·20	10	Two large	8·69

This Table shows, in the first place, that the different values obtained for the length of a division are not so far removed from each other as to prevent our perceiving a constant value, the uniformity of which is only altered by the influence of slight accidental causes. In the second place, out of twenty experiments, it happened once only that the masses adherent to the bases were both of the small kind.

In the third place, both the perfectly regular results have given identically the same value for the length of a division; this value, expressed approximatively to two decimal places, is 6·67 millims., but its exact expression is $6\frac{2}{3}$ millims., for the operation to be effected consists in the case of the first series, in the division of 90 millims. by 13·5, and in the case of the second series, in the division of 100 millims. by 15. As the two lengths given to the cylinder are considerable in proportion to the diameter, and consequently the numbers of division are tolerably large, this value, $6\frac{2}{3}$ millims., ought very nearly, if not exactly, to constitute that of the normal length of the divisions. It is seen, moreover, that to give the divisions this closely approximative or exact value of the normal length, the transformation has chosen, in one case the first, in the other case the second method.

55. Let us pursue our inquiry into the laws of the phænomenon with which we are engaged; we shall soon make an important application of them, and it will then be understood why so extensive a development is given to this part of our work. It might be regarded as evident *à priori* that two cylinders formed of the same liquid and placed in the same circumstances, but differing in diameter, would tend to become divided in the

ame manner, i e that the respective normal lengths of the divisions would be to each other in the proportion of the diameters of these cylinders

In order to verify this law by experiment, I procured some copper wires, the diameter of which was exactly double that of the first, therefore equal to 2 1 millims , and I made with them a new series of ten experiments, giving the cylinder a length of 100 millims This series also furnished me with only a single perfectly regular result, which I have denoted as before by an A placed opposite the corresponding number of isolated spheres The following is the Table relating to this series

N[illegible]	M[illegible]	L[illegible]gth [illegible]
		mm
7	Two small	13 13
6	Two large	13 33
6	One large and one small	11 28
7	One large and one small	12 50
A 6	Two large	13 33
6	Two large	13 33
6	One large and one small	11 8
8	One large and one small	11 11
8	Two small	11 76
6	One large and one small	11 8

By stopping at the second decimal place, we have, as is evident, 13 33 millims for the value of the length of a division corresponding to the perfectly regular result, but as the operation which yields it consists in the division of 100 by 7 5, the value when perfectly expressed is 13⅓ millims This then is very nearly, if not exactly the normal length of the divisions of this new cylinder, now this length 13⅓ millims , is exactly twice the length, 6⅔ millims , which belongs to the divisions of the cylinder of the preceding paragraph, these two lengths are therefore, in fact, in the proportion to each other of the diameters of the two cylinders

As the perfectly regular result of the above Table has given a mass of the larger kind to each base, it follows that to enable the divisions of the cylinder itself to assume their normal length, or the nearest possible length to this, the transformation has necessarily ensued according to the former method whilst in regard to a cylinder the diameter of which is a half less, and the total length of which is the same, 100 millims the transformation ensued according to the second method (§ 51)

Here also, the case in which there are two masses of the small kind to the solid bases is the least frequent, although it occurred twice. Lastly, the different values of the length of a division are more concordant than in the second series relating to the first diameter, and consequently show the tendency towards a constant value better, we also see that the normal length is that which is most frequently reproduced.

56. According to the law which we have just established, when the nature of the liquid and external circumstances do not change, the normal length of the divisions is proportional to the diameter of the cylinder; or in other words, the proportion of the normal length of the divisions to the diameter of the cylinder is constant.

As we have seen, the diameter of the cylinder in paragraph 54 was 1·05 millim., and the normal length of its divisions was very little less than 6·67 millims.; consequently, when the liquid used is mercury and the cylinder rests upon a plate of glass, the value of the constant proportion in question is $\frac{6{\cdot}67}{1{\cdot}05} = 6{\cdot}35$, which approximates closely.

To ascertain whether the nature of the liquid and external circumstances exert any influence upon this proportion, we shall now determine the value of the latter in the case of a cylinder of oil formed in the alcoholic mixture, which may be effected, at least approximatively, with the aid of the result of the experiment in paragraph 47. To simplify the considerations, we shall suppose that the transformation does not commence until the rapidity of transference has entirely ceased. The point of the funnel, on the one hand, and the section by which the imperfect liquid cylinder is in contact with the mass which collects at the bottom of the vessel, on the other hand, may then be regarded as playing the part of the two bases of the figure. Now it is evident that, as regards the second of these bases, the last portion of the figure which is transformed should be a constriction; for if it constituted a dilatation, there would be discontinuity of the curvature at the junction of the respective surfaces of the latter and the large mass, which is inadmissible. But the same reason does not apply to the other base, and experiment shows that in this case a dilatation is formed, because after the termination of the phænomenon, we always find at the point of the funnel a mass comparable to the isolated spheres,

nce in this experiment the transformation ensues according the second method Therefore as the whole length of the ure is about 200 millims and as the transformation constantly lds two isolated spheres, the mean length of the divisions has 53) for its approximative value $\frac{200}{3}$ millims $=66.7$ millims

say the mean length, because, as the diameter of the figure reases slightly from the summit towards the base the divi ns are probably not exactly equal in length It must be ded here, that the transformation ensues under circumstances nich are always identical and consequently in the absence of idental disturbing causes the above quantity ought to repre nt the normal length of the divisions, or the nearest possible ngth to the latter Now I estimate the mean diameter of the ure before the transformation at about 4 millims, we should nsequently have $\frac{66.7}{4}=16.7$ as the approximative value of the

oportion between the normal length of the divisions and the meter of the cylinder This is therefore approximatively the nstant proportion sought in the case of a cylinder of oil formed the alcoholic mixture now this proportion, as is evident is uch greater than that which belongs to the case of a cylinder mercury resting upon a plate of glass

In fact, the length 66.7 millims may differ somewhat mate lly from the normal length for if on the one hand, the ole length of the figure of oil is considerable in regard to its meter, on the other hand, the number of divisions which m there is very small Let us then see for instance, what is least value which the normal length of these divisions may ve We must in the first place remark, that in this case, not hstanding the absence of disturbing causes, the third method transformation is possible, in fact, as the lower constriction adherent to a liquid base, nothing can prevent the oil which loses from traversing this base to reach the large mass, so at in the third method also, the direction of the movements transport may be the same in regard to all the constrictions 52) This granted, as the denominator of the expression which es the length of one division can only vary by half units (53), l as the length which we have found resulted from the divi n of 200 millims by 3, it follows that the length immediately

below would be $\frac{200}{3.5}$ millims. = 57.1 millims., which would correspond to three isolated spheres and a transformation disposed according to the third method. But as matters do not take place in this manner, since there are never more than two isolated spheres formed, and the transformation always ensues according to the second method, we must conclude that the normal length of the divisions approximates more closely to the length found, 66.7 millims., than the length 57.1 millims.; if then the normal length is greater than the first of these two quantities, it must at least be more than their mean, *i. e.* 61.9 millims.; consequently the relation of the normal length of the divisions and the diameter of the cylinder is necessarily greater than $\frac{61.9}{4}$ =15.5; now this latter number considerably exceeds the number 6.35 which corresponds to the mercurial cylinder.

Thus, the proportion of the normal length of the divisions to the diameter of the cylinder varies, sometimes according to the nature of the liquid, sometimes according to external circumstances, at others according to both these elements.

57. But I say that there is a limit below which this proportion cannot descend, and that this is exactly the limit of stability. Let us imagine a liquid cylinder of sufficient length in proportion to its diameter, comprised between two solid bases, and the transformation of which is taking place with perfect regularity. Suppose, for the sake of clearness, that the phænomenon ensues according to the second method, or in other words, that the terminal portions of the figure consist one of a constriction, the other of a dilatation; then, as we have seen (52), the regularity of the transformation will extend to these latter portions, *i. e.* the terminal constriction and the dilatation will be respectively identical with the portions of the same kind of the rest of the figure. Let us then take the figure at that period of the phænomenon at which it still presents constrictions and dilatations, and let us again consider the sections, the diameter of which is equal to that of the cylinder (§ 52). Let us start from the terminal constricted portion; the solid base upon which this rests, and which constitutes the first of the sections in question, will occupy, as we have shown, the origin of the constriction itself; we shall then have a second section at the origin of the

first dilatation; a third at the origin of the second constriction; a fourth at the origin of the second dilatation, and so on: so that all the sections of the even series will occupy the origins of the dilatations, all those of the odd series the origins of the constrictions. The interval comprised between two consecutive sections of the odd series will therefore include a constriction and a dilatation; and as the figure begins with a constriction and terminates with a dilatation, it is clear that its entire length will be divided into a whole number of similar intervals. In consequence of the exact regularity which we have supposed to exist in the transformation, all the intervals in question will be equal in length; and as the moment at which we enter upon the consideration of the figure may be taken arbitrarily from the origin of the phænomenon to the maximum of the depth of the constrictions, it follows that the equality of length of the intervals subsists during the whole of this period, and that, consequently, the sections which terminate these intervals preserve during this period perfectly fixed positions. Besides the parts of the figure respectively contained in each of the intervals undergoing identically and simultaneously the same modifications, the volumes of all these parts remain equal to each other; and as their sum is always equal to the total volume of the liquid, it follows that from the origin of the transformation to the maximum of depth of the constrictions, each of these partial volumes remains invariable, or in other words, no portion of liquid passes from any one interval into the adjacent ones. Thus, at the instant at which we consider the figure, on the one hand, the two sections which terminate any one interval will have preserved their primitive positions and their diameters; and on the other hand, these sections will not have been traversed by any portion of liquid. Matters will then have occurred in each interval in the same manner as if the two sections by which it is terminated had been solid discs. But the transformation cannot ensue between two solid discs, if the proportion of the distance which separates the discs to the diameter of the cylinder is less than the limit of stability; the proportion of the length of our intervals and the diameter of the cylinder cannot then be less than this limit. Now, the length of an interval is evidently equal to that of a division; for the first, in accordance with what we have seen above, is the sum of the lengths of a dilatation and a constriction, and the second is the sum of the lengths of a dilatation and two semi

constrictions (§ 53); the proportion of the length of a division to the diameter of the cylinder cannot then be less than the limit of stability, and we may remark here that this conclusion is equally true, whether the divisions are able or not to assume exactly their normal length.

58. Let us now attempt to ascertain the influence of the nature of the liquid and that of external circumstances, commencing with the latter Our liquid cylinder of mercury, along the whole of the line at which it touches the plate of glass, must contract a slight adherence to this plate, which adherence must more or less impede the transformation. To discover whether this resistance exerted any influence upon the normal length of the divisions, consequently upon the proportion of the latter to the diameter of the cylinder, a simple means presented itself, viz to augment this resistance. To arrive at this result, I arranged the apparatus in such a manner as to remove only one of the strips of glass, so that the liquid figure then remained simultaneously in contact with the plate and the other strip. I again repeated the experiment ten times, using copper wires 1·05 millim. in diameter, and giving the cylinder a length of 100 millims. The following were the results:—

Number of isolated spheres	Masses adherent to the bases	Length of a division
		millims
9	One large and one small	10·00
8	One large and one small	11 11
9	One large and one small	10 00
8	One large and one small	11 11
11	Two small ...	8 69
8	One large and one small	11 11
8	One large and one small ...	11 11
8	Two large	10 53
8	One large and one small ...	11 11
6	Two large.	13 33

It is evident that the different values of the length of a division, with a single exception, are all obviously greater than all those which relate to a cylinder of the same diameter, the surface of which only touches the glass by a single line (§ 54). We must thence conclude, that, all other things being the same, the length of the divisions increases with the external resistance, consequently, under the action of the same resistance this length is necessarily greater than it would be if the convex surface of the cylinder had been perfectly free

In the above series, neither of the results appears to be very regular; but we can readily understand that the mean of the values of the third column will approach the normal length of the divisions. This is moreover confirmed by the tables in §§ 54 and 55: if we take in the former the respective means of the values of the two series, we find for one 6.77 millims., and for the other 7.17 millims., quantities, the first of which is nearly equal to the length 6.67 millims., which may be considered as the normal length, and from which the second does not differ much; and if likewise we take the relative mean in the following table, we find 13.15 millims., a quantity very near the length 13.39 millims., which in the case of the second table may also be regarded as the normal length. Now, the corresponding mean in the above table is 10.81 millims.: consequently, in the case of two lines of contact we have $\frac{10.81}{1.05}=10.29$ as the approximate value of the proportion of the normal length of the divisions to the diameter of the cylinder, whilst in the case of a single line of contact we found only 6.35. Hence the proportion between the normal length of the divisions and the diameter of the cylinder increases by the effect of an external resistance.

59. Let us proceed to the influence of the nature of the liquid. All liquids are more or less viscid, i.e. their molecules do not enjoy perfect mobility with regard to each other. Now this gives rise to an internal resistance, which must also render the transformation less easy; and as external resistances increase the length of the divisions, we can understand that the viscidity will act in the same manner; consequently all other things being equal, the proportion now under consideration will increase with this viscidity. But on the other hand, with equal curvatures, the intensities of the forces which produce the transformation vary with the nature of the liquid, for these intensities depend in the case of each liquid, upon that of the mutual attraction of the molecules. Now it is clear that the viscidity will exert so much more influence upon the length of the divisions as the intensities of the forces in question are less. Thus, leaving external resistances out of the question, the proportions of the normal length of the divisions to the diameter of the cylinder will be greater in proportion to the viscidity of the liquid and the feebleness of the configuring forces.

The intensities of the configuring forces corresponding to different liquids may be compared numerically for the same curvatures. In fact, let us first bear in mind that the pressure corresponding to one element of the superficial layer and reduced to unity of the surface, is expressed by (§ 4),

$$P+\frac{A}{2}\left(\frac{1}{R}+\frac{1}{R'}\right).$$

Now, the value of the part P of this pressure being the same for all the elements of the superficial layer, and the pressures being transmitted throughout the mass, this part P will always be destroyed, whether equilibrium exists in the liquid figure or not; so that the active part of the pressure, that which constitutes the configuring force, will have for its measure simply $\frac{A}{2}\left(\frac{1}{R}+\frac{1}{R'}\right)$. Hence it is evident that when the curvatures are equal, the intensity of the configuring force arising from one element of the superficial layer is proportional to the coefficient A. Now this coefficient is the same as that which enters into the known expression of the elevation or depression of a liquid in a capillary tube, consequently the measures relating to this elevation or depression will give us, in the case of each liquid, the value of the coefficient in question. Hence we may also say that the proportion of the normal length of the divisions to the diameter of the cylinder will be greater as the liquid is more viscid and as the value of A which corresponds to the latter diminishes For instance, oil is much more viscid than mercury; on the other hand, it would be easy to show that the value of A is much less for the first than for the second of these two liquids; lastly, this value must be much diminished in regard to our figure of oil by the presence of the surrounding alcoholic liquid, the mutual attraction of the molecules of the two liquids in contact diminishing the intensities of the pressures (§ 8). This is why the proportion belonging to a cylinder of oil formed in the alcoholic mixture considerably exceeds that belonging to a cylinder of mercury resting upon a plate of glass, notwithstanding the slight external resistance to which the latter is subjected.

60. It follows from this discussion concerning the resistances, that the smallest value which the proportion of the normal length of the divisions to the diameter of the cylinder could be supposed to have, corresponds to that case in which there is simultaneously complete absence of external resistance and of vis-

cidity, and, after the demonstration given in § 57, this least value will be at least equal to the limit of stability. Now as all liquids are more or less viscid, it follows that, even on the hypothesis of the annihilation of all external resistance, the proportion in question will always exceed the limit of stability; and since this is more than 3, this proportion will *a fortiori* be always more than 3.

It is conceivable that the least value considered above, *i. e.* that which the proportion would have in the case of complete absence of resistance both internal as well as external, would be equal to the limit of stability itself, or would very slightly exceed it. In fact, on the one hand, the proportion approximates to this limit as the resistances diminish, and on the other hand, if it exceeds it, the transformation becomes possible (§ 57); hence we see no reason why it should differ sensibly from it if the resistances were absolutely null. The results of our experiments, moreover, tend to confirm this view. First, since the proportion belonging to our cylinder of mercury descends from 10.29 to 6.35, passing from that case in which the cylinder touches the glass at two lines to that where it touches it at a single one only (§ 58), it is clear that if this latter contact itself could be suppressed, which would leave the influence of the viscidity alone remaining, the proportion would become much less than 6.35; and as, on the other hand, it must exceed 3, we might admit that it would at least lie between the latter number and 4, so that it would closely approximate the limit of stability. If then it were possible to exclude the viscidity also, the new decrease which the proportion would then experience, would very probably bring the latter to the very limit in question, or at least to a value differing but exceedingly little from it. Thus, on the one hand, the least value of the proportion that corresponding to the complete absence of resistances, would not differ, or scarcely so, from the limit of stability; and on the other hand, under the influence of viscidity alone, the proportion appertaining to the mercury would be but little removed from this least value. Hence it is evident that the influence of the viscidity of mercury is small, which is moreover explained by the well known feebleness of this same viscidity.

We can now understand in the case of other but very slightly viscid liquids, such as water, alcohol, &c., where the viscidity is not able to form more than a minimum resistance, that this viscidity, notwithstanding the differences in the intensities of the

configuring forces, will also exert only a feeble influence upon the proportion in question. Hence it results that in the absence of all external resistance, the values of this proportion respectively corresponding to the various very slightly viscid liquids, cannot be very far removed from the limit of stability; and as the smallest whole number above this is 4, we may in regard to these liquids adopt this number as representing the mean approximative probable value of the proportion in question.

Starting from this value, calculation gives us the number 1·82 as the proportion of the diameter of the isolated spheres which result from the transformation to the diameter of the cylinder, and the number 2·18 for the proportion between the distance of two adjacent spheres and this diameter.

61. There is another consequence arising from our discussion. For the sake of simplicity, let the diameter of the cylinder be taken as unity. The proportion of the normal length of the division to the diameter will then express this normal length itself, and the proportion constituting the limit of stability will express the length corresponding to this limit. This admitted, let us resume the conclusion at which we arrived at the commencement of the preceding section, which conclusion we shall consequently express here by stating that in the case of all liquids the normal length of the divisions always exceeds the limit of stability; we must recollect in the second place, that the sum of the lengths of one constriction and one dilatation is equal to that of a division (§ 57); and thirdly, at the first moment of the transformation, the length of one constriction is equal to that of a dilatation (§ 46). Now, it follows from all these propositions, that when the transformation of a cylinder begins to take place, the length of a single portion, whether constricted or dilated, is necessarily greater than half the limit of stability; consequently the sum of the lengths of three contiguous portions, for instance two dilatations and the intermediate constriction, is once and a half greater than this same limit. Hence, lastly, if the distance of the solid bases is comprised between once and once and a half the limit of stability, it is impossible for the limit of stability to give rise to three portions, and it will consequently only be able to produce a single dilatation in juxtaposition with a single constriction. This, in fact, as we have seen, always took place in regard to the cylinder in § 46, which was evidently in the above condition, and the want of symmetry in its transformation now becomes explicable.

62. As stated at the conclusion of § 18, we have yet to describe a remarkable fact which always accompanies the end of the phenomenon of the transformation of a liquid cylinder into isolated masses.

In the transformation of large cylinders of oil, whether imperfect or exact (§ 14 to 46), when the constricted part is considerably narrowed, and the separation seems on the point of occurring, the two masses are seen to flow back rapidly towards the rings or the discs; but they leave between them a cylindrical line which still establishes, for a very short time, the continuity of the one with the other (fig. 28): this line then resolves itself into partial masses. It generally divides into three parts, the two extreme ones of which become lost in the two large masses, the intermediate one forming a spherule, some millimetres in diameter, which remains isolated in the middle of the interval which separates the large masses; moreover, in each of the intervals between this spherule and the two large masses another very much smaller spherule is seen, which indicates that the separation of the parts of the above line is also effected by attenuated lines; fig. 29 (Pl. VIII.) represents this ultimate state of the liquid system. The same effects are produced when the resolution of the thin and elongated cylinder of oil of § 17 into spheres occurs; only there is in one or the other of the intervals between the spheres frequently a larger number of spherules; and besides, the formation of the principal line is less easily observed, in consequence of the more rapid progress of the phenomena. Lastly, in the case of our cylinders of mercury, the resolution into spheres takes place also in too short a time to allow of our perceiving the formation of the lines; but we always find, in several of the intervals between the spheres, one or two very minute spherules, whence we may conclude that the separation is effected in the same manner[*].

[*] We cannot avoid recognizing an analogy between the phenomenon of the formation of liquid lines and that of the formation of laminæ. In fact in the experiment in § [illegible], for instance, the plane layer begins to be formed when the two opposite concave surfaces are almost in contact with each other at their summits; and in the resolution of a cylinder into spheres, the formation of the lines commences when all the meridional sections of the figure almost touch each other by the summits of their concave portions.

When treating of the layers we have considered their formation as indicating a kind of tendency towards a particular state of equilibrium which results from the circumstance that in the case of the thin part of the liquid system the ordinary law of pressures is modified. For the analogy between the two orders of phenomena to be complete, it would therefore be necessary that excessively delicate liquid lines should connect thick masses and should

As we are now acquainted with the entire course which the transformation of a liquid cylinder into isolated spheres must take, we can represent it graphically; fig. 30 represents the successive forms through which the liquid figure passes, commencing with the cylinder up to the system of isolated spheres

thus form with these masses a system *in equilibrio*, notwithstanding the incompatibility of this equilibrium with the ordinary law of pressures Now we shall show that this equilibrium is in reality possible, at least theoretically Let us always take as example, the resolution of our unstable cylinder into partial masses When the cylindrical lines form, their diameter is even then very small in comparison with the dimensions of the thick masses, consequently their curvature in the direction perpendicular to the axis is very great in comparison with the curvature of these masses The pressure corresponding to the lines is then originally much greater than those corresponding to the thick masses, whence it follows that the liquid must be driven from the interior of the lines towards these same masses, and that the lines, like the layers, ought to continue diminishing Moreover, their curvatures, and consequently their pressure augmenting in proportion as they become more attenuated, their tendency to diminish in thickness will go on increasing, and consequently if we disregard the instability of the cylindrical form, we see that they must become of an excessive tenuity But I say that the augmentation of the pressure will have a limit, beyond which this pressure will progressively diminish, so that it may become equal to that which corresponds to the thick parts of the liquid system

In fact, without having recourse to theoretical developments, it is readily seen that if the diameter of the line becomes less than that of the sphere of the sensible activity of the molecular attraction, the law of the pressure must become modified, and the diameter continuing to decrease, the pressure must finish by also progressively diminishing, notwithstanding the increase of the curvatures, in consequence of the diminution in the number of attracting molecules Hence the pressure may diminish indefinitely, for it is clear that it would entirely vanish if the diameter of the line became reduced to the thickness of a single molecule Those geometricians who study the theory of capillary action know, that the formulæ of this theory cease to be applicable in the case of very great curvatures, or those the radii of which are comparable to that of molecular attraction Now it follows from what has been stated, that we may always suppose the thinness of the line to be such that the corresponding pressure may be equal to that existing in thick masses which have attained a state of equilibrium. In this case, admitting that the lines are mathematically regular, so that the pressure there may be everywhere rigorously the same, consequently that they have no tendency to resolve themselves into small partial masses, equilibrium will necessarily exist in the system In this case, the form of the thick masses will not be mathematically spherical, for their surface must become slightly raised at the junctures with the lines, by presenting concave curvatures in the meridional direction This form will be the same as that of an isolated mass, traversed diametrically by an excessively minute solid line (§ 10) This system, like those into the composition of which layers enter, is composed of surfaces of a different nature, but this heterogeneity of form becomes possible here, as in the case of the layers, in consequence of the change which the law of pressures undergoes in passing from one to another kind of surface

We can moreover understand, that the equilibrium in question, although possible theoretically, as we have shown, can never be realized, in consequence of the cylindrical form of the lines The same does not apply to the case of the plane layers, for as we shall show in the following series, the plane surfaces are always surfaces of stable equilibrium, whatever may be their extent.

and of spherules. This figure refers to the case of a very slightly viscid liquid, such as water, alcohol, &c., and where the convex surface of the cylinder is perfectly free: consequently, in accordance with the probable conclusion with which § 60 terminates, the proportion of the length of the divisions to the diameter has been taken as equal to 4.

The phænomenon of the formation of lines and their resolution into spherules is not confined to the case of the rupture of the equilibrium of liquid cylinders: it is always manifested when one of our liquid masses, whatever may be its figure, is divided into partial masses: this is the manner in which, for instance, in § 29 of the preceding memoir, the minute masses which were then compared to satellites are formed*. The phænomenon under consideration is also produced when liquids are submitted to the free action of gravity, although it is then less easily shown. For instance, if the rounded end of a glass rod be dipped in æther and then withdrawn carefully in a perpendicular direction, at the instant at which the small quantity of liquid remaining adherent to the rod separates from the mass an extremely minute spherule is seen to roll upon the surface of the latter. Lastly, the phænomenon in question is of the same nature as that which occurs when very viscid bodies are drawn into threads, as glass softened by heat; except that in this case the great viscidity of the substance, and moreover the action of cold, which solidifies the thread formed, maintains the cylindrical form of the latter and allows of its acquiring an indefinite length.

63. To complete the study of the transformation of liquid cylinders into isolated spheres, it still remains for us to discover the law according to which the duration of the phænomenon varies with the diameter of the cylinder, and to endeavour to obtain at least some indications relative to the absolute value of this duration in the case of a cylinder of a given diameter, composed of a given liquid, and placed in given circumstances.

We can understand *à priori*, that when the liquid and the external circumstances are the same, and supposing the length of the cylinder to be always such that the divisions assume exactly their normal length (§ 53), the duration of the phænomenon must increase with the diameter; for the greater this is, the greater is

* It is clear that this mode of formation is entirely foreign to Laplace's cosmogonic hypothesis; therefore we have had no idea of deducing from this little experiment, which only refers to the effects of molecular attraction and not to those of gravitation, any argument in favour of the hypothesis in question, an hypothesis which in other respects we do not adopt.

the mass of each of the divisions, and, on the other hand, the less the curvatures upon which the intensities of the configuring forces depend. It is true that the surface of each of the divisions increases also with the diameter of the cylinder, consequently it is the same with the number of the elementary configuring forces; but this augmentation takes place in a less proportion than that of the mass; this we shall proceed to show more distinctly. Under the above conditions, two cylinders, the diameters of which are different, will become divided in the same manner, *i. e.* the proportion of the length of a division to the diameter will be the same in both parts (§ 55). Now it may be considered as evident that the similitude in figure will be maintained in all the phases of the transformation, this is moreover confirmed by experiment, as we shall soon see. Hence it follows at each homologous instant of the transformations of the two cylinders, the respective surfaces of the divisions will always be to each other as the squares of the diameters of these cylinders, whilst the masses, which evidently remain invariable throughout the entire duration of the phænomena, will always be to each other as the cubes of these diameters. Thus, at each homologous instant of the respective transformations, the extent of the superficial layer of a division, consequently the number of the configuring forces which emanate from each of the elements of this layer, change from one figure to the other only in the proportion of the squares of the primitive diameters of these figures, whilst the mass of a division, all the parts of which mass receive, under the action of the forces in question, the movements constituting the transformation, changes in the proportion of the cubes of these diameters. As regards the intensities of the configuring forces, we must remember first that the measure of that which corresponds to one element of the superficial layer has (§ 59) for its expression $\frac{A}{2}\left(\frac{1}{R}+\frac{1}{R'}\right)$. This granted, if, at an homologous instant in the transformations of the two figures, we take upon one of the divisions of each of the latter any point similarly placed, it is clear from the similitude of these figures, that the principal radii of curvature corresponding to the point taken upon the second, will be to those corresponding to the point taken upon the first, in the proportion of the diameters of the original cylinders, so that if this proportion be n, and the radii relating to the point of the first figure be R and R', those belonging to the point of the second will be nR and nR', whence

it follows that the measure of the two configuring forces corresponding to these points will be respectively $\frac{A}{2}\left(\frac{1}{R}+\frac{1}{R'}\right)$, and $\frac{A}{2}\left(\frac{1}{nR}+\frac{1}{nR'}\right)=\frac{1}{n}\cdot\frac{A}{2}\left(\frac{1}{R}+\frac{1}{R'}\right)$. Thus, in passing from the first to the second figure, the intensities of the elementary configuring forces in all the phases of the transformation will be to each other in the inverse proportion of the diameters of the cylinders.

I have convinced myself, by means of cylinders of mercury 1.05 millim. and 2.1 millims. in diameter (§ 54 and 55), that the duration of the phenomenon increases, in fact, with the diameter; although the transformation of these cylinders is effected very rapidly, yet we have no difficulty in recognising that the duration relating to the greater diameter is greater than that which refers to the least.

61. As regards the law which governs this increase in the duration, it would undoubtedly be almost impossible to arrive at its experimental determination in a direct manner, i. e. by measuring the times which the accomplishment of the phenomenon would require in the case of two cylinders of sufficient length to allow of their being respectively converted into several complete isolated spherules, and of their satisfying the conditions indicated at the commencement of the preceding section. In fact I can hardly see any method of realizing such cylinders without giving them very minute diameters, like those of our cylinders of mercury, and then their duration is too short to allow of our obtaining the proportion with sufficient exactness.

But we may be able to arrive at the same result, but with certain restrictions which we shall mention presently, by means of two short cylinders of oil formed between two discs (§ 46); there is nothing to prevent these cylinders from being obtained of such diameters as to render the exact measure of the durations easy. In the transformation of a cylinder of this kind, only a single constriction and a single dilatation are produced; but as in the transformation of cylinders which are sufficiently long to furnish several complete isolated spheres, the phases through which the constrictions and the dilatations pass are the same for all, we need only consider one constriction and one dilatation. We can understand that the relative dimensions of the two solid systems ought to be such, that the relation between the distance of the

discs and their diameters is the same in both parts, in order that similitude may exist between the two liquid figures, at their origin and at each homologous instant of their transformations.

Before giving an account of the employment of these figures of oil for the determination of the law of the durations, we shall take this opportunity of making several important remarks. We shall only require to make use of the law in question, in that case, which in other respects is the most simple, where the cylinders are formed *in vacuo* or in air, and are free from all external resistance, or, in other words, free upon the whole of their convex surface. Now our short cylinders of oil are formed in the alcoholic liquid, and it might be asked whether this circumstance does not exert some influence upon the proportion of the durations corresponding to a given proportion between the diameters of these cylinders. At first, a greater or less portion of the alcoholic liquid must be displaced by the modifications of the figures, so that the total mass to be moved in a transformation is composed of the mass of oil and this portion of the alcoholic liquid; but it is clear that in virtue of the similitude of the two figures of oil and that of their movements, the quantities of surrounding liquid respectively displaced will be to each other exactly, or at least apparently, as the two masses of oil, so that the relation of the two entire masses will not be altered by this circumstance. Hence it is very probable that this circumstance will no longer exert any influence upon the proportion of the durations, except that the absolute values of these durations will be greater. On the other hand, the mutual attraction of the two liquids in contact diminishes the intensities of the pressures (§ 8), and consequently the configuring forces; but it is easy to see that this diminution does not alter the relation of these intensities in the two figures. For let us imagine that at an homologous instant of the two transformations the alcoholic liquid becomes suddenly replaced by the oil, and let us conceive in the latter the surfaces of the two figures as they were at that instant. The configuring forces will then be completely destroyed by the attraction of the oil outside these surfaces, or, in other words, the external attraction will be at each point equal and opposite to the internal configuring force. If we now replace the alcoholic liquid, the intensities of the external attractions will change, but they will evidently retain the same relations to each other, whence it follows that

those corresponding to two homologous points taken upon both the figures will still be to each other as the configuring forces commencing at these points, so that in fact the respective resultants of the external and internal actions at these two same points will be to each other in the same proportion as the two internal forces alone. Thus the attractions exerted upon the oil by the surrounding alcoholic liquid will certainly diminish the absolute intensities of the configuring forces, but they will not change the relations of these intensities; consequently they may be considered as not exerting any influence upon the durations. But it is clear that this cause will nevertheless greatly increase the absolute values of the latter. For the two reasons which we have explained, the presence of the alcoholic liquid will then increase the absolute values of the two durations to a considerable extent; but we may admit that it will not alter the relation of these values, so that this proportion will be the same whether the phenomenon takes place *in vacuo* or in air. We shall therefore consider the law which we deduce from our experiments upon short cylinders of oil as independent of the presence of the surrounding alcoholic liquid, and this will be found to be supported by the nature of the law itself.

But the exact formation of our short cylinders of oil requires (§ 16) that in these cylinders the proportion between the length and the diameter, or what comes to the same thing, between the sum of the lengths of the constriction and the dilatation and the diameter, exceeds but little the limit of stability. Now in the transformation of cylinders sufficiently long to furnish several spheres, which would be formed *in vacuo* or in the air, and free upon their entire convex surface, and the divisions of which have their normal length, the proportion of the sums of the lengths of one constriction and one dilatation to the diameter, which proportion is the same as that of the length of one division to the diameter, would vary with the nature of the liquid (§ 59), and we are ignorant whether the law of the durations is independent of the value of this proportion. The law which we shall obtain in regard to short cylinders of oil can only therefore be legitimately applied to cylinders of sufficient length to furnish several spheres supposed to be in the above conditions, in the case where these latter cylinders are formed of such a liquid that they would give for the proportion in question a value but little greater than that of the limit of stability.

Now this is the case of mercury (§ 60), and it is also very

probably that of all other very slightly viscid liquids (§ 60). Thus the law given by the short cylinders of oil will be exactly or apparently that which would apply to cylinders of mercury of sufficient length to furnish several spheres, supposing the latter to be produced *in vacuo* or in air, free at the whole of their convex surface, and of such length that the divisions in each of them would assume their normal length. Moreover, the same law would be undoubtedly applicable to cylinders formed of any other very slightly viscid liquid, and supposed to be in the same conditions as the preceding.

The law may possibly be completely general, *i. e.* it may apply to cylinders formed, always under the same circumstances, of any liquid whatever; but our experiments do not furnish us with the elements necessary to decide this question. Lastly, the transformation of our short cylinders presents a peculiarity which entails another restriction. The two final masses into which a cylinder of this kind resolves itself being unequal, the smallest acquires its form of equilibrium considerably before the other, so that the duration of the phænomenon is not the same. Hence we can only determine its duration up to the moment of the rupture of the line; consequently the proportion which we thus obtain for both cylinders will only be that of the durations of two homologous portions of the entire transformations. Moreover, the proportion of these partial durations is exactly that of which we shall have hereafter to make use.

65. I made the experiments in question by employing two systems of discs, the respective dimensions of which were to each other as one to two; in the former, the diameter of the discs was 15 millims., and they were 54 millims. apart; and in the second their diameter was 30 millims., and their distance apart 108 millims. The cylinders formed respectively in these two systems were therefore alike, and, as I have previously stated (§ 63), these two figures exactly maintained their similarity, as far as the eye was capable of judging, in all the phases of their transformations. It sometimes happened that the cylinder, when apparently well formed, was not at all persistent and immediately began to alter; this circumstance being attributable to some slight remaining irregularity in the figure, I immediately re-established the cylindrical form*, and the time was only taken into account when the figure appeared to maintain this form for a few moments. Another anomaly then some-

* See the second note to paragraph 16.

times presented itself, which consisted in the simultaneous formation of two constrictions with an intermediate dilatation; this modification ceased when it had attained a very slightly marked degree, and the figure appeared to remain in the same state for a considerable period*; then one of the constrictions became gradually more marked, whilst the other disappeared, and the transformation afterwards went on in the usual manner. As this peculiarity constituted an exception to the regular course of the phenomenon, I ceased to reckon as soon as it showed itself, and I again re-established the cylindrical form. The estimation of the time was only definitively continued in those cases in which, after some persistence in the cylindrical form, a single constriction only was produced.

I repeated the experiment upon each of the two cylinders twenty times, in order to obtain a mean result. As soon as one transformation was completed, I reunited the two masses to which it had given rise, and again formed the cylinder †, in order to proceed to a new measure of the time.

The number of seconds are given below; each expresses the time which elapsed from the moment of the transformation of the cylinder to that of the rupture of the line. These periods were determined by means of a watch, which beat the ⅕ths of a second.

Cylinder 15 millims. in diameter		Cylinder 30 millims. in diameter	
25.0	36.1	59.6	51.6
26.6	32.0	73.0	68.0
28.0	30.1	57.0	73.6
30.0	21.6	61.0	61.8
21.8	32.6	67.8	53.0
35.2	33.8	60.0	58.0
27.0	33.8	63.6	63.8
30.0	20.2	51.2	60.0
30.1	28.6	61.0	52.6
29.8	32.6	52.6	55.2
Mean 29″.59		Mean 60″.38	

* We shall see in the following series to what this singular modification in the figure is owing.

† This was effected by conducting the large mass towards the small one by means of the ring, of which I spoke in the first note to paragraph 10. But care must be taken to prevent the ring, on separating it from the liquid figure, from

It is evident that the numbers relating to the same diameter do not differ sufficiently from each other to prevent our regarding the proportion of the two means as closely approximating to the true proportion of the durations. Now the proportion of these two means is 2·01, *i. e.* almost exactly equal to that of the two diameters. Moreover, it is evident that in the case of each of the latter the greatest of the numbers obtained must correspond to that case where the cylinder is formed in the most perfect manner; consequently it is probable that the proportion of these two greatest numbers also closely approximates to the true proportion of the durations. Now, these two numbers are, on the one hand 36·4, and on the other 73·6, and their proportion is 2·02, which number differs still less from 2, or from the proportion of the diameters.

We may therefore admit that the durations relating to these two cylinders are to each other as their diameters, whence we deduce this law, that the partial duration of the transformation of a cylinder of the same kind is in proportion to its diameter.

I have said (§ 61) that the law thus obtained would of itself furnish a new motive for believing that it would not change if our short cylinders of oil were produced *in vacuo* or in air. In fact the proportionality to the diameter is the simplest possible law, and, on the other hand, the circumstances under which the phænomenon is produced are less simple in the case of the presence of the alcoholic liquid than they would be in that of its absence; consequently, if the law changed from the first to the second, it would follow that a simplification in the circumstances would on the contrary induce a complication of the law, which is not very probable.

We may therefore, I think, legitimately generalize the above law in accordance with the whole of the remarks made in the preceding section, and deduce the following conclusions:—

1. If we conceive a cylinder of mercury formed *in vacuo* or

carrying away with it any perceptible quantity of oil; for this purpose, instead of making the entire ring adhere to the great mass, I left a small portion of the latter free, and as its action was then insufficient to make the large mass reach the other, I aided it by gently pushing the oil with the extremity of the point of the syringe. On withdrawing the ring after the reunion of the two masses, only a very small spherule of oil separated from it in the alcoholic liquid, which in the next experiment I again united to the rest of the oil by means of the ring itself, as also the largest of the spherules arising from the transformation of the line.

in air, of sufficient length to furnish several spheres, its convex surface being entirely free, and its length such that the divisions assume exactly their normal length, the time which will elapse from the origin of the transformation to the instant of the rupture of the lines will be exactly or apparently proportional to the diameter of this cylinder.

2. The same very probably applies to a cylinder formed of any other very slightly viscid liquid, as water, alcohol, &c., and supposed to exist under the same circumstances.

3. It is possible that this law is completely general, *i. e.* applicable to a cylinder formed always under the same circumstances of any kind of liquid whatever; but our experiments leave us in doubt on this point.

66. Let us now enter upon the consideration of the absolute value of the time in question for a given diameter, the cylinder always being supposed to be produced *in vacuo* or in air, of sufficient length to furnish several spheres, its entire convex surface free, and its length such that its divisions assume their normal length. It is clear that this absolute value must vary according to the nature of the liquid; for it evidently depends upon the density of the latter, upon the intensity of its configuring forces, and lastly upon its viscidity. The experiments which we have detailed give with regard to oil a very remote superior limit; this results first, from the two causes which we have mentioned in § 64, and which are due to the presence of the alcoholic liquid; but with these two causes is connected a third which we must make known. If we imagine a cylinder of oil formed under the above conditions, the sum of the lengths of a constriction and a dilatation will necessarily be much greater in regard to this cylinder than in regard to one of our short cylinders of oil of the same diameter; for in the former this sum is equivalent to the length of a division, and in consequence of the great viscidity of the oil, this latter quantity must greatly exceed the length corresponding to the limit of stability. Now, it may be laid down as a principle, that, all other things being equal, an increase in the sum of the lengths of a constriction and a dilatation tends to render the transformation more rapid, and consequently to abbreviate the total and partial durations of the phænomenon. In fact, for a given diameter, the more the sum in question differs from the length corresponding to the limit of stability, the more the forces which produce the transformation

must act with energy, moreover, as the transformation ceases to take place immediately above the limit of stability, the duration of the phenomenon may then be considered as infinite, whence it follows that when this limit is exceeded, the duration passes from an infinite to a finite value, consequently it must decrease rapidly as it deviates from this limit; lastly, this is also confirmed by the results of observation, as we shall show hereafter. Thus, even if it were possible to form *in vacuo* or in air one of our very short cylinders of oil, consequently to eliminate the two causes of retardation due to the presence of the alcoholic liquid, the duration relative to the cylinder would still exceed that which would relate to a cylinder of oil of the same diameter formed under the conditions we have supposed.

I have said that the principle above established is confirmed by experiment, *i. e.* for the same diameter, the same liquid, and the same external actions, if any exist, when from any cause, the sum of the lengths of a constriction and a dilatation augments, the total and partial durations of the transformation become less. We shall proceed to make this evident. In the experiments of the preceding section, the partial duration relating to the cylinder, the diameter of which was 15 millims., was for instance about 30 seconds, the mean, as shown by the table. Consequently, if we were to form in the alcoholic liquid a similar cylinder of oil, the diameter of which is 4 millims., the partial duration of this, in virtue of the law which we have found, would be nearly equal to $\frac{30'' \times 4}{15} = 8''$. Now, in the nearly cylindrical figure of oil of § 47, which figure is also formed in the alcoholic liquid, the mean diameter was (56) about 4 millims. In this and the preceding figure, the diameter, the liquid and the external actions then are the same; but in the former, the sum of the lengths of the constriction and the dilatation would only be equal to 4 millims. $\times 3{\cdot}6 = 14{\cdot}4$ millims., whilst in the second, this sum, which is equivalent to the length of a division, was (§ 56) approximatively 66·7 millims.; now on observing this latter figure, we recognise easily that the duration of its transformation is much less than 8″. In truth, from the nature of the experiment, it is impossible with regard to this same figure, to fix upon the commencement of the formation of a given constriction or dilatation, so that the complete duration should considerably exceed that which would b deduced by the simple inspection of the phenomenon;

but the latter does not amount to one second, and there cannot be any doubt that it would be going too far to extend the complete duration and *a fortiori*, the portion which terminates at the rupture of the lines, to two seconds. Thus in the case we have just considered, the sum of the length of a constriction and a dilatation becoming about four and a half times greater, the partial duration becomes at least four times less.

67. But if in reckoning the absolute duration in the case of one of our short cylinders of oil, we only obtain with regard to this liquid one upper limit, and this much too high, the cylinder of mercury in § 53 (which cylinder is formed in the air, and the length of which in proportion to the diameter is sufficient for the divisions to have assumed exactly, or very nearly, their normal length), will furnish us, on the contrary, in regard to this latter liquid, with a limit which is probably more approximative and which will be very useful to us.

First, in the case of this cylinder the diameter of which, as we have said, was 2.1 millims., the transformation does not take place in a sufficiently short time for us to estimate with any exactitude the total duration of the phenomenon. I say the total duration, because in so rapid a transformation it would be very difficult to determine the instant at which the rupture of the lines occurs. To approximate as closely as possible to the value of this total duration, I have had recourse to the following process.

By successive trials, I regulated the beats of a metronome in such a manner, that on rapidly raising, at the exact instant at which a beat occurs, the system of glass strips belonging to the apparatus serving to form the cylinder (§ 50 and 51) the succeeding beat appeared to me to coincide with the termination of the transformation; then having satisfied myself several times that this coincidence appeared very exact, I determined the duration of the interval between two beats by counting the oscillations made by the instrument during two minutes, and dividing this time by the number of oscillations. I thus found the value 0″.39 for the interval in question. The total duration of the transformation of our cylinder of mercury may therefore be valued approximatively at 0″.39, or more simply, at 0″.4.

But the entire convex surface of this cylinder is not free, and its contact with the plate of glass must exert an influence upon its duration, both directly as well as by the increase which it

produces in the length of the divisions. Let us examine th influence in question under this double point of view.

The direct action of the contact with the plate is undoubtedl very slight, for as soon as the transformation commences, th liquid must detach itself from the glass at all the intervals be tween the dilated parts, so as only to touch the solid plane by series of very minute surfaces belonging to these dilated parts consequently, if the direct action of the contact of the plate wer alone eliminated, *i. e.* if we could manage so that the entire con vex surface of the cylinder should be free, but that the division formed in it should acquire the same length as before, the tota duration would scarcely be at all diminished.

There still remains the effect of the elongation of the divisions The length of the divisions of our cylinder is equal to 6·35 time the diameter (§ 56), whilst, according to the hypothesis of th complete freedom of the convex surface, this length would ver probably be less than four times the diameter (§ 60); now in virtu of the principle established in the preceding section, this in crease in the length of the divisions necessarily entails a dimi nution in the duration, which diminution is more considerabl in proportion as it occurs in the vicinity of the limit of stability consequently, if it could be managed so that the elongation in question should not exist, the total duration would be very considerably increased. Thus the suppression of the direct action of the contact of the plate would only produce a very slight diminution of the total duration; and the annihilation of the elongation of the divisions would produce, on the other hand, a very considerable increase in this same duration; if then these two influences were simultaneously eliminated, or in other words, if the entire convex surface of our cylinder were free, the total duration of our transformation would be very considerably greater than the direct result of observation.

Now the quantity which we have to consider, is the partial and not the total duration; but under the same circumstances, the first must be but little less than the second, for when the lines are about to break, the masses between which they extend even then approximate to the spherical form; consequently, in accordance with the conclusion obtained above, we must admit that the partial duration under our present consideration, *i. e.* that referring to the case of the complete freedom of the convex

surface of the cylinder, would still exceed considerably the total duration observed *i e* 0″ 1

In starting from this value 0″ 1 as constituting the lower limit corresponding to a diameter of 2 1 millims the law of the proportionality of the partial duration to the diameter will immediately give the lower limit corresponding to any other diameter we shall find, *e g* that for 6 millimetres this limit would be $\frac{0'' 4 \times 10}{2 1} = 1'' 9$, or more simply 2″

If then we imagine a cylinder of mercury a centimetre in diameter, formed *in vacuo* or in air, of sufficient length to furnish several spheres, entirely free at its convex surface, and of such a length that its divisions assume their normal length, the time which will elapse from the origin of the transformation of this cylinder to the instant of the rupture of the lines will considerably exceed two seconds

68 It will not be superfluous to present here a *résumé* of the facts and laws which the experiments we have described have led us to establish with respect to unstable liquid cylinders

1 When a liquid cylinder is formed between two solid bases, if the proportion of its length to its diameter exceeds a certain limit, the exact value of which is comprised between 3 and 3 6, the cylinder constitutes an unstable figure of equilibrium

The exact value in question is that which we denominate *the limit of stability of the cylinder*

2 If the length of the cylinder is considerable in proportion to its diameter it becomes spontaneously converted, by the rupture of equilibrium, into a series of isolated spheres, of equal diameter, equally distant, having their centres upon the right line forming the axis of the cylinder, and in the intervals of which in the direction of this axis, spherules of different diameters are placed except that each of the solid bases retains a portion of a sphere adherent to its surface

3 The course of the phenomenon is as follows —The cylinder at first gradually swells at those portions of its length which are situated at equal distances from each other, whilst it becomes thinner at the intermediate portions, and the length of the dilatations thus formed is equal, or nearly so, to that of the constrictions, these modifications become gradually more marked, ensuing with accelerated rapidity, until the middle of the constrictions has become very thin; then, commencing at the middle,

the liquid rapidly retires in both directions, still however leaving the masses united two and two by an apparently cylindrical line; the latter then experiences the same modifications as the cylinder, except that there are in general only two constrictions formed, which consequently include a dilatation between them; each of these little constrictions becomes in its turn converted into a thinner line, which breaks at two points and gives rise to a very minute isolated spherule, whilst the above dilatation becomes transformed into a larger spherule; lastly, after the rupture of the latter lines, the large masses assume completely the spherical form. All these phænomena occur symmetrically as regards the axis, so that, throughout their duration, the figure is always a figure of revolution.

4. We denominate *divisions* of a liquid cylinder, those portions of the cylinder, each of which must furnish a sphere, whether we conceive these portions to exist in the cylinder itself, before they have begun to be apparent, or whether we take them during the transformation, *i. e.* whilst each of them is becoming modified so as to arrive at the spherical form. The length of a division consequently measures the constant distance which, during the transformation, is included between the necks of two adjacent constrictions.

Moreover, by *normal length of the divisions*, we denominate that which the divisions would assume, if the length of the cylinder to which they belong were infinite.

In the case of a cylinder which is limited by solid bases, the divisions also assume the normal length when the length of the cylinder is equal to the product of this normal length by a whole number, or rather a whole number and a half. Then, if the second factor is a whole number, the transformation becomes disposed in such a manner that during its accomplishment the figure terminates on one side with a constriction, and on the other with a dilatation; if the second factor is composed of a whole number and a half, the figure terminates on each side in a dilatation. When the length of the cylinder fulfils neither of these conditions, the divisions assume that length which approximates the most closely possible to the normal length, and the transformation adopts that of the two above dispositions which is most suitable for the attainment of this end.

5. In the case of a cylinder of a given diameter, the normal length of the divisions varies with the nature of the liquid, and

with certain external circumstances such as the presence of a surrounding liquid, or the contact of the convex surface of the cylinder with a solid plane. In all the subsequent statements we shall take the simplest case, i.e. that of the absence of external circumstances; in other words, we shall always suppose that the cylinders are produced *in vacuo* or in air, and that they are free as regards their entire convex surface.

6. Two cylinders of different diameters, but formed in the same liquid, and the lengths of which are such that the divisions assume in each of them their normal length, become subdivided in the same manner, i.e. the respective normal lengths of the divisions are to each other as the diameters of these cylinders. In other words, when the nature of the liquid does not change, the normal length of the divisions of a cylinder is proportional to the diameter of the latter.

The same consequently applies to the diameter of the isolated spheres into which the normal divisions become converted, and to the length of the intervals which separate these spheres.

7. The proportion of the normal length of the divisions to the diameter of the cylinder always exceeds the limit of stability.

8. This proportion is greater as the liquid is more viscid and as the configuring forces in it are weaker.

9. In the case of a cylinder of mercury, this proportion is much less than 6, and we may admit that it is less than 4.

In the case of a cylinder composed of any other very slightly viscid liquid, such as water, alcohol, &c., it is very probable that the proportion in question is very nearly 4. Hence, in the case of the latter liquids we have for the probable approximative value of the proportion of the diameter of the isolated spheres resulting from the transformation and the diameter of the cylinder, the number 1.82, and for that of the proportion of the distance of two adjacent spheres to this same diameter, the number 2.18.

10. If mercury is the liquid, and the divisions have their normal length, the time which elapses between the origin of the transformation and the instant of the rupture of the lines, is exactly or apparently proportional to the diameter of the cylinder.

This law very probably applies also to each of the other very slightly viscid liquids.

This same law may possibly be general, i.e. it may be appli

cable to all liquids, but our experiments leave this point uncertain.

11. For the same diameter, and when the divisions are always of their normal length, the absolute value of the time in question varies with the nature of the liquid.

12. In the case of mercury, and with a diameter of a centimetre, this absolute value is considerably more than two seconds.

13. When a cylinder is formed between two solid bases sufficiently approximated for the proportion of the normal length of the cylinder to the diameter to be comprised between once and once and a half the limit of stability, the transformation gives only a single constriction and a single dilatation, we then obtain for the final result, only two portions of a sphere which are unequal in volume and curvature, respectively adherent to solid bases, besides interposed spherules.

Application of the properties of liquid cylinders. theory of the constitution of liquid veins emitted from circular apertures.

69. Let us now pass to the application which we have announced of most of the above facts and laws,

Let us consider a liquid vein flowing freely by the action of gravity from a circular orifice perforating the thin wall of the horizontal bottom of a vessel. The molecules of the liquid within the vessel, which flow from all sides towards the orifice, as we know, still retain, immediately after their exit, directions which are oblique to the plane of this orifice; whence there is produced a rapid constriction of the vein, commencing at the orifice and extending as far as a horizontal section, which has been improperly denominated the contracted section. When the molecules have arrived at this section, which is very near the orifice, they all tend to assume a common vertical direction, with a velocity corresponding to the height of the liquid in the vessel, and they are, moreover, urged in this direction by their individual gravity. Hence, supposing the orifice to be circular, the vein commencing at the contracted section tends to form an almost perfect cylinder, of any length; but this form is modified, as we now know, by the acceleration which gravity imparts to the velocity of the liquid, and the diameter of the vein, instead of being everywhere the same, decreases more or less in proportion as we recede from the contracted section.

If the causes which we have detailed were alone in action, the

vein would appear simply more and more attenuated in proportion as it is considered more distant from the contracted section without losing either its limpidity or its continuity. But it results from our experiments that a liquid figure of this kind, the form of which approximates to that of a very elongated cylinder, must become transformed into a series of isolated spheres, the centres of which are arranged upon the axis of the figure. In fact, we have here a liquid submitted to the action of gravity; but it is evident that during the free descent of a liquid, gravity no longer presents any obstacle to the play of the molecular attractions, and that the latter must then exert the same configuring actions upon the mass as if this mass were free from gravity and in a state of rest; this is the manner in which, for instance, drops of rain, during their fall acquire the spherical form. But, for the preceding conclusion to be perfectly rigorous, it would be requisite for all parts of the mass to be actuated by the same velocity, which is not the case with the vein; we can, however, understand that, although this difference may be capable of producing some modifications in the phænomenon, it cannot prevent its production.

The liquid of the vein, therefore, during its movement must necessarily gradually form a series of isolated spheres. But as this liquid is constantly being renewed, the phænomenon of transformation must also continue to be renewed. In the second place, as each portion of the liquid begins to be subjected to the configuring forces as soon as it forms a part of the imperfect cylinder which the vein tends to form, i. e. from the moment at which it passes the contracted section, and subsequently remains during its course under the continued action of these forces, it is evident that each of the *divisions* of the vein must begin to be formed at the contracted section and to descend, conveyed by the movement of translation of the liquid, modifying itself by degrees so as to arrive at the state of an isolated sphere. Hence it follows that at any given instant the divisions of the vein must exist in a more advanced phase of transformation in proportion as they are considered at a greater distance from the contracted section, at least as far as that at which the transformation into spheres is completely effected. From the orifice to the distance where the separation of the masses occurs, the vein must evidently be continuous; but at a greater distance, the

portions of liquid which pass must be isolated from each other

If, then, the movements of the liquid, both that of translation and that of transformation, were sufficiently slow to allow of our following them with our eyes, the vein would appear to be formed of two distinct parts, the one superior and continuous, the other inferior and discontinuous The surface of the former would present a series of dilatations and constrictions, which would descend with the liquid, becoming constantly renewed after passing the contracted section, and which, although very feebly indicated at their origin near this section, would become more and more marked during their movement of transference, the dilatations becoming more prominent and the constrictions narrower these divisions of the vein arriving one after the other, in their greatest development, at the lower extremity of the continuous part, would be seen to become detached from it, and immediately to complete their assumption of the spherical form. Moreover, the separation of each of these masses would necessarily be preceded by the formation of a line which would resolve itself into spherules of different diameters, so that each isolated sphere would be succeeded by similar spherules. The discontinuous part of the vein would then be seen to consist of isolated spheres of the same size and of unequal spherules arranged in the intervals of the former, both of them being conveyed by the movement of translation, and being unceasingly renewed at the extremity of the continuous part.

Now Savart's beautiful investigations* have taught us that this is, in fact, the real constitution of the vein, except that under ordinary circumstances an extraneous cause, which was also pointed out by Savart, more or less modifies the form of the divisions of the continuous part, and alters the sphericity of the isolated masses composing the discontinuous part; but Savart has given the means of excluding this influence, of which we shall speak hereafter.

70. Now as the movement of transference is too rapid to allow of the phænomena which are produced in the vein being recognised by direct observation, certain peculiar appearances ought to be the result of this. We must remember here, that when a liquid cylinder becomes resolved into spheres, the rapidity with which the transformation takes place is accelerated, and conse-

* *Annales de Chimie et de Physique*, Août 1833.

quently at the commencement is extremely small. In consequence then of this original minuteness and of the velocity of the movement of transference in the vein, the effects of the gradual transformation cannot begin to become obvious until a greater or less distance from the contracted section has been attained. Up to this distance the rapid passage of the dilatations and constrictions before the eye cannot give rise to any effect visible to the simple sight; so that this portion of the vein will appear in the form which it would affect if it had no tendency to become divided. Beyond this distance the dilatations will begin to acquire considerable development; the vein will appear to continue enlarging until another distance has been attained, beyond which the diameter will appear constant. Such is, in fact, as the observations of Savart have shown, the form presented to direct observation by a vein withdrawn from the influence of any disturbing cause.

Lastly, we know that from the orifice to the point at which it appears to begin to enlarge, the vein is seen to be limpid, whilst further on it appears more or less turbid; and Savart has perfectly explained these two different aspects, as also some other curious appearances which the troubled part presents, by attributing the limpidity of the upper portion to the slight development of dilatations and constrictions which are propagated in it, and the turbidity, as also the other appearances of the remainder of the vein, to the rapid passage before the eye, at first of the dilatations and constrictions which have become more marked, then lower down, of the isolated spheres and the interposed spherules. We must refer for the details to the memoir quoted above.

71. But we may go further: two consequences spring directly from our explanation of the constitution of the vein. In the first place, as the divisions become transformed during their descent, it is clear that the space traversed by a division during the time it is effecting a given part of its transformation, will be as much greater as it descends more rapidly, or in other words, as the charge, *i.e.* the height of the liquid in the vessel, is more considerable; whence it follows clearly, that, the orifice being the same, the length of the continuous part of the vein must increase with the charge. Now this has been confirmed by Savart's observations. In the second place, since the transformation of a cylinder is slower in proportion to the size of its diameter, the

time which a division of the vein will occupy in effecting any one and the same part of its transformation, will be as much longer as the vein is thicker, whence it follows, that if the rapidity of the flow does not change, the space which the division will traverse during this time will be as much greater as the diameter of the orifice is greater, consequently, for the same charge, the length of the continuous part must increase with the diameter of the orifice, and this is also verified by the observations detailed in the memoir quoted.

With regard to the laws which regulate these variations in the length of the continuous part, Savart deduces from his observations, which were made by employing veins of water, that for the same orifice this length is nearly proportional to the square root of the charge, and that for the same charge it is nearly in proportion to the diameter of the orifice.

Let us now examine whether these two laws also emanate from our explanation

72 Imagine for a moment that gravity ceases to act upon the liquid as soon as the latter passes the contracted section. Then, commencing at this section, the rapidity of translation will simply be that which is due to the charge, and the value of which, as we know, is $\sqrt{2gh}$, g denoting gravity and h the charge. This velocity will be uniform, consequently, if the vein had no tendency to divide, it would remain exactly cylindrical throughout any extent (§ 69). Now all parts of the liquid being actuated by the same velocity of transference, this common movement cannot exert any influence upon the effect of the configuring actions; so that, for instance, the gradual modifications which each of the constrictions undergoes, and the time which it takes in their accomplishment, will be independent of the rapidity of transference.

This admitted, let us consider the infinitely thin section which constitutes the neck of a constriction, at the moment at which it quits the contracted section This section will descend with a constant velocity, and at the same time its diameter will continually diminish until the constriction to which it belongs becomes transformed into a line, and then the section in question will occupy the middle of this line, the line will become disunited, to be converted into spherules. As we have shown above, the time employed in the accomplishment of these phænomena, and during which the liquid section we have con-

sidered has traversed the distance comprised between the contracted section and the place which the middle of the line occupies at the precise instant of rupture is independent of the velocity of transference; consequently if the diameter of the orifice does not change this time will be constant whatever may be the charge. Now when the movement is uniform, the space traversed during a determinate time being in proportion to the velocity, the above distance will be in proportion to $\sqrt{2gh}$, consequently to $\sqrt{h}$. As we shall frequently have occasion to make use of this distance, we shall represent it, for the sake of brevity, by D.

Now it is easily understood that in our vein the length of the continuous part does not differ sensibly from the distance D. In fact, the continuous part terminates at the exact place at which, in each line, the most elevated of the points of rupture of the latter is produced; for at the instant at which the rupture takes place the phases of transformation of all that portion which is above the unit in question are less advanced (§ 69), and therefore it still possesses continuity, whilst all that below this point is necessarily already discontinuous. Thus on the one hand, the continuous part of the vein commences at the orifice and terminates at the place at which the most elevated point of rupture of each filament is produced; and on the other hand, the distance D commences at the contracted section, and terminates at the point corresponding to the middle of the length of each of the lines at the instant of their rupture. The continuous part then takes its origin rather higher up, but also terminates a little above the distance D; the differences in the origins of these two magnitudes and that of their terminations must consequently partially compensate each other, and as these differences are both very minute, the excess of one over the other will *à fortiori* be very small, so that the two magnitudes to which they refer may, as I have stated, be regarded, without any sensible error, as equal to each other.* In virtue of this equality, the length of the continuous part of the vein which we are considering will then apparently follow the same law as the distance D, *i. e.* it will be very nearly proportional to $\sqrt{h}$.

Thus in the imaginary case of uniform velocity of transference, we again recognise the first of the laws given by Savart. Now it is clear that in a real vein the velocity will deviate from

* We shall recur to this point, and shall then establish it more clearly.

uniformity so much the less as the charge is greater, whence we may infer, that for sufficiently great charges, the length of the continuous part of the real vein must still exactly follow this law. We shall, moreover, demonstrate this in a rigorous manner.

73. Let us then take the real case, *i. e.* let us consider a vein submitted to the action of gravity, in which consequently the movement of transference is accelerated. Then the velocity possessed, after any time t, by a horizontal section of the liquid conveyed by the movement of transference, will have for its value $\sqrt{2gh}+gt$, the first term representing the portion of the velocity due to the charge, the second the portion due to the action of gravity upon the vein, and t being reckoned from the moment at which the liquid section passes the contracted section. It must be borne in mind, that in virtue of the acceleration of the velocity, the vein, if it did not become divided, would continue to become indefinitely thinner from above downwards (§ 69).

This admitted, let us imagine that another vein of the same liquid, placed under the imaginary condition of the preceding paragraph, flows off with the same charge from another orifice of the same diameter, in the same time as the true vein in question. Let θ denote the time occupied by this second vein in traversing the distance which we have denoted by D, *i. e.* that which is comprised between the instant at which the liquid section that constitutes the neck of a constriction passes to the contracted section, and the instant of the rupture of the line into which this constriction becomes transformed. In the expression of the velocity relative to the first vein, let $t=\theta$, which gives for this velocity, after the time θ, the value $\sqrt{2gh}+g\theta$; in other words, let us consider the velocity of a liquid section belonging to the true vein, after the time necessary for a section belonging to the imaginary vein to have traversed the distance D. According to what we have seen in the preceding section, if the orifice remains the same, this time is constant whatever the charge may be, so that in the above expression the term $g\theta$ remains invariable when h is made to vary. Hence whatever may be the value of θ, we may suppose the charge h to be sufficiently large for the term $\sqrt{2gh}$ to be very great in proportion to the term $g\theta$, and that the latter consequently may be neglected without any sensible error. In the case of a value of h which will realize this condition, and *à fortiori* in the case of all still greater values, the velocity of a section of the true vein during the time θ may

be regarded as constant and equal to that of a section of the imaginary vein so that throughout the entire space traversed by the first during this time commencing at the contracted section the real vein if it did not become divided, would preserve exactly the same diameter, and might be regarded as identical with the imaginary vein also assumed to be free from divisions

Now it necessarily follows from this approximative identity, that during the time θ the same will apparently occur in like manner in both veins consequently the time θ will be very nearly that which, in the true vein, the liquid section corresponding to the neck of a construction would employ in accomplishing the modifications which we have considered and the space which it will traverse during these modifications may be regarded as equal to the distance D relative to the imaginary vein

Now as the continuous part of the true vein terminates a little below this space, and is consequently included in the same portion of the vein it follows from the above approximative identity, that this continuous part will be exactly equal in length to that of the imaginary vein and therefore commencing with the least of the charges considered above, the lengths of the continuous parts of both veins must be very nearly governed by the same law

We arrive then, lastly, at this conclusion that for the same orifice, and commencing with a low but sufficient charge, the length of the continuous part of the true vein must be in proportion to the square root of the charge

In accordance with the preceding demonstration the low charge in question is that at which the movement of transference of the liquid begins to remain apparently uniform in all that portion of the true vein which is comprised between the contracted section and the point occupied by the middle of each line at the instant of rupture, but as the extremity of the continuous part is very little distant from this point (§ 7 ?), we may neglect the small difference, and say simply that the low charge in question is that which begins to render the movement of transference of the liquid exactly uniform as far as the extremity of the continuous part of the vein

Thus, under the condition of a low charge sufficient to produce this approximative uniformity, which condition is always

realizable, the law indicated by Savart as establishing the relation between the length of the continuous part and the charge, necessarily follows from the properties of liquid cylinders. To discover whether this law is also true when weaker charges are employed, we must start from other considerations; but it is evident so far, that if in the latter case the law is different, it must at least necessarily converge towards the proportionality in question, in proportion as the charge increases.

We must remark here, that in the case of a given liquid, the charge with which the vein begins to exist under the condition which we have determined, must be as much less as the diameter of the orifice is smaller In fact, since, all other things being equal, the transformation of a liquid cylinder occurs with a rapidity proportionate to the diminution in size of the diameter of the cylinder, it follows that the value of θ will diminish with the value of the orifice, and therefore the smaller the latter is, so much the less will the value of h become to allow of the term $g\theta$ in the expression $\sqrt{2gh}+g\theta$, placed at the commencement of this section, being neglected in comparison with the term $\sqrt{2gh}$, and consequently for the vein to exist under the condition in question.

Moreover, as the time θ varies with the nature of the liquid, the same will necessarily apply to the charge under consideration.

74 Let us now investigate the second law, namely that which establishes the approximative proportion of the length of the continuous part of the vein and the diameter of the orifice, when the charge remains the same.

Let us resume, for an instant, the imaginary case of an absolutely uniform movement of transference The vein, leaving its divisions out of consideration, will then constitute a true cylinder commencing at the contracted section (§ 72), which cylinder will be formed in the air, and the entire convex surface of which will be free, moreover, as the movement of transference of the liquid does not exert any influence upon the effect of the configuring forces (§ 72), and as there is no extraneous cause tending to modify the length of the divisions, the latter will necessarily assume their normal length It is evident, therefore, that excepting that the formation of its divisions is not simultaneous (§ 69), our imaginary vein will exist under exactly the same circumstances as the cylinders to which the laws recapitulated in section 68 refer, consequently, if we consider in particular one

of the constrictions of this vein, it must pass through the same forms and accomplish its modifications in the same time as any one of the constrictions which would result from the transformation of a cylinder of the same diameter as the vein, formed of the same liquid, and placed under the conditions in question.

Now in the case of a cylinder of mercury, the time comprised between the origin of the transformation and the instant of the rupture of the lines is, in accordance with one of our laws, exactly or apparently in proportion to the diameter of the cylinder; and it is clear that this law is equally applicable to any one of the constrictions in particular, or even simply to its neck, as to the entire figure. If then, we suppose our imaginary vein to be formed of mercury, the time which the neck of each of its constrictions will occupy in arriving at the instant of the rupture of the line will be exactly or apparently in proportion to the diameter which the vein would possess if the divisions in it were not formed, *i. e.* to that of the contracted section. Now as the cylindrical form of the vein supposed to exist without divisions only begins at the contracted section, it is only from this part that the configuring actions arising from the instability of this cylindrical form commence. We must therefore admit that the liquid section which constitutes the neck of a constriction, does not begin to undergo the modifications which result from the transformation until the instant at which it passes the contracted section; thus the interval under consideration commences at this very instant.

But this interval, comprised between the instant at which the liquid section of which the neck of a constriction is formed, passes the contracted section, and the instant of the rupture of the line into which this constriction becomes converted, is that which we have designated by θ, and in which the liquid section traverses the distance D; in our imaginary vein of mercury, the time θ will therefore be in proportion to the diameter of the contracted section.

Now we know that in a liquid vein, the diameter of the contracted section may be regarded as proportional to that of the orifice when the latter exceeds 6 millims.; and that above this limit the proportionality does not alter very appreciably except when the diameter of the orifice becomes less than a millimetre*.

* In fact, the results obtained by Hachette show (*Ann. de Chim. et de Phys.* t. iii. p. 78) that when the diameter of the orifice is equal to or greater than

Moreover, as this alteration is attributed to the influence which the thickness of the edges of the orifice, although very slight, exerts, it is probable that it may be rendered still less by employing, as Savart has done, orifices expanded outwardly, and which may be shaped so that their edges may be very sharp. Thus, with properly made orifices, we may undoubtedly admit, without appreciable error, that commencing with a diameter equal at most to a millimetre, the diameter of the contracted section is proportional to that of the orifice.

Hence, as the length of the continuous part of our imaginary vein is in proportion to the diameter of the contracted section, it will also be in proportion to the diameter of the orifice, at least starting from a low value of the latter, which must not be much less than a millimetre.

We have only considered the case of mercury; but the principle with which we set out, *i. e.* the proportionality between the partial duration of the transformation of a cylinder and the diameter of the latter, very probably applies also, as we are already aware, to all other very slightly viscid liquids; consequently, in the case of any of the latter liquids, it is very probable that the length of the continuous part of the imaginary vein will also be in proportion to the diameter of the orifice The law may also be true in regard to all liquids; but it may be the case that this general application does not hold good.

If we now pass from the imaginary to the true vein, we have only to suppose that the value of the constant charge is sufficiently considerable to allow of the condition assumed in the preceding section being satisfied throughout the entire extent which we assign to the variations in the diameter of the orifice, so that, for each of the values given to this diameter, the continuous part of the true vein is apparently of the same length as that of the corresponding imaginary vein. The law which regulates this length may then be regarded as the same in both kinds of veins. In accordance with the two remarks terminating the preceding section, it is evident that if the common charge fulfills the condition in question with regard to the greater value assigned to the diameter of the orifice, it will *à fortiori* fulfill it with regard to all the others

10 millims., the mean proportion of the diameter of the contracted section to that of the orifice is 0·78, that in passing from 10 millims to 1 millim, the proportion only increases 0·83, and lastly, when the diameter is equal to 0·55 millim, the proportion becomes 0·88

We are therefore led to the following definitive conclusion:—

In the case of mercury, and very probably also in that of all other very slightly viscid liquids, such as water, if for the same charge increasing values are given to the diameter of the orifice, from a value slightly less than a millimetre to some other determinate value, and if the common charge be sufficiently great, the length of the continuous part of the vein will be proportionate to the diameter of the orifice.

This conclusion is perhaps true in the case of any liquid whatsoever; but the elements for deciding this question are wanting.

Thus, with the restrictions contained in the above enunciation, the second law given by Savart results necessarily from the properties of liquid cylinders; and it is also evident, that if, in the case of a common inconsiderable charge, the law becomes modified, it must approximate towards that of Savart in proportion as the value given to this charge is greater.

75. We said (note to § 72) that we should return to the closely approximative principle of equality between the length of the continuous part of an imaginary vein and the corresponding distance D: in order to establish this principle more clearly, we shall now do this.

Let L be the length of the continuous part, and C the portion common to this length and the distance D; let also s be the interval between the origins of the lengths L and D, *i. e.* the small distance comprised between the orifice and the contracted section; and lastly, let i be the interval between the terminations of these same lengths, *i. e.* the distance comprised between the uppermost point of the rupture of the line and the middle of this line: we shall then have

$$L=C+s,$$
$$D=C+i;$$

consequently

$$L-D=s-i,$$

whence

$$\frac{L}{D}=1+\frac{s-i}{D}. \qquad (1.)$$

Let us now first approximatively value the quantity i in the case of some particular liquid; and let us again take mercury. After what was shown at the commencement of the preceding section, the length of the divisions of an imaginary vein is equal to the normal length of those of a cylinder of the same diameter

and of the same liquid which would be formed in the air, and the entire convex surface of which is free; now in the case of mercury, we know that the proportion of this normal length to the diameter of the cylinder must be less than 4; consequently, in our imaginary vein of mercury, the proportion of the length of the divisions to the diameter of the contracted section will also be less than 4, but in our state of ignorance of the exact value of this proportion, we will first suppose it to be equal to the above number. If we then denote the diameter of the contracted section by k, the diameter of the isolated spheres composing the discontinuous part of the vein will be (§ 60) equal to $1{\cdot}82 \,.\, k$, and the length of the interval between two successive spheres will be $2{\cdot}18 \,.\, k$. But the line into which a constriction is converted is necessarily shorter than this interval; for so long as the rupture does not take place, the two masses united together by the filament must still be slightly elongated; and, moreover, each of them must present a slight elongation of the line, so as to be connected to the latter by concave curvatures. Judging from the comparison of the aspects presented immediately after the rupture of the line, and after the entire completion of the phænomena, by the figure resulting from the transformation of one of our short cylinders of oil (see figs. 28 and 29), I should estimate that for each of the two masses connected by a line, the elongation towards the latter *plus* the slight concave prolongation form about two-tenths of the diameter which these masses acquire after their transition to the state of spheres. To obtain the approximative value of the line belonging to our vein, we must therefore deduct from the interval $2{\cdot}18 \,.\, k$, four-tenths of the diameter $1{\cdot}82 \,.\, k$, which gives $1{\cdot}45 \,.\, k$. On the other hand, if we denote the diameter of the orifice by K, we have (note to the preceding section) very nearly $K = 0{\cdot}8 \,.\, K$; whence it follows that the approximate value of the length of our line is equal to $1{\cdot}45 \times 0{\cdot}8 \,.\, K = 1{\cdot}16$ K. Lastly, the uppermost point of rupture of the line must be very near the upper extremity of the latter; if we suppose it to be at this extremity itself, the quantity z will be half the length of the line, and we shall consequently have

$$z = 0{\cdot}58 \,.\, K.$$

Let us pass to the quantity s. We know that the distance between the orifice and the contracted section, although not entirely independent of the charge, always differs but little from

the semi-diameter of the orifice, so that we should have very nearly $s=0.50$ K, and therefore

$$s-i=0.50\ \text{K}-0.58\ \text{K}=-0.08\ \text{K},$$

evidently a very slight difference.

We have assumed 4 as the value of the proportion of the length of the divisions of our vein to the diameter k; this value is undoubtedly too great; but as the exact value must necessarily exceed the limit of stability, which is itself more than 3, we may admit that this exact value is considerably more than the latter number. Suppose it, however, to be equal to this number 3; calculation will then give for the diameter of the isolated spheres the quantity $1.65\ k$, and for the interval between two consecutive spheres the quantity $1.35\ k$. Completing the operations with these data in the same manner as above, we obtain as the final result

$$s-i=0.23\ \text{K},$$

also a very slight difference.

Now as the true value of the difference $s-i$ must be comprised between the two limits which we have just found, *i.e.* -0.08 K and $+0.23$ K, and as we cannot ascertain either the one or the other, we shall obtain a sufficient approximation to this true value by taking the mean of the two above limits, which gives, lastly,

$$s-i=0.07\ \text{K} \qquad (2.)$$

Let the distance remain D. As this is traversed by a uniform movement during the time θ and with the velocity $\sqrt{2gh}$, we shall first have

$$\text{D}=\theta\sqrt{2gh}.$$

Now as the time θ is equal (preceding section) to the partial duration of the transformation of a cylinder of the same diameter and of the same liquid as the vein, and which would be formed under the conditions of the results summed up in § 68, it follows from one of the latter, that if the diameter of the contracted section of our imaginary vein of mercury were a centimetre, the time θ would be considerably more than 2 seconds; however, in order to place ourselves intentionally under unfavourable circumstances, let us suppose that, in the above case, the time in question were only equal to 2 seconds. But the time θ is proportionate to the diameter of the contracted section (preceding section); if then we take the second as the unit of time and the

centimetre as the unit of length, we shall have for any value k of this diameter

$$\theta = 2k;$$

and if we replace k by its approximative value 0·8 . K, it will become

$$\theta = 1{\cdot}6 \,.\, \mathrm{K};$$

consequently

$$\mathrm{D} = 1{\cdot}6 \,.\, \mathrm{K} \sqrt{2gh}.$$

As we have taken the second and the centimetre as the units of time and length, g will be equal to 980·9; and this value being substituted in the above expression, it will finally become

$$\mathrm{D} = 70{\cdot}87 \,.\, \mathrm{K} \sqrt{h}.$$

From this expression, and that of $s-i$ given by the formula (2.), we deduce

$$\frac{s-i}{\mathrm{D}} = \frac{0{\cdot}07}{70{\cdot}87 \sqrt{h}} = 0{\cdot}001 \frac{1}{\sqrt{h}}.$$

Now according to the equation (1.) this quantity represents the error we commit in supposing $\frac{\mathrm{L}}{\mathrm{D}} = 1$, or $\mathrm{L} = d$; it is evident that this error is independent of the diameter of the orifice, but that it varies with the charge, and that it is less in proportion as the strength of the charge is greater, it is also evident, that for it not to be very small, an extremely small value must be given to the charge, for when the charge is too small, either the flow does not take place, or it ensues drop by drop, in both which cases the nature of the phænomenon is changed, and cannot be referred to the transformation of a cylinder. We shall therefore suppose that the value of the charge is 4 centims. for instance, which is certainly a small value, and which is slightly greater than the least of the values employed by Savart in his experiments. We shall then have

$$\frac{s-i}{\mathrm{D}} = 0{\cdot}0005;$$

and transferring this value to the equation (1.), we shall find

$$\frac{\mathrm{L}}{\mathrm{D}} = 1 + 0{\cdot}0005,$$

or rather

$$\mathrm{L} - \mathrm{D} = 0{\cdot}0005 \,.\, \mathrm{D}.$$

Thus, according to this result, whatever the diameter of the orifice may be with the feeble charge of 4 centims., the length

of the continuous part of an imaginary vein of mercury only exceeds the distance D by a quantity equal to 6 ten thousandths of the latter so that for instance, if the diameter of the orifice were such that the distance D were a metre, the length of the continuous part would only differ from it by half a millimetre and in consequence of the very small value we have attributed to θ even this probably exceeds the true difference Lastly, if we pass from mercury to some other liquid, the difference between L and D, or rather the proportion of this difference to D, would necessarily vary in magnitude and direction with the nature of the liquid but this proportion as we have shown is so small that we may safely admit that it will always be very small in regard to any other liquid

76 Let us now go within the limit commencing with which the real vein may be compared, in its continuous part, to the corresponding imaginary vein (§ 73 and 74) in other words let us suppose the charge to be so inconsiderable, or the diameter of the orifice to be so great that the movement of transference, in the extent of the continuous part of the real vein, is not perfectly uniform The vein will also then tend to become thinner from above downwards, and this diminution in thickness will become visible upon the limpid portion The question of the laws which under these circumstances must regulate the length of the continuous part is very complicated, we shall however attempt to elucidate it to a certain point

Let us consider a division of the vein at the instant at which its upper extremity passes the contracted section The two liquid sections between which the division in question is comprised separate from this position with different velocities for in the short path which the inferior section has traversed its velocity is even slightly augmented by the action of gravity Now it follows from this excess of velocity and the acceleration of the motion, that the two sections will continue to separate from each other more and more in proportion as they descend or, in other words, that the portion of the liquid included between them will gradually become elongated during its motion of transference Consequently if no other cause intervened each of the divisions, conveyed by the accelerated velocity of the liquid, would gradually increase in length up to the instant of the rupture of the line, and would preserve a constant volume during its descent

But there is a cause which acts in an opposite manner upon the divisions. If we imagine the divisions of the continuous part to be suddenly effaced, the small portion of the vein thus modified which replaces, at this instant, any given division, will be smaller in proportion as the division in question is more distant from the contracted section. Consequently we may consider each of the divisions which at a determinate instant are arranged upon the entire length of the continuous part, as arising respectively from the transformation of a different cylinder; and as the minute portion of the vein which replaces, in the above hypothesis, any given division would continue slightly diminishing in thickness from above downwards, we should exactly obtain the diameter of the corresponding cylinder by taking the mean diameter of this portion. Now we know that for any liquid, the normal length of the divisions of a cylinder supposed to be formed in the air, and the entire convex surface of which is free, is in proportion to the diameter of this cylinder; consequently if nothing opposed the action of the configuring forces upon the vein, the proportion of the length of a division to the above mean diameter corresponding to it would be the same for all the divisions, and as this mean diameter diminishes at each division from the top to the bottom of the continuous portion, it follows that the length of the divisions would continue to decrease in the same proportion. If then the cause with which we are engaged were alone in action, each division would gradually diminish in length and volume in proportion as it descended in the continuous portion. But then the divisions starting from the contracted section with the velocity of the liquid, would necessarily follow in their movement of transference a different law. We shall show that this movement would be retarded, so that the liquid, which descends on the contrary with an accelerated velocity, must pass from one division to the other, and that the latter must simply constitute, upon the surface of the vein, a sort of undulation, which would be propagated according to a particular law.

Let us assume the hypothesis of the entirely free action of the configuring forces, and let us commence with the moment at which the section of the surface of the vein which constitutes the neck of a constriction passes to the contracted section. After a brief interval, another superficial section, corresponding to the next neck, will pass in its turn, and

these two sections will include a division between them. After another interval of time equal to the first another division will have passed to the contracted section, but the first will even then be shortened so that its lower neck, in this second interval of time will have traversed a less space than the first. For the same reason, the space traversed in a third interval of time equal to the two others will be still smaller, and so on afterwards. The movement of transference of the necks, and therefore that of the divisions which they include two and two, will then constitute, as I have stated, a retarded movement.

Now the two causes which we have mentioned, and which act concurrently upon the divisions will necessarily combine their effects. Consequently the velocity of transference of the divisions will be intermediate between the accelerated velocity of the liquid and the retarded velocity which would result from the second cause alone; in the second place, the divisions will gradually diminish in volume during their descent along the continuous portion, but according to a less rapid law than would be the case under the isolated action of this second cause; lastly, the length of the divisions will follow a law intermediate between the gradual increase determined by the first cause and the decrease produced by the second.

77. We shall now investigate the manner in which these modifications in the volume, length, and velocity of the divisions, are capable of exerting an influence upon the laws regulating the length of the continuous portion of the vein.

We must first draw attention to the fact, that in our imaginary veins where the movement of transference of the liquid is supposed to be uniform with all charges, the causes producing the above modifications do not exist; consequently the divisions must always descend with the same velocity as the liquid without varying in either volume or length in the course of the continuous part. Moreover we must recollect, that after what has been detailed in §§ 72, 74 and 75, Savart's laws are already satisfied with regard to these veins commencing with very feeble charges, the first law in the case of any liquid whatever, and the second in the case of mercury, very probably also in that of any other very slightly viscid liquid, and perhaps even in that of all liquids.

Let us now return to the true vein of the preceding section, and let us begin by examining the influence exerted by the

diminution of the volume of its divisions. Since a cylinder, supposed to exist under the conditions of our laws and formed of a given liquid, becomes transformed with rapidity proportionate to the smallness of its diameter, it necessarily follows that as the volume of its divisions is smaller, the gradual diminution in the volume of the divisions of the vein tends to render the velocity of their transformation more accelerated than it would be in the imaginary vein of the same liquid if it flowed under the same charge, and from an orifice of the same diameter. Under the isolated influence of this modification of the volume, the time which the portion of the phænomenon corresponding to the course of the continuous portion requires would therefore be shorter, consequently the length of this portion would be less than in the imaginary vein. Now if the charge under consideration were replaced by a charge very nearly sufficient to annihilate the acceleration of the movement of transference of the liquid in the continuous part, this portion of the vein would then be equal in length to that of the corresponding imaginary vein (§ 73); therefore in passing from the first charge to the second, the continuous part of the true vein would augment more than that of the imaginary vein, *i. e.* would consequently augment in greater proportion than that of the square roots of the two charges Thus the gradual diminution in the volume of the divisions tends to render the law regulating the length of the continuous part of the vein, when the charge is made to vary, more rapid than that of Savart.

Let us pass on to what relates to the length of the divisions. As the acceleration of the velocity of the transference of the liquid forms an obstacle to the free shortening of the divisions, the latter must be gradually extended in the direction of their length, in proportion as they descend upon the continuous part. Now this gives rise to an influence exerted in the same direction as the preceding; for in consequence of their less thickness, the constricted portions will yield more readily to this traction than the dilated portions, which will necessarily increase the rapidity with which the former become diminished in thickness, and will therefore tend to produce, in each of them, the formation and rupture of the line sooner than in the corresponding imaginary vein. But the difference of the laws which the divisions and the liquid follow in their respective movements of transference, engenders an influence which acts in a contrary direction to the two

preceding. In virtue of the excess which the velocity of the liquid acquires above that of the divisions, the liquid passes, as we have seen, from one division to the other; so that any one portion traverses successively sometimes the narrower canal of a constriction, sometimes the larger space of a dilatation. But as the liquid thus moves in a conduit the dimensions of which are alternately smaller and larger, its velocity must be greater in the constricted parts, and less in the dilated parts, than if the divisions did not exist; whence this singular consequence results, that the velocity of transference of the liquid, instead of being uniformly accelerated, is subjected, in the course of the continuous part, to a series of particular variations which render it alternately greater and less than that which a solid body falling from a point situated at the elevation of the liquid in the vessel would have. Moreover, the liquid molecules, instead of moving in the direction of lines presenting a very slight curvature, and always in the same direction, as they would do if the divisions were absent, will necessarily describe sinuous lines in their passages from division to division. Now the configuring forces emanating from the superficial layer of the vein, and which produce the divisions, cannot force the molecules of the liquid to undergo these alternate changes of direction and velocity without expending a part of their own action; so that things will go on as if these forces experienced a loss in intensity. If then the influence in question were alone exerted, the transformation would be effected with less rapidity, and therefore the continuous portion would be longer than in the corresponding imaginary vein; whence it follows, that in passing from the charge under consideration to a charge which would establish the approximative uniformity of the movement of transference of the liquid in the continuous portion, the length of this portion of the vein would increase in a less proportion than that of the square roots of the two charges.

With regard to the transference of the divisions separately considered, we are well aware that it must be intermediate between the retarded velocity which would result from the free shortening of these divisions, and the accelerated velocity of the liquid; but it would be difficult to decide *a priori* whether this intermediate velocity preserves any retardation, or whether it presents any acceleration. However, admitting that retardation exists, the latter, tending evidently to diminish the length of the continuous portion, would produce an influence in the same direction as

the above two former; and supposing, on the contrary, that acceleration occurred, this would produce an influence in the same direction as the third.

78. To sum up, then: when the charges are less considerable than those which would render the movement of transference of the liquid perfectly uniform in the continuous part of the vein, two opposite kinds of influences affect the law, according to which the length of this continuous portion varies with the charge, the first tending to make this length increase more rapidly than the square root of the charge, whilst the second, on the contrary, tends to make it increase less rapidly. Now in virtue of their opposition, these two kinds of influences will mutually neutralize each other to a greater or less extent; but in accordance with the diversity of the immediate causes which respectively produce each of these influences, complete neutralization must be regarded as very improbable, which leads us to the former conclusion, that, when the charges are sufficiently weak, the law in question will differ from that of Savart; but it will be impossible to decide *à priori* in what direction.

In the second place, the primary cause of all the influences which we have mentioned being the acceleration of the movement of the liquid, it is clear that the resulting action of those which act in the same direction, considered separately, decreases in proportion to the augmentation of the charge, and may be neglected, *commencing* with the first of the charges under which the movement of the liquid becomes perfectly uniform in the continuous portion. Now what remains of the mutual neutralization of the two resulting opposed actions is necessarily less, and probably considerably so, than each of them in particular; whence we must believe that this excess may be neglected, commencing with a much less charge. We then arrive at this second conclusion, that Savart's first law will undoubtedly begin to be true in the case of a charge which will still leave a very marked acceleration in the movement of transference of the liquid in the continuous portion.

Lastly, this result, in connexion with a principle which we have established at the end of § 73, furnishes us with a third conclusion, viz. that the charge at which the vein begins in reality to satisfy Savart's first law will be less in proportion to the size of the orifice; for it is evident that, in passing from one orifice to the other, this charge must vary in the same manner

as that at which the acceleration of the movement of the liquid may be neglected. But I say further, that the variation in question will very probably take place in a much greater proportion than that of the diameters of the orifices.

For let h' be the charge with which the approximative uniformity of the movement of transference begins in the case of a given orifice and liquid, and θ' the corresponding value of θ. The charge h', as we have seen, should be such that $\sqrt{2gh'}$ may be very considerable in regard to $g\theta'$, or, in other words, that the proportion $\frac{\sqrt{2gh'}}{g\theta'}$ may be very great. Let us now take an orifice of less diameter, and let h'' denote the charge which fulfills in regard to this second orifice the same condition as h' with regard to the former; let also θ'' denote what θ becomes in the case of the new orifice. If we wish, in the movement of the liquid, in the continuous portion of the vein which flows from the latter to have the same degree of uniformity as in the continuous portion of the preceding one, we must evidently make

$$\frac{\sqrt{2gh'}}{g\theta'} = \frac{\sqrt{2gh''}}{g\theta''},$$

which gives

$$\frac{\sqrt{h'}}{\sqrt{h''}} = \frac{\theta'}{\theta''};$$

consequently

$$\frac{h'}{h''} = \frac{\theta'^2}{\theta''^2}.$$

But the time θ, at least in the case of mercury, is proportionate to the diameter of the contracted section, consequently to that of the orifice (§ 71); hence, in the case of this liquid, we may substitute for $\frac{\theta'^2}{\theta''^2}$ that of the squares of the diameters of the two orifices, whence it follows that in passing from any determinate orifice to one which is less, the charge under consideration will decrease as the square of the diameter of the orifice. Now it must be considered as very probable that the weakest charge at which Savart's law begins to be realized will decrease in an analogous manner, *i. e.* in a much greater proportion than that of the diameters. As we have several times stated, we are not aware whether the considerations relative to mercury are applicable or not to all other liquids, but we know at least that they are very probably so to all those the viscosity of which

is very slight; consequently the above conclusion is very probably also true in regard to any of the latter liquids, such for instance as water.

79. Let us provisionally admit the preceding conclusions as perfectly demonstrated, and let us pass to the other law, *i. e.* that which governs the length of the continuous portion when the diameter of the orifice is made to vary. I say, in the first place, that, in the case of mercury, this law will coincide with the second of those of Savart, when we give to the common charge the value at which the vein escaping from the largest of the orifices employed would begin in reality to satisfy the first of these laws. In fact, let us remark first, that with the charge in question, and which we shall denote by h_1, the veins escaping from all the lesser orifices will exist *à fortiori* in the effective conditions of the first law. Consequently, if for a moment we substitute for this charge h_1 a sufficiently considerable charge to render the velocity of the liquid sensibly uniform throughout all the continuous parts, and if we again pass from this second charge to the preceding, the respective lengths of the continuous parts will all decrease in the same proportion, *i. e.* in that of the square roots of the two charges. Now, with the largest of the latter, the lengths in question were to each other as the diameters of the corresponding orifices (§ 74); it will also be the same with the charge h_1, consequently with this charge the second of Savart's laws will be satisfied.

In the second place, I say that with a lower charge than h_1 the same will not hold good. To show this, let h_2 be this new charge; and let us denote by h_3 the charge which plays the same part with regard to the vein escaping from the smallest orifice as that which h_1 plays with regard to that which escapes from the larger one. It must be borne in mind that h_3 is less than h_1, and let us suppose h_2 to be comprised between the two latter. With the charges h_1 and h_2 the vein escaping from the smallest orifice will therefore then still exist under the effective conditions of Savart's first law, whilst as regards the vein which escapes from the larger orifice, these conditions will only commence at h_1, if then we pass from h_1 to h_2, the continuous portion of the first vein will decrease in proportion to the square roots of these two charges, but that of the latter vein will decrease in a different proportion. Now with the charge h_1 these two lengths were to each other as the diameters of the corresponding orifices; with the charge h_2 then they would exist in another proportion;

consequently the second law of Savart would no longer be satisfied, at least as regards the two extreme veins of the series brought into comparison.

The following new conclusions result from all this:—With a sufficiently weak common charge the proportionality of the length of the continuous portion of the mercurial column to the diameter of the orifice does not exist throughout the entire extent assigned to the variations of this diameter, but it begins to manifest itself when that value is given to the common charge at which the vein escaping from the largest of the orifices commences to exist under the effective conditions of Savart's first law.

Respecting these conclusions, we must repeat what we stated with regard to that terminating the preceding section, viz., that they are very probably applicable at least to all very slightly viscid liquids, consequently to water.

Now we shall see that these same conclusions, as also those of the preceding section, are in accordance with the results of Savart's experiments, which results relate to water.

80. Savart has made two series of observations upon veins of water withdrawn from all extraneous influences, one with an orifice 6 millims., the other with an orifice 3 millims. in diameter; the successive charges were the same in both series. The two following tables represent the results obtained, i. e. the lengths of the continuous part corresponding to the successive charges; both the lengths and the charges are expressed in centimetres. I have inserted in each table a third column, containing, in regard to each of the lengths of the continuous part, the proportion of the latter to the square root of the corresponding charge.

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1 [illegible]	107	[illegible]	1 [illegible]	[illegible]	11 [illegible]
12	120	[illegible] 1	12	[illegible]	11 [illegible]
7	113	7 5	27	58	11 2
17	158	21 0	17	78	11 1

Before discussing these tables, we may remark here, that all the lengths of the continuous portions are expressed in whole numbers, which shows that Savart has taken for each of them the nearest approximative whole number in centimetres, disregarding the fraction; hence it follows that the lengths given in these tables cannot in general be perfectly exact.

This being established, let us now begin by examining the table relating to the orifice of 6 millims. It is evident that the proportion of the length of the continuous portion to the square root of the charge diminishes considerably from the first charge to the last; whence it follows, that in the case of a vein of water escaping from an orifice 6 millims. in diameter, if the charge be not made to exceed 47 centims., Savart's first law is far from being satisfied. Thus the first conclusion of § 78 is conformable with experiment. Moreover, the diminution of the proportion determines the direction in which the true law differs from that of Savart, within the limit at which this begins to be sufficiently approximative; it is evident that the length of the continuous portion then augments less rapidly than the square root of the charge. In the second place, as the proportion in question increases, we find that the latter converges towards a certain limit, which must be a little less than 23, *i. e.* the value corresponding to the charge of 47 centims. In fact, whilst the charge receives successive augmentations of 7·5, 15 and 20 centims., the proportion diminishes successively by 14, 8·9 and 4·5 units, and the latter difference is still tolerably slight in regard to the value of the latter proportion, whence we may presume, that if the charge were still further increased, the further diminution of the proportion would be very small, and that a sensibly constant limit would soon be attained, at which limit Savart's first law would be satisfied.

Let us now find the proportion of the velocity of transference of the liquid at the extremity of the continuous part to that at the contracted section, in the case of the vein escaping under a charge of 47 centims. We shall disregard here the small alternate variations which have been treated of in § 77, and shall therefore consider the velocity of transference of a horizontal section of the liquid of the vein as being also that which this section would have if it had fallen freely and in a state of isolation from the height of the level of the liquid in the vessel. Then, on neglecting the small interval comprised between the orifice and the contracted section, we shall have for the velocity in question, at any distance l of this section, the value $\sqrt{2g.(h+l)}$; if then l denotes the length of the continuous portion, the proportion of the velocity at the end of this length to that at the contracted section will be expressed generally by $\frac{\sqrt{2g(h+l)}}{\sqrt{2gh}}$, or more simply by $\sqrt{\frac{h+l}{h}}$. On now substituting,

in this expression for h and l the values relative to the vein in question, *i. e.* 47 and 158, we find for the relation between the extreme velocities the value 2.1. Thus, although under a charge of 47 centims., the vein escaping from an orifice of 6 millims. may probably nearly exist under the effective conditions of Savart's first law, the velocity at the end of the continuous portion is even more than double the velocity at the contracted section, so that the movement of transference of the liquid is still more considerably accelerated. The second conclusion of § 78 therefore appears so far to agree, like the first, with the results of experiment.

Let us pass to the table relating to the orifice of 3 millims. Here it is evident that the proportion of the length of the continuous portion to the square root of the charge is very nearly the same for all the charges; whence it follows, that with this orifice the vein already begins to come within the effective conditions of Savart's first law under a charge of 1.5 centims. But, according to what we have stated, the orifice being 6 millims., the vein does not satisfy these conditions except under a charge at least equal to 17 centims.; the charge at which Savart's first law begins to be realized, then, augments and diminishes with the diameter of the orifice, and much more rapidly than this diameter. Now this is the substance of the conclusion of § 78.

Lastly, if in the general expression of the relation of the extreme velocities found above, we replace h and l by the values 1.5 and 2.1 relative to the first vein of the table under consideration, we shall find for this relation the value 2.5, which shows that with the charge 1.5, under which the vein is already placed in the effective conditions of Savart's law, the velocity of transference of the liquid is still very notably accelerated. No doubt can therefore remain of the legitimacy of the second conclusion of § 78.

Let us now calculate, for each of the four charges, the proportion of the lengths of the continuous parts corresponding respectively to the two orifices; we shall thus form the following table:—

Charges.	[illegible]
4.5	1.40
12	3.21
27	2.40
17	2.03

This table shows, that for charges below 47 centims., the relation between the respective lengths of the continuous portions of two veins of water escaping, one from an orifice 6 millims. in diameter, and the other from an orifice of half this diameter, is far from being the same as those of the diameters; whence it follows, that, under these charges, Savart's second law is not satisfied. But it is evident, at the same time, that this relation converges towards that of the diameters in proportion as the charge is augmented, and that, under the charge of 47 centims., it nearly attains it; now according to what we have seen above, under this same charge of 47 centims., the vein escaping from the larger of the two orifices very probably nearly attains the effective conditions of Savart's first law. The conclusions of the preceding section appear then to agree, as those of § 78, with the results of observation. We shall now however see this agreement confirmed by the results obtained with veins of water when not withdrawn from extraneous influences.

81. These extraneous influences, which consist of certain more or less regular vibratory movements transmitted to the veins, do not appear to alter the laws under consideration considered generally; but they produce a curtailment of the continuous portions, and thus produce the same effect as a diminution of the diameters of the orifices, so that under their influence Savart's laws begin to be realized with weaker charges.

I have just stated that the complete laws which govern the continuous portion do not appear to be changed by the extraneous influences in question; this will be readily seen, when for each of the series made by Savart under the influence of these actions, in which series the orifices, the charges, and the liquid are the same as before, we construct a table of the proportions of the length of the continuous part and the square root of the charge. Notwithstanding the slight differences arising on the one hand from the irregularities inherent to the extraneous influences, and on the other hand from Savart always having given the lengths in whole numbers, we shall see, that with an orifice of 6 millims. the proportion still begins to diminish, and converge towards a certain limit; only here the limit is less, for the reason I have given above, and the limit appears to be attained under a less charge than 47 centims.; 2nd, that with an orifice of 3 millims. the proportion is perfectly constant.

Hence the series in question may also serve for the discussion

of the laws which govern the length of the continuous part I shall limit myself here to the production of two of these series they consist of those which Savart adopted as his type, and from which he deduced his laws The following are the tables containing them —

D t f tl fl 6 lli			D t f th fl 8 illi		
Cl g	Le gtl f tl t l t	P tl l t q t f tl l g	Cl ge	L gtl f tl tl p t	P tl p tl q t t tl h g
4 5	10	18 9	4 5	16	7 5
12	60	17 0	12	25	7 2
27	82	15 8	27	41	7 9
47	112	16 3	47	5	8 0

and the first shows, that with an orifice of 6 millims, the proportion of the length of the continuous portion to the square root of the charge appears to have attained its limit even with a charge of 27 centims the slight increase manifested in the case of the succeeding charge is undoubtedly due to the causes of irregularity which I have mentioned

Let us further calculate, for these two series, the proportions of the lengths corresponding respectively to the two orifices, which gives us the following table —

Cl g	P l t
4 5	2 50
12	2 36
27	2 00
47	2 04

It is therefore also under the charge of 27 centims that the proportion of the lengths of the continuous portions attains that of the diameters of the orifices, which completes the establishment of the conformity of the conclusions of § 79 to the results of observation

Lastly with an orifice of 3 millims, Savart has made a series of observations corresponding to four larger charges than the preceding and the proportion of the length of the continuous portion to the square root of the charge still appeared perfectly constant, the first of these new charges was 51 and the last 459 centims

82 Thus, as we have been taught by Savart's investigations, the vein gives rise to the production of a continuous sound,

principally arising from the periodical shock of the isolated masses of which the discontinuous portion is composed against the body upon which they fall, and this sound may be made to acquire great intensity by receiving the discontinuous portion upon a tense membrane. On comparing the sounds thus produced by veins of water under different charges and with orifices of different diameters, Savart found that, for the same orifice, the number of vibrations made in a given time is proportionate to the square root of the charge, and that for the same charge, this number is in inverse proportion to the diameter of the orifice. We shall now see that these two laws also result from our principles.

Let us again have recourse to imaginary veins. In these the length of the divisions is equal, as we have seen (§ 74), to the normal length of those of a cylinder of the same liquid, formed under the conditions of our laws, and having for its diameter that of the contracted section of the vein; thus this length depends only upon the diameter of the orifice and the nature of the liquid, and does not vary with the velocity of the flow. Now it follows from this, that for the same liquid and the same orifice, the number of divisions which pass in a given time to the contracted section is in proportion to this velocity, *i. e.* to $\sqrt{2gh}$, consequently to $\sqrt{h}$. But each of these divisions furnishes lower down an isolated mass, and each of these subsequently strikes the membrane, the number of impulses produced in a given time is equal then to that of the divisions which pass in the same time to the contracted section, and is consequently proportionate to the square root of the charge

In the second place, as the normal length of the divisions of a cylinder, supposed to exist under the conditions of our laws and composed of a given liquid, is proportionate to the diameter of this cylinder, it follows, that for any liquid, the length of the divisions of the imaginary vein is proportionate to the diameter of the contracted section, and therefore exactly proportionate to that of the orifice. Now for a given velocity of escape, the number of divisions which pass in a given time to the contracted section is evidently in inverse ratio to the length of these divisions; if then the liquid remains the same, this number is exactly in inverse ratio to the diameter of the orifice.

Thus the two laws which, according to Savart, regulate the sounds produced by the veins, would necessarily be satisfied

with regard to our imaginary veins Now I say that the sound produced by a true vein will not differ from that which the corresponding imaginary vein would produce, if the charge is sufficient relatively to the diameter of the orifice for the velocity of transference of the liquid to augment very slightly from the contracted section to a distance equal to the length of the divisions of the imaginary vein Then, in fact, within this extent the two causes which tend to modify the length of the divisions (§ 76), *i e* the acceleration of the velocity of the liquid and the resulting diminution in the diameter of the vein, will both be very small and as they act in opposite directions, their resulting action will be insensible, so that the divisions will freely acquire at their origin the length corresponding to that of the corresponding imaginary vein, now it is clear that in this case the number of divisions which will pass in a given time to the contracted section will be the same in the real and the imaginary vein, consequently the sounds produced by both the veins will also be identical

But in confining ourselves to very slightly viscid liquids, as water, we know that the relation between the normal length of the divisions of a cylinder imagined to exist under the conditions of our laws and the diameter of this cylinder, must very probably differ but little from 4, consequently the same applies to the relation between the length of the divisions of an imaginary vein formed of one of these liquids and the diameter of the contracted section of this vein If, then, in a true vein formed of one of these liquids, the increase in the velocity of transference is very slight at a distance from the contracted section equal to 4 times the diameter of this section, the condition laid down above will very probably be satisfied, however, to avoid any chance of being deceived, we will take, for instance, 6 times this diameter

It is moreover clear, that if the condition, thus rendered precise, is fulfilled with regard to a given charge and orifice, it will be so *à fortiori* for the same orifice and greater charges, and for the same charge and smaller orifices We arrive then at the following conclusions —

1 When a series of veins formed of a very slightly viscid liquid, flow successively from the same orifice and under different charges, if the least of them is sufficient for the velocity of transference of the liquid to augment very slightly, as far as a distance

from the contracted section equal to about 6 times the diameter of this section, the number of vibrations corresponding respectively to the sounds produced by each of the veins of the series will necessarily satisfy the first of the two laws discovered by Savart.

2. When a series of veins, formed of a very slightly viscid liquid, escapes under a common charge and from orifices of different diameters, if the common charge is sufficient for the same condition to be fulfilled with regard to the vein which escapes from the larger orifice, the number of vibrations corresponding respectively to the sounds produced by each of the veins of the series will necessarily satisfy the second law. It now remains for us to show that the above condition was satisfied in the experiments from which Savart deduced the two laws under consideration

In the series relating to the first of these laws, the diameter of the common orifice was 3 millims., and the smallest charge was 51 centims.; and in the series which refers to the second law, the value of the common charge was the same, 51 centims., and the diameter of the largest orifice was 6 millims. For our condition to be fulfilled with regard to both series, it was therefore evidently sufficient that it was so in the vein which escaped under the charge of 51 centims., and from the orifice the diameter of which was 6 millims. Now on multiplying this diameter by 0·8, we obtain for the approximative value of that of the contracted section of the vein in question 4·8 millims., and 6 times the latter quantity gives us 28·8 millims., or nearly 3 centims. Now if in the expression $\sqrt{\frac{h+l}{h}}$, which gives the general value of the relative proportions of the velocities of transference at a distance l from the contracted section and at this section (§ 80), we make $h=51$ and $l=3$, we obtain for this proportion the value 1·03; whence it is evident, that from the contracted section to a distance equal to about 6 times the diameter of this section, the velocity of transference of the liquid of the vein in question only increased 3 centims. more than its original value.

83. Let us imagine a vein of water, and let us call a division considered immediately after its passage to the contracted section, *i. e.* at the instant at which its upper extremity passes this section, the nascent division. It follows from what we have detailed in the preceding section, starting with a sufficient charge,

that the proportion of the length of the nascent divisions of the vein in question to the diameter of the contracted section will assume a constant value, i.e. independent of the charge, and that this value will very probably differ but little from 1.

Now the results obtained by Savart in the experiments relative to the laws which we have just discussed, allow us, as we shall see presently, to verify the consequences of our principles.

The two opposite causes which tend to modify the length of the divisions, are also those which exert an influence upon the velocity of transference, or, more precisely, upon the velocity of the transference of the necks which terminate them (§ 76). Now in the case under consideration, these same causes both remaining very small throughout the extent corresponding to a nascent division, their resulting action upon the velocity of transference of the necks will be insensible throughout this extent; consequently the velocity with which a neck descends may be regarded as exactly uniform and equal to the velocity of the flow $\sqrt{2gh}$, from the contracted section to a distance equal to the length of a nascent division.

If, then, for an orifice of a given diameter, λ denotes the length of a nascent division, and t the time occupied by a neck to traverse it, we shall have

$$\lambda = t\sqrt{2gh}.$$

Moreover, let n represent the number of divisions which pass to the contracted section in a second of time; as the time t evidently measures the duration of the passage of one of them, we shall have, taking the second as the unit of time, $t=\frac{1}{n}$, and therefore

$$\lambda=\frac{1}{n}\sqrt{2gh}.$$

Lastly, let k denote the diameter of the contracted section corresponding to the same orifice; to represent the proportion of the length of the nascent division to this diameter, we shall have the formula

$$\frac{\lambda}{k}=\frac{1}{kn}\sqrt{2gh}. \qquad (a)$$

Now to obtain, by means of this formula, the numerical value of the proportion $\frac{\lambda}{k}$ relative to a determined charge and orifice, we have only to ascertain by experiment the number of vibra-

tions per second corresponding to this charge and this orifice; for then the value of h will be given, that of k may be deduced from the diameter of the orifice employed, we shall find that of n by taking (see preceding section) half the number of vibrations found, and lastly, that of g is known. It is unnecessary to remark, that the values of h, k, and g must be reduced to the same unit of length. Now Savart's observations relative to the first law, give us, for an orifice of 3 millims., the number of vibrations per second corresponding respectively to four different charges; we can calculate then, for each of these observations, the value of the proportion $\frac{\lambda}{k}$.

The following table contains these numbers, with the charges to which they refer. The latter are expressed in centimetres:—

Diameter of the orifice, 3 millims	
Charges	Number of vibrations
51	600
102	853
153	1024
459	1843

We may conclude, from the results detailed in the note to § 74, that when the diameter of the orifice amounts to 3 millims., that of the contracted section is almost exactly eight-tenths of this quantity, consequently, if we retain the centimetre as the unit of length, which gives 0·3 for the value of the diameter of the orifice in question, we shall have

$$k=0{\cdot}3\times 0{\cdot}8=0{\cdot}24.$$

Lastly, the numbers of vibrations, and therefore the values of n, supposing the second taken as the unit of time, and the values of h and k being reduced to the centimetre as the unit of length, we must make $g=980{\cdot}9$.

Substituting in the formula (a) these values of k and g, as also those of h taken from the above table, and those of n obtained by taking the respective halves of the numbers of vibrations contained in the same table, we shall find, for the proportion $\frac{\lambda}{k}$, the four following numbers:—

4·39
4·37
4·46
4·29

and we see that, in fact, these numbers closely approximate to each other, and very nearly amount to 4. The mean of these numbers, *i. e.* 4.38, gives us then very nearly the constant value which, commencing with a suitable charge, the proportion of the length of the nascent divisions of a vein of water to the diameter of the contracted section of this vein assumes.

This is also evidently the value of the proportion of the length of all the divisions of the continuous portion of a vein of water to the diameter of the contracted section, when the charges are sufficiently considerable for the movement of translation of the liquid to be perfectly uniform throughout the whole extent of this continuous portion. In experimentally determining, in the case of any other liquid, the number of vibrations corresponding to a given charge and orifice, the value of $\frac{\lambda}{k}$ referring to this liquid is also obtained by means of the formula (a). If we confine ourselves to liquids the viscosity of which is very slight, the values would very probably be found to differ but little from the preceding; and it may consequently be considered, that, with the same charge and the same orifice, the sounds produced by the veins formed respectively of these various liquids are very nearly of the same pitch; but the case would undoubtedly be different, at least in general, if we passed to liquids of considerable viscosity.

Savart says, that the nature of the liquid appears to exert no influence upon the number of vibrations corresponding to a given charge and orifice, but he does not point out what the liquids were which he compared in this respect. From what we have stated, it may be presumed that these liquids were some of those the viscosity of which is very slight.

81. Since the partial duration of the transformation of a cylinder may evidently be taken into account, as we have already remarked, by considering only one of the constrictions of the figure, or simply the neck of the latter, and, on the other hand, as this duration varies, for the same diameter, with the nature of the liquid, it follows that in the vein the time comprised between the instant at which the superficial section which constitutes the neck of a constriction passes to the contracted section, and the instant of the rupture of the line into which this constriction is converted, will also vary, all other things being equal, with the nature of the liquid. Now it necessarily follows

from this, that for the same charge and the same orifice the length of the continuous part of the vein will vary according to the nature of the liquid, and this conclusion is also in conformity with the results of experiment. In fact, as is well known, Savart has measured the continuous portion of four veins flowing under identical circumstances, and formed respectively of sulphuric æther, alcohol, water and a solution of caustic ammonia, and he found the following lengths:—

Æther	90
Alcohol	85
Water	70
Ammonia	46

85 Hitherto we have only entered upon the consideration of veins projected vertically from above downwards. Let us now consider veins projected in other than vertical directions. These are incurved by the action of gravity, and cannot therefore be any further compared to cylinders; but we must remark, that the phænomenon of the conversion into isolated spheres is not the result of a property belonging exclusively to the cylindrical form; it appears that this phænomenon must be produced in the case of every liquid figure, one dimension of which is considerable with regard to the two others; we have, in fact, seen the liquid ring formed in the experiment described in § 19 become converted into a series of small isolated masses, which would constitute so many spheres if their form were not slightly modified by the action of the metallic wire which traverses them. We can understand, then, that in curved veins divisions passing gradually to the state of isolated spheres ought also to be produced; consequently the constitution of veins projected either horizontally or obliquely must be analogous to that of veins projected vertically from above downwards, which conclusion agrees, in fact, with Savart's observations.

This analogy of constitution must evidently extend to the ascending portion of the veins projected vertically from below upwards; only in the case of the latter veins the phænomena are disturbed by the liquid which is thrown back.

86 The properties of those liquid figures, one dimension of which is considerable with regard to the two others, and particularly of cylinders, furnishes then the complete explanation of the constitution of liquid veins projected from circular orifices,

and accounts for all the details and all the laws of the phenomenon, at least so long as the modifications produced in it by extraneous causes, i.e. by the vibratory movements transmitted to the liquid, are excluded.

As regards the mode of action of these vibratory movements, it is evident that the properties of the liquid cylinders cannot make us acquainted with them. These movements constitute a totally different cause from the configuring forces, consequently one which is foreign to the general object of our treatise; however, to avoid leaving a deficiency in the theory, we will also examine, relying upon other considerations, the manner in which the vibratory movements act upon the vein, and we shall thus arrive at the complete explanation of the modifications which result from it, and the constitution of the latter; but we shall reserve this subject for the following series.

The influence exerted by the vibratory movements communicated to the liquid, led Savart to regard the constitution of the vein as being itself the result of certain vibratory movements inherent in the phenomenon of the flow. From this assumption, Savart has endeavoured to explain how the kind of disturbance occasioned in the mass of the liquid by the emission of the latter, might in reality give rise to vibration, and he has shown that the existence of the latter would entail the alternate formation of dilatations and constrictions in the vein. It has been shown, in the exposition of our theory, that the constitution of the vein is explained in a necessary manner by facts, quite independently of all hypothesis. We may then, I think, dispense with a detailed discussion of the ingenious ideas which we have mentioned, ideas for the complete comprehension of which we must refer to Savart's memoir itself. We shall merely remark that it is difficult to admit the kind of disturbance supposed by Savart to occur, except during the first moments after the orifice is opened; moreover, that it is not very evident how the vibrations in question, after having traced upon the surface of the vein a nascent division, would produce the further development of the latter, so as to make it pass gradually, during its descent, to the state of an isolated mass; lastly, that to remove these difficulties, we should again be obliged to have recourse to additional hypotheses, to arrive at the laws governing the length of the continuous portion, and those to which the numbers of vibrations corresponding to the

sounds produced by the shock of the disturbed portion are subject. However, it is by borrowing one of Savart's ideas, which becomes applicable when, from some external cause, vibrations are really excited in the liquid, that we find the elements requisite for entering upon the latter part of the theory.

87. In the next series, after having concluded what relates to the vein, we shall return to the liquid masses free from gravity; and we shall study the other figures of revolution besides the sphere and the cylinder, as also those figures which do not belong to this class, for which the equation of equilibrium may be interpreted in a rigorous manner.

ARTICLE XIX.

On the Determination of the Intensity of Magnetic and Diamagnetic Forces. By Professor PLÜCKER *of Bonn*.†

[From Poggendorff's *Annalen* for July 1848.]

§ 1. *General Considerations.*

1. BY the intensity of the magnetism of a substance, I understand the intensity of that force with which this substance, when near one of the poles, is attracted by it in consequence of magnetic induction. We must first establish some point of view, in which we may compare this magnetism, which is specifically dependent upon the nature of the substance, as it occurs in the case of different substances. In so doing, by commencing with any one substance, its intensities may then be expressed by absolute numbers, as has been done for instance with specific heat.

2. If we take a watch-glass and grind its margin to fit a flat glass plate so that the latter accurately closes it, we may fill it with a liquid above the margin, and then skim this off with the flat glass forming the cover. We are then certain that any inclosed liquid having *the same form* occupies exactly the same volume. If we fill the watch-glass with two different fluids successively, and if this is then equally attracted by the pole of a magnet, to which it is in each case exposed in a similar manner, both fluids are *in the same degree* magnetic. If the attraction of the two fluids is at all different, we consider the intensity of their magnetism as *proportional to this attraction*. The proof of this will be theoretically and experimentally given in the next paragraphs.

3. If for instance the two fluids are solutions of different iron compounds, and if equal volumes of each contain the same number of atoms of iron, in both cases these atoms are distributed in the same manner within the watch-glass, and have exactly the same position as regards the pole of the magnet; the proportion of the attraction of the whole volumes of the liquid may then be regarded as the proportion of the magnetism of the atom of iron in both the chemical compounds. For when two extremely

† Translated by Dr. J. W. Griffith.

minute particles of any magnetic substances, placed in succession at the same spot, experience attractions, which stand in any relation, this relation is not altered when both the particles are placed in succession in *any other spot, provided it be the same for both*; an admission which must necessarily be made, if magnetic forces diminish in any definite way with the distance. It then follows mathematically from this admission, that the relation of the attraction of the entire mass is also the relation of the attraction of the individual atom, supposing merely *that the magnetic attraction of each individual atom is not disturbed by the magnetic excitement of the remaining atoms, and that the attracted mass does not by its reaction increase the magnetism of the pole of the magnet.* On this supposition, the relation of the attraction of the masses remains unchanged even when the form of the watch-glass is exchanged for any other form, provided it remain *the same* during the comparison of the attractions with each other.

If, where previously there existed only a single atom of iron, there are now two or three of the same atoms of iron, or in other words, if in the same space twice or thrice as much iron is *uniformly distributed* in a definite chemical compound, according to the above method of deciding the magnitude of the attraction, inasmuch as it emanates immediately from the pole of the magnet, it is evidently twice or thrice as great

4. When the substance to be examined as regards magnetism is of a greasy or waxy consistence, the watch-glass may be completely filled with it in the same manner as with fluids, as may also be effected when it is susceptible of reduction to fine powder. In the latter case, for the purpose of diminishing the attraction, the powder may be mixed with extreme uniformity with fresh hog's lard, and the mixture placed in the watch-glass

If, for instance, we take on the one hand finely divided iron and on the other finely divided nickel, both *at first* in the same atomic number, or *secondly* in equal weight, and mix it with a given quantity of lard, and then fill the watch-glass with the mixture, the relative of the attractions in the first case gives the *relative* of the *magnetism of the atoms* of the two metals, or, in the second supposition, *the relative magnetism of these metals when of the same weight*

5. To determine the strength of the attraction, I place the watch-glass with its contents and its cover in a thin ring of brass,

suspended by three silk threads about 200 millim. in length from a balance which is sufficiently delicate to indicate a milligramme, and which, excepting the axis of the beam, contains no iron. To increase the action when the forces are weak, the glass is not brought into contact with one only of the two poles of the great electro-magnet, but the two keepers (C)[1] are applied to it, and these are approximated by their rounded ends in such a manner that their least distance apart amounts to 6 millim.; and the balance is so adjusted that the watch-glass in the ring, when the balance is counterpoised, simultaneously touches each half of the keeper, and thus at a single point. After the excitation of the magnetism the watch-glass is attracted. In the scale suspended at the other end of the beam small leaden shot and then fine sand or thin paper in small fragments are placed, until the watch-glass is drawn away from the halves of the keeper. This takes place with the greatest uniformity, and when the forces are small, after some practice, the results of the different weighings do not differ from each other by more than 2 milligrammes. The weight of the shot and sand or paper added is the measure of the magnetic force in each case.

6. As we are able to compare the intensity of the magnetism of different magnetic substances, so we can also determine the relative intensities of the diamagnetism of different diamagnetic substances. For this purpose we require merely to measure the repulsion which such substances experience from the pole of a magnet; and here we may again most conveniently make use of the balance with the arrangement described in the previous paragraph. With this view we may at once counterpoise the substance to be tested, so that it comes into contact with the two portions of the keeper, and after it has been repelled by the excitation of magnetism in the electro-magnet, it may be gradually loaded until it again comes into contact with the two portions of the keeper; or when the magnetism is excited, we might adjust the balance as above, and when the substance, in consequence of the interruption of the current, ceases to be repelled by the portions of the keeper and comes to rest upon them, place weights in the pan until the substance again commences to move from the two portions of the keeper. However, I have decided in favour of another method, *which permits of much more accurate determination*.

[1] Compare page [illegible].

My watch-glass and the brass ring in which it is suspended are both magnetic; therefore if I place any diamagnetic substance in the former, the attraction which we observe is the excess of the magnetic attraction of the two former over the diamagnetic repulsion of the latter. This attraction was stronger than the diamagnetic repulsion of almost all the substances I examined, so that the filled glass was always retained by the two portions of the keeper, and could be pulled off like a magnetic body. If then we subtract from the attraction of the empty glass the smaller attraction of the glass filled with a diamagnetic substance, we obtain the diamagnetic repulsion, which the latter experiences by the electro-magnet. In this manner we are able to compare the diamagnetism of different fluids of the same form and volume, and of any bodies to which by fusion or otherwise we can impart the form of the interior of the watch-glass.

7. To give the idea of the determination of a molecular magnetism laid down in paragraphs 2 to 4, and its relative intensity in different substances a sure basis, we must first of all show experimentally that when in the same volume having the same boundaries in one case m times as many magnetic molecules of the same substance are uniformly distributed as in any other case, the resulting magnetic attraction in the one case is also m times as great as in the other, so long at least as the magnetic particles are not so close together that magnetic excitation of one portion of the mass can exert a perceptible influence upon the magnetic excitation of the other portion.

8. I first took a somewhat concentrated *solution of protochloride of iron*, and mixed one part of it with an equal volume of distilled water, so that the mixture in the same volume contained only half the original solution of the chloride; hence also only half the original quantity of the chloride and half only of the original quantity of iron. This mixture was again diluted to twice its bulk, and the solution thus obtained again diluted to two volumes. Hence in the four solutions which we shall denote by I. II. III and IV., the quantities of the uniformly distributed magnetic substance were in the following proportions:—

$$8 : 4 : 2 : 1.$$

The watch-glass previously mentioned was first used in the empty state, then filled with distilled water, and lastly with the four solutions in succession; the adjustment being the same as

that described in paragraph 5, the force with which the mass in each case, when exposed to the two half-keepers, was attracted by them, was determined. To excite the magnetism in the large electro-magnet, six platinum elements were used, the exciting liquid consisted of commercial nitric acid, and of sulphuric acid, the latter being diluted in the proportion of 1 : 12 according to volume.[1] The intensity of the current during the continuance of the experiment was constant. The weights required for the withdrawal were for—

		grms.
The empty watch-glass (with the cover and ring)		0·10
The watch-glass with distilled water		0·28
" " the solution	I	3·01
	II	2·11
	III	1·23
	IV	0·72

If we deduct from the three weights last determined the attraction of the watch-glass, cover and brass ring, we get the attraction of the four solutions I to IV. But whilst in these solutions the protochloride of iron is *magnetically attracted*, the water they contain is *diamagnetically repelled*, and the attraction determined above is the excess of the attraction over the repulsion.

From the two first weighings we find for the diamagnetic repulsion of the water filling the entire cavity of the watch-glass

0·12 grm.

Thus if in all the solutions we neglect the volume of the protochloride of iron in comparison with the volume of water, in doing which the greatest error occurs with the strongest solutions, in each case 0·28 grm. must be deducted instead of 0·10 grm. But we proceed more accurately when, instead of the protochloride of iron, we regard the solution I as the original magnetic substance, to which in the following solutions water is added in given proportions. The volumes of the water added amount to $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$ of the whole volume, and hence the corresponding diamagnetic repulsions, when the water is uniformly diffused through the entire space, are as follow:

0·06 grm. 0·09 grm. 0·105 grm.

[1] About the same proportion was used in all the experiments described in this memoir.

Thus we obtain the following numbers as representing the attraction of the original solution of the protochloride of iron in I. to IV.:—

	grms	grms	grms	grms.
I.	3·94	−0·40		=3·54
II.	2·14	−0·40	+0·06	=1·80
III.	1·23	−0·40	+0·09	=0·920
IV.	0·72	−0·40	+0·105	=0·425.

The attraction of I. is exactly eight times that of IV., and in general the attraction is almost in proportion to the amount of the magnetic substance. Assuming this proportionality as a basis, if we calculate the attraction of IV. by dividing the sum of the attractions by 15, and then calculate the attraction of the other solutions, we have—

		Difference
I.	3·566	−0·026
II.	1·783	+0·017
III.	0·891	+0·029
IV.	0·446	−0·021

The differences are so small that they fall within the limits of errors of observation, and thus it is confirmed *that the attraction of the solution of the protochloride is in proportion to the quantity of the latter, presupposing that it is uniformly distributed through the same space.*

9. In a second experiment, very finely divided iron was procured from a chemist's shop, and

1·6 grm. 0·8 grm. 0·4 grm. 0·2 grm. 0·1 grm.

of it in each case triturated in a mortar with 25 grms. of fresh lard until it formed a mass which was homogeneous in appearance. We shall denote the five mixtures by I. II. III. IV. and V. The watch-glass was first filled with pure lard, and was then attracted by the electro-magnet, which was adjusted in exactly the same manner as in the experiments detailed in the last paragraph, with a force of

0·25 grm.

The watch-glass was then filled with each of the five mixtures in succession, and the weight of the mixture in the watch-glass (from which the amount of iron contained in it was calculated), and lastly, the weight necessary to overcome the attraction of the watch-glass were determined. By these means the following results were obtained:—

	Weight of the mixture grms	Quantity of iron grm	Attraction grms
I	10.70	0.6818	259.95
II	10.65	0.3108	133.60
III	10.55	0.1621	61.73
IV	10.35	0.0828	31.65
V	10.15	0.0406	15.95

The numbers in the last vertical column give the attraction of the iron in the different fatty mixtures; they represent the weights requisite for the separation of the watch glass *minus* 0.25 grm, whereby, without incurring a perceptible error, we have assumed that the amount of the diamagnetic fluid remained the same throughout.

If we start from the assumption that the attraction of the iron in the different fatty mixtures is in proportion to the mass of iron, we need only divide the numbers in the third vertical column by those in the second to obtain the force with which, in the above experiments, a gramme of iron is attracted. In this way we obtain the following numbers:—

	grms
I	379.3
II	392.0
III	398.5
IV	406.5
V	394.8

As there is ground for believing that the differences do not arise from errors in weighing, we take the mean of these weights, which amounts to

394.2 grms.

If we now calculate the attraction of the different fatty mixtures, we obtain the following numbers instead of those previously obtained:—

	grms		grms
I	269.95	Difference	−10.0
II	134.34		− 0.74
III	61.02		+ 0.71
IV	32.61		+ 1.01
V	16.00		− 0.05

The differences, which can by no means be attributed to errors in weighing, become less in consequence of the observation, that the intensity of the current at first increased and at last diminished. The increase and decrease was certainly not directly

measured, but the approximative estimation explained the above deviations, and the weight of the first weighing only would *remain slightly below that* given by calculation, when we assume the subsequent weighings as a basis.

10. Instead of controlling the intensity of the current by which the magnetism was excited in the electro-magnet by the insertion of a galvanometer in the ordinary manner, another method of proceeding appeared to me far preferable for our peculiar object.

During the weighings described in paragraph 8, the intensity of the current remained unchanged, which was known by the magnetism excited in the large electro-magnet remaining the same. Since the two portions of the keeper when applied could not be removed or even disturbed during the entire continuance of the experiment, an iron cylinder, the upper end of which was pointed conically, 27 millim. in height and 25 millim. in diameter, was placed upon one of the keepers at any accurately determined spot, and its magnetism estimated by the weight which was requisite to withdraw from its apex a small and also pointed iron cylinder weighing 1·7 grm., and which was 16 millim long and 4·5 millim. thick. This determination was effected by the aid of a balance. In these experiments, when the larger cylinder was supported upon one half of the keeper, in contact with the centre of that upper edge of it which was parallel to the equatorial plane and at the greatest distance from it, it amounted to 352 grms.; and this weight did not vary throughout the entire duration of the observation more than 1 or at the most 2 grammes.

The determination in question can be effected in two ways. When more weight is gradually added until at last the small iron cylinder is drawn away, the magnet now no longer supports the same weight as it was in a condition to support before the separation, if it is all applied at once. To find the intensity of the magnetism, we may take either the former weight, which corresponds to the gradual loading, or the weight which it immediately supports. I prefer the former, because it allows of a more accurate determination. In the above instance, the difference in the two determinations amounts to some grammes; whilst the error, of which the first method of determination is susceptible, is at the most two decigrammes*.

11. In the different points of the approximated halves of the

* M. vom Kolke, in his Inaugural Dissertation, which has just appeared, entitled *De nova magnetismi metiendi methodo ac de rebus quibusdam hac methodo*

keeper, the intensity of the magnetism produced by polar induction by no means increases in the same proportion as the intensity of the current. Hence if the object be not to determine merely whether the magnetic attraction remains the same, but also *to correct* the forces which are requisite for the withdrawal of the watch-glass from the halves of the keeper with regard to slight variations in the intensity of the current, the method of proceeding described in the last paragraph is no longer applicable. We must then substitute for the small pointed iron cylinder a watch-glass filled with some magnetic substance, resembling as much as possible that in which the other substances were placed when tested in regard to magnetism; and whilst in the balance, it must be allowed to be withdrawn from time to time from the two halves of the keeper in exactly the same manner. The weights requisite to effect this evidently afford a measure of the intensity of the magnetism in action during the experiments in question; they may evidently be considered as proportional to these weights; and hence when the intensity of the current varies they may be corrected.

The necessity of the new method of determination is evident from the weighings detailed below, which were made for this purpose. Once under the same conditions as in the previous paragraph, the force which was requisite for withdrawing the small iron cylinder was determined; and on another occasion, the force, with which a watch-glass containing the fatty mixture III., consisting of 1000 parts of lard to 16 of iron, when applied to the approximated halves of the keeper, was retained by them. When four freshly filled cells were used in succession to excite the magnetism, the following results were obtained:—

Number of cells.	Attraction of the cylinder.	Attraction of the watch-glass.
	grms.	grm.
1	100.1	15.16
2	178.9	31.15
3	239.6	50.15
4	291.8	60.10

12. The supporting power of a magnet is a completely indefinite expression, principally because the mass of the keeper sup-

recently applies this method to determine numerically the distribution of the magnetism in the surfaces of the poles of the large electro-magnet, in keepers and in steel bars, to measure the influence of the inducing action of poles of the same and of different names, and in my opinion has obtained results which decidedly deserve the preference over those obtainable by other methods, especially those of Coulomb by steel magnets.

ported exerts the most decided influence upon it; and by varying the mass, this supporting power may be increased a hundred or a thousand times. And how can we determine this mass in different magnets so as to be enabled to compare their supporting power? Moreover, so long as the magnetic polarity excited in the iron of the keeper or the entire body attracted reacts to the augmentation of the power of the electro-magnet, and lastly, so long as one portion of the attracted body acts upon the other so as to excite magnetism, so long will a *comparison* of the intensity of the attractive forces, which the magnet exerts upon the different magnetic substances, be out of the question. I believe, however, that after the previous remarks we may admit without hesitation, that the disturbing influences in question are not present when iron or nickel, in a state of fine division, is uniformly diffused *in not too large quantity* through a substance which is but little susceptible of the influence of the magnet, as lard; or when the solution of a salt of iron or nickel is used. I believe that I am justified in assuming *that the attraction of the entire mass is then equal to the sum of those attractions, which, when we divide this mass into parts, the magnet would exert upon the parts individually, even if the other portions were not present.*

But our method of determining the *relative* magnetic intensities of different substances would retain its full value, even if the action of the reciprocally inducing portion of the attracted mass did not vanish; but the volume and the limits remaining the same, is proportional to that force with which the magnet attracts the different substances.

13. We may easily become satisfied, by a simple experiment, that the attraction of a compact mass of iron by a magnet is not the sum of those attractions which are emitted by the magnet to the separate parts of the mass, *but that the disturbing effects of induction are also present.* Thus if we place an iron rod upon the pole of a magnet, a certain weight A is requisite to withdraw it. If we then cut the rod into two pieces, and place the lowest piece upon the pole exactly as before, but the upper piece upon any non-magnetic support, which keeps it at its former distance from the pole, we can again determine two weights B and C by the balance, which are requisite for the withdrawal of the two parts. We then find,—

$$A > B + C.$$

By the inductive action of the two portions upon each other, the attraction is here also *increased*.

14. Our views allow of our determining *the interfering effects of induction* in each of the present cases.

For the purpose of measuring the more powerful attractions I had a brass cup made of the shape of a watch glass, and its upper margin was ground, so that it might be filled with liquids and powders, exactly like the above described watch glass. A massive piece of iron accurately fitted its cavity; this could be removed and replaced by other substances; for the present purpose these were finely divided iron, and a fatty mixture consisting of twenty five parts of lard to one part of the iron filings. The adjustment was as before, the distance of the heavy rounded keepers 6 millim.; the only difference was that the cup was not immediately laid upon the keepers, but to diminish the force, a glass plate 1 millim. in thickness was first placed upon the keepers, and the attraction at this point measured. The magnetism was excited by one Grove's element with nitric acid which had been once used. The following are the results:—

		grms.
I.	Weight of the iron in the cup	81.0
	Its attraction	2187.5
II.	Weight of iron filings in the cup	32.85
	Its attraction	996.0
III.	Weight of the fatty mixture in the cup	10.00
	Its attraction	12.80

The attraction of the cup itself with the glass cover, which amounted to 8.36 grms., has been deducted throughout.

Hence, when we calculate the attraction which *one* gramme of iron experiences in the three separate weighings, we find—

		grms.
I.	for the massive piece of iron	27.00
II.	for the iron filings	30.32
III.	for the same in the fatty mixture	33.28

It is thus seen that the disturbing action of induction diminishes the total attraction of the molecule of iron. If we admit that this disturbing action vanishes in the case of the fatty mixture, which is at least approximatively correct, the attraction of the piece of iron and of the iron powder, independently of the disturbing effect of the induction, as it immediately arises

from the electro-magnet, and which we shall call the *normal attraction*, would amount respectively to

2795·68 grms. 1114·25 grms.;

consequently the disturbing effect of induction is respectively

−608·18 grms. −118·25 grms.

If we consider the normal attraction as equal to unity, this amounts relatively to

0·186 0·089.

15 In the example given in paragraph 13, the disturbing action of induction *increases* the normal attraction; in the example contained in the previous paragraph, it *diminishes* it. If in the latter instance we had withdrawn the cup from the surface of one of the poles instead of from the two portions of the keeper, we should, on the other hand, have obtained a disturbing effect of induction, which would have *increased* the normal attraction. Experiment evidently confirms this; I shall not, however, detail any numbers, because the *exact* estimation of the weight is attended with some difficulty arising from disturbing influences.

If we imagine two iron bars to be placed one upon the other on the same pole, in consequence of the original action of the electro-magnet, poles of different names become excited at the place of contact, and *mutually strengthen each other*. But if two rods, forming a bridge from one pole to the other, are superimposed, the poles of the same name are excited at the corresponding ends, in consequence of the original action, and these poles become *weakened* by their mutual action. Thus, while in the first instance the disturbing action of induction necessarily augments the magnetism excited in the iron, in the second case it must weaken it.

In this way the whole appears to be perfectly explicable*.

§ 2. *Comparison of the Intensity of the Magnetism of different substances*

16. The method of determination used consists, as stated in the preceding paragraphs, in placing different magnetic substances in the same space in a watch-glass closed by a cover, then, the

* Hence it is very probable, that in the experiments with the lard, detailed in paragraph 9, the difference occurring in the first fatty mixture really arises in part from the disturbing action of induction.

intensity of the current being constant, the force with which these substances are attracted by the electro-magnet is the relative measure of their magnetism. If we divide it by the weight of the substances, we obtain numbers which represent the relative intensities of the magnetism of these substances *for equal weights*.

We here enter upon a new field of physical investigations, and a number of questions arise, the answers to which are of manifold interest,—questions which partly encroach upon the province of chemistry. Hitherto I have only been able to procure pure oxide of nickel, but not pure nickel itself, nor the other magnetic metals except iron. My investigations must therefore first be confined to iron and its chemical compounds. In what proportion is the original magnetic force of the iron diminished when oxygen is added to it to form the peroxide? how, again, when water is added to the oxide as in the hydrate? when the oxide has combined with different acids to form salts? What relations do the salts of the peroxide hold to those of the protoxide? What must the chemical composition of a salt containing iron be so that it may cease to be magnetic?

17. I shall first communicate the results of two series of experiments, which may be condensed into one, because the electro-magnetism, which was excited by six Grove's elements, was in each case of the same intensity, and made but very slight variations. The weighings were not made until in each case the battery had been in action for some time, and after each weighing the circuit was opened. The *first* series of experiments related to aqueous solutions of salts of iron. I took—1, pernitrate of iron, which had been prepared by pouring excess of concentrated nitric acid upon the peroxide of iron denoted by II.; 2, perchloride of iron prepared from the same oxide with concentrated hydrochloric acid; 3, dry neutral persulphate of iron from the chemical laboratory, which dissolved very slowly in water; 4 and 5, protochloride and protosulphate of iron, prepared on the morning of the performance of the experiment by pouring hydrochloric and sulphuric acid upon finely divided iron. All the solutions except the latter were saturated. The watch-glass described in the second paragraph was filled with the different solutions in succession, and subsequently the amount of iron contained in that quantity of each of these solutions which was used in the experiment was determined. The attraction of the solutions by the electro-magnet was compared with the attrac-

tion of a fatty mixture containing 2 parts by weight of iron to 100 of lard.

With this mixture, in the second series of experiments, different peroxides of iron were first compared; the first (I.) was prepared in a chemical laboratory; the second (II.) in the chemical manufactory in this town, which is specially devoted to this purpose, the third was fibrous red hæmatite (reniform); the fourth a beautiful crystal of micaceous iron ore from Elba: next three hydrated peroxides of iron; first, that from which the oxide I. was procured, and which, by a direct determination, contained 24·24 per cent. of water; secondly, brown hæmatite; and thirdly, artificial blood-stone, which, according to a subsequent determination, contained 11·55 per cent. of water; moreover, a beautiful crystal of pyrites; lastly, protoxide of nickel and its hydrate, the latter containing 24·75 per cent. of water, according to an approximative determination. All these substances were finely powdered, and when compressed as uniformly as possible, were placed in the watch-glass and their weight determined. Both the peroxides of iron, the red ironstone and the protoxide of nickel, after having been powdered, were dried at a temperature of 212° F. immediately before use.

18. The results obtained are collected in the following table. The *first* column (A) contains the directly determined weight of the various substances examined; the *second* (B) the quantity of metal they contain. These quantities were determined in the case of the five solutions by chemical analysis, but were calculated for other metallic compounds. The *third* column (C) contains the total attraction of the substances examined. Here in each case 0·41 grm., *i. e.* that weight by which the empty watch-glass was withdrawn, was deducted from the weight which caused the filled watch-glass to separate. The *fourth* column (D) gives the quotients which are obtained by dividing the total attraction by the weight of the substance; hence the relative proportion of the magnetism of the substance for the same weight. The *fifth* column (E) gives the relative proportion of the magnetism of the iron or nickel in the various chemical compounds. This is immediately obtained for the solid matters by dividing the total attraction by the weight of the metal they contain. With regard to the solutions, however, bearing in mind the diamagnetism of the water they contain, a slight correction must be made in the total attraction C. But instead of calculating, as in the eighth

paragraph, the quantity of water in each solution and the corresponding diamagnetic repulsion, we shall for the sake of simplicity approximatively increase the total attraction of each of these solutions by 0 1 grm

	A	B	C	D	E
	g	g	g	g	g
Pernitrate of iron Solution	11 5	1 1	2 502	0 172	081
Perchloride of iron Solution	16 175	2 11	8 10	0 500	37 1
Persulphate of iron Solution	18 [illegible]	2 13	5 335	0 0	170
Protochloride of iron Solution	1[illegible] 35	2 8 5	7 09[illegible]	0 120	2 501
Protosulphate of iron Solution		0 115	1 350		1 011
Liquor ferri 50 1 Solution	8 [illegible]	0 161	8 37		510 71
Peroxide of iron I Powdered	12 188	8 513	11 10	581	1 0 8
Peroxide of iron II Powdered	11 82	10 377	21 000	1 103	2 080
Red iron stone Powdered	28 [illegible]5	1[illegible] 385	10 501	0 080	0 081
Micaceous iron ore Powdered	33 72	[illegible]3 001	11 755	[illegible]21	8 887
Hydrated peroxide of iron Powdered	16 50	8 750	13 238	0 800	1 513
Brown ironstone Powdered	22 70		8 210	0 303	
Artificial blood stone Powdered	1 15	7 708	9 018	0 773	1 235
Sulphuret of iron Powdered	25 [illegible]2	11 770	10 117	0 770	1 0 0
Protoxide of nickel Powdered	11 05	11 511	2[illegible]30	0 180	0 228
Hydrated protoxide of nickel Powdered	11 125	6 571	0 0 5	0 514	0 0 1

19 In a preliminary experiment, the magnetism of four of the above substances had been already compared with the magnetism of the iron To control the accuracy of the method, I shall give the results here, so as to allow of the comparison of the more recent with the earlier results at the same time I shall limit myself in the following table to the columns A, C and D

	A	C	D
	g	g	g
Liquid mixture 25 1	11 57	[illegible]32 11	521 75
Peroxide of iron II Powdered	1 13	2 01	1 17
Red haematite Powdered	8 07	10 53	0 080
Micaceous iron ore Powdered	31 17	91 0	080
Brown haematite Powdered	[illegible]7[illegible]	0 37	0 378

20 Still earlier with a current of less intensity, I had compared the magnetism of the hydrated peroxide of iron of the

The weights of the quantities of the peroxide obtained from the five solutions mentioned in the above series amounted to

g
1 731
3 16
3 01
1 036
0 030

I am indebted for these determinations as also those detailed in the note to paragraph 2[illegible] to the kindness of M D Bhandis

table in the eighteenth paragraph with the magnetism of the oxide obtained from it *by heating it to strong redness in a furnace*, which I shall designate by III., as also with the magnetism of the neutral persulphate of iron in powder, also prepared from it, and which was afterwards dissolved in water. The oxide I. was subsequently prepared from the same hydrate. The results found are contained in the following table.—

	A	C	D
	grms	grms	grms
Hydrated peroxide of iron	14·17	7·94	0·534
Peroxide of iron	12·35	650·68	52·687
Persulphate of iron	13·10	4·95	0·379

21. Since according to the table in the last number the relative magnetic deportment of the *protoxide of nickel* and its *hydrate*, according to our present notions of magnetism, appear contradictory, a separate examination was subsequently again made On using six elements and the same watch-glass, the following results were then obtained·—

	A	C	D.
	grms	grms	grm
Oxide of nickel Powdered	11·96	2·58	0·173
Hydrated oxide of nickel Powdered	11·07	6·00	0·512

From the earlier experiments we obtain for the relation of the magnetism of the protoxide of nickel and its hydrate,—

$$\frac{544}{180}=3{\cdot}017\,;$$

we now find for the same,—

$$\frac{542}{173}=3{\cdot}132.$$

The numbers agree sufficiently

Although the protoxide was obtained from the hydrate, it nevertheless appeared to me desirable to convert *the same* hydrate, which had been used in the experiments, into oxide, and then to examine it The above 11·07 grms., immediately after the determination of their magnetism, were heated to redness in a platinum crucible for a long time, whereby the weight became reduced to 8·38 grms. The protoxide thus obtained was placed in the same watch glass, but this it did not now completely

fill it experienced an attraction of 2·94 grms., although, if the protoxide were the only magnetic agent in the hydrate, this attraction should have exceeded 6 grms., because the protoxide was now comparatively nearer the magnet.

However the attraction of the freshly prepared protoxide of nickel exceeded the attraction of the original, perhaps because the expulsion of the water was still not complete.

22. Lastly, a third series of experiments was made, the results of which I shall collect in the following table, which corresponds to the former one. The magnetism in this case also was excited by six Grove's elements.

	A	B	C	D	E
	g	g	g	g	g
1 part iron, 25 lard	10·850		10·800		50·300
1 part magnetic iron ore, 25 lard	11·000		86·135	0·3502	
Gray oxide of manganese before being heated to redness, powdered	23·301		8·470	0·3[illegible]4	
Gray oxide of manganese after being heated to redness, powdered	22·860		18·570	0·812	
Crystals of protosulphate of iron, powdered	15·978		6·300	0·391	
Protonitrate of nickel. Solution		1·377	1·150		1·03[illegible]
Protochloride of nickel. Solution		1·571	1·830		1·006

The attraction of the empty watch-glass, the former with another ground-glass cover, simultaneously with this and the brass ring, amounted to

0·375 grm.,

and when filled with lard only, to

0·210 grm.

From the attractions found by the direct weighings the latter number has been deducted in the case of the lard mixture in the third column, and the former in the case of the powdered substances; in the solutions, making an approximate allowance for the diamagnetism of the water, we deducted

0·275 grm.

The attraction given in the column D refers, in the case of the (as finely as possible) powdered magnetic iron ore, to this independently of the lard mixed with it.

The weights of the quantities of protoxide of nickel obtained from the two solutions were respectively—

1·751 grm.
2·133 grms.

The gray oxide of manganese consisted of very beautiful crystals in powder, in which iron could not be detected by ferrocyanide of potassium. It appeared, in correspondence with the well-known analysis, to consist of pure hydrated oxide of manganese. Immediately after the experiment, it was heated to strong redness over a spirit-lamp to remove its water of hydration. 9·388 grms. lost 1·164 grm. in weight. The loss in weight corresponding to an atom of water would have amounted to exactly 10 per cent., that found amounts to 12·40 per cent. This agrees well with the statement of Berzelius, that hydrated oxide of manganese, when heated to redness, becomes converted into protoxide of manganese, which would require a loss of 13 2 per cent Since the heating to redness between the weighings should take place upon the same quantities, it could not be perfectly complete.

The crystals of the protosulphate of iron were taken out of the solution in which they had formed immediately before the experiment, dried in the air upon bibulous paper for half an hour, and then powdered The powder, moist as it was, was placed in a watch-glass, hence but very little of the water which the crystals contained could have escaped.

The solutions of the salts of nickel used in the experiments were prepared by dissolving the hydrated oxide of nickel already mentioned in acids.

23. From the two previous tables I have calculated the following one for the comparison of the intensity of the magnetism of different substances, both separately, as also in chemical combination with others, placing the intensity of the magnetism of iron as = 100,000:—

1.	Iron	100,000
2.	Magnetic iron ore	40,227
3.	Peroxide of iron I.	500
4.	 II	286
5.	Red hæmatite	134
6.	Micaceous iron ore	533
7.	Hydrated peroxide of iron	156
8.	Brown hæmatite	71
9.	Artificial blood-stone	151
10.	Dry persulphate of iron	111
11.	Green vitriol	78

12	Saturated solution of pernitrate of iron	34
13	perchloride of iron	98
14	persulphate of iron	58
15	protochloride of iron	81
16	Green vitriol in solution	126
17	Protosulphate of iron in the form of green vitriol	112
18	Pernitrate of iron in solution	95
19	Perchloride of iron	221
20	Persulphate of iron	133
21	Protochloride of iron	190
22	Protosulphate of iron	219
23	Perchloride of iron in solution	251
24	Protochloride of iron	216
25	Iron pyrites	150
26	Protoxide of iron in a muriatic solution	381
27	sulphuric solution	162
28	Peroxide of iron as a hydrate	206
29	in the form of blood stone	168
30	in a nitric solution	287
31	in a muriatic solution	516
32	in a sulphuric solution	332
33	Iron in magnetic iron ore	55,55[illegible]
34	peroxide I	711
35	peroxide II	109
36	red hæmatite	191
37	micaceous iron ore	761
38	hydrated peroxide of iron	296
39	blood stone	210
40	pyrites	321
41	persulphate of iron	349
42	protosulphate of iron	385
43	a solution of pernitrate of iron	110
44	a solution of permuriate of iron	737
45	a solution of persulphate of iron	474
46	a solution of protomuriate of iron	190
47	a solution of protosulphate of iron	591
48	Protoxide of nickel	35
49	Hydrated protoxide of nickel	106
50	Protonitrate of nickel in solution	65
51	Protomuriate of nickel in solution	100
52	Protochloride of nickel in solution	111

53	Protoxide of nickel in the hydrate	142
54	Protoxide of nickel in a nitric solution	164
55	Protoxide of nickel in a muriatic solution . . .	171
56.	Nickel in the protoxide	45
57	. . . the hydrated protoxide	180
58	. . . a nitric solution	208
59.	... a muriatic solution	217
60.	Hydrated oxide of manganese	70
61.	Protoperoxide of manganese . . .	167
62	Oxide of manganese as hydrate	78
63	Manganese as hydrated oxide	112
64	. . . protoperoxide . . .	2,32

24 If we reduce in the same manner the results of the table in paragraph 19, and arrange them beside the corresponding ones which we have previously obtained, we have—

	I	II
Iron	100,000	100,000
Peroxide of iron II	286	289
Red hæmatite	134	133 5
Micaceous iron ore	533	512
Brown hæmatite	71	72

The agreement of the observations, which were instituted under different circumstances, leaves nothing to be desired if we exclude the micaceous iron ore. As regards powders, a source of error consists in their unequal pressure into the watch-glass; and since it cannot be admitted that in the two months, during which the powdered micaceous iron ore was exposed to the air, it might have undergone a chemical change, I am inclined to attribute the difference to the former source of error

25. Although iron is of itself so strongly magnetic, it loses its magnetism in most of its chemical combinations in so great a degree, that until quite recently, as these were *not* attracted by the magnet, they were regarded as *not magnetic*. I have not yet been able to examine the deportment of the *protoxide of iron*, nor *to determine accurately in what proportion the intensity of the magnetism of iron is diminished in the pure peroxide* On taking different kinds of the peroxide of iron occurring in nature and prepared in laboratories, the results were extremely discordant It might be imagined that the different intensity of the magnetism in the various kinds of peroxide of iron might be con-

nected with their very different appearances, and also to the corresponding very different molecular states in which the peroxide is formed both in nature and in the laboratory. Without wishing to speak positively upon this point, the supposition appears to me however as yet best founded, that the different intensity of the magnetism arises from an admixture of protoxide. The first peroxide of iron which I examined, and which is denoted by III. in the 21st paragraph, was procured from the hydrate which occurs in the table in the 18th paragraph, by heating it strongly to redness in a furnace: it was so strongly magnetic, that it was taken up by a very weak magnet. In comparison with the hydrate it was a hundred times stronger, hence the intensity of its magnetism was

15201.

This peroxide evidently contains a considerable quantity of *protoxide* in admixture. Hence also I think it probable that the oxide I., the magnetism of which is = 500, is not free from protoxide, and contains more of it than the oxide II. I dare not yet venture to determine, from the above data, the number which corresponds to the pure peroxide. Red hæmatite is much less magnetic than micaceous iron ore: that which I examined has not been subjected to a chemical analysis; if it were chemically pure, I should consider 131 as about the magnetism of the peroxide.

Were we to deduce the magnetism of the peroxide of iron from the magnetism of the hydrated peroxide of iron prepared in the chemical laboratory, for which we found 156, on the supposition that the water which is added to the peroxide in the hydrate exerts no influence upon its magnetism, we should obtain the number 206, which by the last assumption, that the red hæmatite possessed the normal magnetism, would be too great. But this supposition is entirely unsupported, and the two numbers would not be inconsistent if (as in the case of nickel, only not in the same degree) the water added to the hydrate increased the magnetism of the oxide (protoxide).

26. The powerful magnetism of the magnetic iron ore is remarkable in more than one respect. That which I examined, and which I obtained through the kindness of M. Noggerath, together with other minerals from the Poppelsdorf collection, came from Sweden, and as stated, was pure protoperoxide, containing therefore nearly 31 per cent. of protoxide of iron and 69 per cent. of the peroxide. If we were to regard it as a me-

chanical mixture of peroxide with protoxide of iron, we should obtain for the magnetism of the peroxide in the mixture,

$$69 \quad 1{\cdot}34 = 92\,;$$

consequently, on deducting this number from the magnetism of the magnetic iron ore, we have the number 40135 for the magnetism of the 31 per cent. of protoxide of iron. If we reduce this number to our unity of weight, we get

132694

for the magnetism of the protoxide, which would consequently exceed the magnetism of iron itself.

Treating this question as a mathematical problem, we should obtain a more probable result, were we to convert the chemical formula for magnetic iron ore ($FeO + Fe^2O^3$) into the quantitative equivalent ($2FeO + FeO^2$); for if the magnetic iron ore contained 62·01 per cent. of protoxide of iron, and we entirely neglected the magnetism of the hypothetical FeO^2, we should have for the magnetism of the protoxide, after reduction to the unit of weight,

64870.

27. Shall we, on the other hand, suppose that in the magnetic iron ore, by the chemical combination of a powerfully magnetic body, the protoxide of iron, with one feebly magnetic, the peroxide of iron, a body is produced which is still more powerfully magnetic than the former?

I shall not at present venture to express an opinion upon the intensity of the magnetism of the protoxide of iron.

28. The most natural supposition is, that in most cases a small quantity of protoperoxide of iron is mixed with the oxides. Adopting this view, 1 per cent. for instance of the former would augment the magnetism of the remaining 99 per cent. to 402; hence, assuming the magnetism of the red hæmatite to be that of the oxide, we should have as its magnetic intensity

535,

which nearly corresponds to that of micaceous iron ore. Thus the increase in the magnetism of the latter would be produced by the admixture of about one-third per cent. of protoxide of iron. This would only make a difference of about one-twelfth per cent. in the total amount of oxygen, which would be difficult to estimate by chemical analysis.

29. In accordance with the view we have taken, we might calculate the admixture of the protoperoxide of iron, if we knew the

magnetism of the peroxide under examination. If, *e. g.*, we take the strongly magnetic oxide III., and premise that it consists of x per cent. of peroxide and y per cent. of protoperoxide, we have

$$x + y = 100$$

$$134\ x + 102.27\ y = 12201$$

hence

$$x = 62.11,\ y = 37.89$$

whence

Peroxide = 88.35
Protoxide = 11.65

30. Chemical analysis usually points out directly the quantities only of the simple substances existing in bodies, but the manner in which these are intimately combined is derived merely from a theoretical combination. As regards iron in particular, the magnetic determination immediately yields a solution of the latter point. Tourmaline, staurolite and basalt, which when suspended between the poles of a magnet, even in the most strongly magnetic liquid, do not cease to act magnetically, could not possess this magnetism if the comparatively small quantity of iron which they contain were mixed with them in the state of peroxide.

In the following paragraph we shall show, at least by a striking example, how considerable quantities of iron (constituting as much as 12 per cent.), when in a state of definite chemical combination, may completely lose their magnetism. The electro-magnet then ceases to prove the presence of iron, or, even supposing this presence, the kind of combination in which it occurs.

31. The great difference which occurs in the magnetic relation of the peroxide to the protoxide, in the same way ceases to be apparent in their *saline compounds*. When in solution, protosulphate of iron is certainly more strongly magnetic than the persulphate, but merely in the proportion of

133 : 219.

In those haloid salts which we have examined, this relation is reversed. The solution of protoxide of iron in hydrochloric acid is more feebly magnetic than the solution of the peroxide in the proportion of

190 : 224,

and the protochloride of iron is more feeble than the perchloride in the proportion of

216 : 251.

32 *In saline solutions the original magnetism of the peroxide is not enfeebled by the addition of acids to the latter*

In the case of its combination with nitric acid, not the slightest alteration occurred provided it was added to that peroxide from which it had been prepared The numbers representing the magnetism in each case are 287 and 286 However, as the latter number, in conformity with the previous investigations (28.), is probably too great, the magnetism of the peroxide has probably increased by the addition of the nitric acid.

The magnetism of the peroxide, when in combination with *sulphuric acid*, is greater than when in combination with *nitric acid*, and more considerable when in combination with *hydrochloric acid* than with *sulphuric acid* The proportion (in the solutions) is

$$287 : 332 : 516.$$

33 According to the tabular sketch given in paragraph 23, the magnetism of green vitriol increases, *when it is dissolved in water*, in the proportion of

$$78 : 126.$$

The same likewise appears to be the case with the anhydrous sulphate of the peroxide of iron In this case we obtain the proportion

$$111 : 133$$

The magnetism of the dry salt is calculated from a previously instituted comparison with the hydrate, with which it had been simultaneously prepared (that detailed in the general sketch). (See the table in paragraph 20.)

34 In the case of nickel, the 21st paragraph proves the perfectly unexpected relation, that *the hydrated protoxide is much more powerfully magnetic than the protoxide itself, the water of hydration added to it increasing the magnetism about fourfold*

The acid added to the oxide, in a solution of the nitrate and hydrochlorate of nickel, as in the case of iron, also augments its magnetism, and the hydrochloric acid more (although not to the same extent as with iron) than nitric acid

35. *Manganese* exhibits a remarkable analogy to iron I shall not venture, in the case of either metal, to decide whether the hydrated oxide, as with nickel, is more powerfully magnetic than the mere oxide But a correspondence with iron consists in *the protoperoxide*, which in the case of manganese is produced by heating the hydrated oxide to redness, being *considerably more magnetic than the hydrate, and probably also than the oxide.*

36. *The relative magnetism of the atoms* of those substances which have been examined may be readily deduced from the tabular sketch given in paragraph 23; for when we have determined the relative magnetism of the iron in the various chemical compounds *for the same weight*, this is also the *relative magnetism of the atoms of these substances*, provided they contain *a single* atom of iron only. When the compound atoms of the substances contain 2 or 3 atoms of iron, we must multiply the numbers given in the table by 2 or 3, to find the magnetism of these atoms. Thus, for instance, if the magnetism of the atom of iron be placed at 100,000, the magnetism of an atom of green vitriol ($FeOSO^3 + 7HO$) is equal to 385, whilst that of an atom of persulphate of iron ($Fe^2O^3\,3SO^3$) is $= 2\cdot349 = 698$.

The tabular sketch moreover gives about 15 as the number representing the magnetism of the nickel in the protoxide of nickel, and the number 180 as that of the hydrated protoxide of nickel. The proportion of these numbers is also the proportion of the magnetism of an atom of protoxide of nickel and that of an atom of hydrated protoxide of nickel. However, to be enabled to compare these numbers with those relating to iron and its compounds, we must multiply them by $\frac{3(\ )\,33}{350\,53}$, the quotient of the atomic weight of iron into that of nickel.

For the same purpose we must first multiply the magnetism of the manganese contained in the hydrated oxide ($Mn^2O^3 + HO$), for which the table gives 116, by 2 on account of the double atom of manganese; the magnetism of the protoperoxide ($MnO + Mn^2O^3$), which was determined to be equal to 230, by 3 on account of the ternary atom of manganese; and then in both cases the quotient of the atomic weight of the iron into the atomic weight of the manganese by $\frac{344\,681}{350\,527}$.*

* When at one time we speak of the magnetism which a given quantity of iron in different chemical compounds possesses, and at another of the magnetism of the atoms of these different compounds, we express ideas which arise from totally different views. It has however already been stated that both ideas stand in exact relation, and this will be still more clearly seen from the following remarks.

If ( ) we take iron and the peroxide ( ), we have as the relation of the magnetism of these substances for the same weights, where about a gramme of each is uniformly diffused within the watch glass, by the table in paragraph 23,

100 000   00

To deduce the relation of the *magnetism of the atoms* from this, we must divide the above numbers respectively by the number of atoms contained in each

37. The following table, which indicates the magnetism of the atoms of some chemical compounds of iron, nickel and manganese, has been calculated in accordance with the preceding paragraph.

		Composition	Magnetism of the atom
1	Iron	Fe	100,000
2	Magnetic iron ore	$FeO + Fe^2 O^3$	166,656
3	Peroxide of iron I	$Fe^2 O^3$	1,128
4	Peroxide of iron II	$Fe^2 O^3$	818
5	Red hæmatite	$Fe^2 O^3$	392
6	Micaceous iron ore	$Fe^2 O^3$	1,522
7	Hydrated oxide of iron	$Fe^2 O^3 + 2HO$	592
8	Blood stone	$Fe^2 O^3 + HO$	480
9	Pyrites	FeS^2	321
10	Persulphate of iron	$Fe^2 O^3 3SO^3$	698
11	Green vitriol	$FeOSO^3 + 7HO$	385
12	Protoxide of nickel	NiO	17
13	Hydrated protoxide of nickel	$NiO + HO$	190
14	Hydrated oxide of manganese	$Mn^2 O^3 + HO$	221
15	Protoperoxide of manganese	$MnO + 2MnO^3$	696
	In solution		
1	Protosulphate of iron	$FeOSO^3$	591
2	Persulphate of iron	$Fe^2 O^3 3SO^3$	938
3	Pernitrate of iron	$Fe^2 O^3 3NO^5$	820
4	Protonitrate of nickel	$NiO NO^5$	219
5	Protochloride of iron	$FeCl^2$	490
6	Perchloride of iron	$Fe^2 Cl^3$	1,171
7	Protochloride of nickel	$NiCl$	229

I need hardly point out expressly, that I cannot regard the numbers in the preceding, as also those in the former table, as by any means definitely fixed. They will certainly undergo cor-

gramme of the two substances, or what is the same, multiply them by their respective atomic weights. In this manner we get in the above example—

$$350 \quad 100{,}000 \quad 1000 \quad 500 = 100{,}000 \quad \frac{1000}{350} . 500$$

Hence, if we again place the atomic magnetism of iron at 100,000, that of the peroxide is 500, multiplied by its atomic weight and divided by the atomic weight of the iron

On the other hand, if we take a given quantity of iron, at one time in its pure state, at another in combination with oxygen, in the form of peroxide, the magnetic attraction is different in both cases. To find the magnetism of the gramme of iron in the peroxide, we must evidently multiply the magnetism of this oxide, for which the table in the 23rd paragraph gives 500, by

$$\frac{1000}{n . 350},$$

in which n denotes the number of the atoms of iron contained in an atom of the compound

From this example we see that the magnetism of iron, in any of its chemical compounds, is equal to the magnetism of the atoms of this iron compound, divided by the number of atoms of iron which an atom of this compound contains, provided that in each case we take pure uncombined iron as the point of comparison

rection and this not so much on account of the method adopted in their determination as on account of the uncertainty regarding the chemical purity of the substances to which point in subsequent determinations of this kind attention should be principally directed

§ 3 *Comparison of the Intensity of the Diamagnetism of different substances*

38 I shall next detail the results of two series of experiments which were performed by the method developed at the end of the 6th paragraph The electro magnetism was in both cases excited by a battery of ten Grove s cells, and in each case nitric acid, which had not been previously used, and a mixture of 1 part of sulphuric acid and 12 parts of water (by volume) were taken The watch glass, with its ground cover, was filled with the various substances in succession and when thus filled was exposed as described in the 5th paragraph, to the two halves of the keeper, the rounded ends of which were turned towards each other, so that it touched each of them at a single point only In the determination of the attraction of the watch glass which then ensued, the error of observation certainly did not amount to 0 01 grm The weight of the solid bodies was determined in each case and usually the specific gravity of the fluids was subsequently taken

In the first series of experiments the following results were obtained —

		grm
Attraction of the empty watch glass with its cover and brass ring		0 19
Attraction of the watch glass filled with		
1	Distilled water	0 3
2	Alcohol I	0 36
3	Sulphuric æther	0 36
4	Solution of ferrocyanide of potassium	0 37
5	Solution of ferridcyanide of potassium	[illegible]
6	Phosphorus	0 215
7	Oxide of bismuth	[illegible]
8	Flowers of sulphur	0 39
9	Sulphuric acid	0 40

39 To test the constancy of the force of the magnet, weighings of a second watch glass, already mentioned in paragraph 11, and

filled with the lard-mixture III. (100 lard, 1 6 iron), were made between the different determinations On first closing the circuit, the watch-glass was attracted by a weight of 69·9 grms , this weight continually increased until it attained its maximum, and then again diminished to 70·0 grms. towards the end. The weighings, which followed each other as rapidly as possible, lasted three hours. After each weighing the circuit was opened, the keepers, however, remained undisturbed.

With regard to the increase of the force of the electro-magnet at first, and its subsequent diminution, I have preferred not reducing the results of the weighings. If merely magnetic substances were concerned, we evidently come nearer to the truth when we consider the attractions as in proportion to the weights requisite for the withdrawal of the normal watch-glass at the various moments. But after numerous observations, and the investigations contained in the following paragraphs, I consider that this proceeding, which in consequence of the great expense of time, augments the inequalities in the intensity of the current, is unjustifiable in this case, where the electro-magnet acts upon a combination of magnetic and diamagnetic substances.

Moreover, the corrections have but little influence upon the result I therefore prefer considering the current as constant throughout.

40. The *second* series of experiments was made on the following day, the halves of the keeper had remained undisturbed; on the addition of fresh acid a similar circuit was set in action. After this had acted for some time, a weight of 70·5 grms. was requisite to withdraw the test watch-glass; and this weight, during the short duration of the weighings, varied 2 decigrms. only The attraction of the watch-glass, both when empty and when filled with distilled water, was also found to be exactly the same as on the previous day, so that we may resolve the two series into one only.

In the second series of experiments, the attraction of the watch-glass, when filled with—

	grm
1. Alcohol II., amounted to	0·32
2. Beaten ox-blood	0·32
3 Mercury	0·05
4 Sulphuret of carbon	0·31
5 Hydrochloric acid	0·33

		grm
6	Nitric acid	0 3[illegible]
7	Oil of turpentine	0 31
8	Powdered ferrideyanide of potassium	1 [illegible]
9	Powdered chloride of sodium	0 3[illegible]

The weights of the solid substances examined were

	g
Phosphorus	3 30
Sublimed sulphur	11 11
Oxide of bismuth	14 10
Chloride of sodium	13 52
Ferrideyanide of potassium	11 0[illegible]

The specific gravities, excepting those of sulphuret of carbon and oil of turpentine, were found by direct determination to be

Alcohol I	0 813
Alcohol II	0 851
Sulphuric æther	0 730
Sulphuret of carbon	1 263
Oil of turpentine	0 870
Sulphuric acid	1 839
Nitric acid	1 [illegible]02
Hydrochloric acid	1 1[illegible]3
Solution of ferrocyanide of potassium	1 [illegible]1
Phosphorus	1 72

41 The following table has been compiled from the determinations in the last paragraph

	[illegible]	[illegible]	[illegible]
Water	g 0 14	100	100
Alcohol I (0 813)	0 13	94	111
Alcohol II (0 851)	0 17	1 [illegible]	111
Sulphuric æther	0 13	93	127
Sulphuret of carbon	0 18	1 [illegible]	102
Sulphuric acid	0 00	81	44
Hydrochloric acid	0 16	114	109
Nitric acid	0 10	71	18
Beaten ox blood	0 17	1 2	
Saturated solution of ferrocyanide of potassium	0 12	80	70
Ignited chloride of sodium powdered	0 13		70
Oxide of bismuth powdered	0 08		95
Sublimed sulphur	0 10		71
Oil of turpentine	0 15	107	123
Mercury	0 44	314	1
Phosphorus	0 245	172	100

The first column in the preceding table exhibits the repulsion which the various substances, when inclosed in the watch-glass, experience from the influence of the electro magnet, expressed in grammes. In the second column the diamagnetic repulsion of the water is placed at 100, and that of the other substances is calculated from it. These numbers have a general signification, and are independent of the volume and the shape of the substance tested; so that when we place the various substances in any other given form, and withdraw this in a corresponding manner from the poles, the same numbers must be found In the case of the powdered substances these numbers are neglected, because they have not a general signification. In the third column, the diamagnetic repulsion which *equal weights* of the substances examined experience, is expressed in numbers, the repulsion of water being placed at 100. Thus these numbers give the corresponding diamagnetic repulsions, when we place equal weights of the various substances, uniformly distributed, in the same given form. The numbers corresponding to the different powders here also have a perfectly definite signification. In each case they were at last pressed into the watch-glass with tolerable force, and as uniformly as possible, by means of the cover Assuming that they were uniformly pressed in, their greater or less density has no influence upon the numbers of the third column.

42. To the preceding table I shall append the following explanations and remarks.

The *mercury* examined was pure. The watch-glass was filled with it, as with the other liquids; but, assuming a convex form at the margin, it elevated the cover, so that at that part most remote from the two halves of the keeper its form became somewhat changed. The diminution of the diamagnetic repulsion arising from this cause is scarcely perceptible.

Impure mercury may exhibit magnetic reactions (70.).

43. The *phosphorus* was fused in water, poured into the watch-glass, and wiped off with the cover. It then solidified within it. With this same mass of phosphorus some preliminary experiments were previously made. With a weak current (the magnetism being excited by only four Grove's cells) the intensities of the diamagnetic repulsion of the phosphorus and of the water were especially compared. The former was repelled with a force of 0·14 grm., the latter with a force of 0·08 grm., hence the

diamagnetic forces *for equal volumes* acting upon water and phosphorus, are in the proportion of

100 : 175.

This result agrees perfectly with that previously detailed. Such conformity in the results which were obtained under different circumstances, is a confirmation of the accuracy of our view which although established as regards magnetism has only been extended by analogy to diamagnetism as far as relates to the comparison of its intensity.

14. The *alcohol* I., which I subjected to examination in the first series of experiments, was found for the same volume to be less strongly diamagnetic than water, although a former but merely preliminary experiment had undoubtedly shown that, on the contrary, alcohol is more strongly diamagnetic than water. This appeared to me more surprising because ordinary spirit lamp alcohol of 0.851 spec. grav. had been substituted for the former alcohol II., whilst the alcohol I. was obtained from a chemical manufactory. To control this result, I again examined the alcohol II. on the following day, and then found the previous result confirmed, as also perfectly the weighing made with alcohol I. on the preceding day. It also appeared so improbable that alcohol, when in combination with a small quantity of water, should be less diamagnetic, and when combined with a larger proportion more so than pure water, that I made a direct experiment to decide this point by adding water to the alcohol I. This caused but little difference in its diamagnetism, as it apparently—very small quantities were used—more approximated to the diamagnetism of the water. Hence we can only imagine that the alcohol I. contained iron, or some other magnetic substance, in admixture, which it had probably taken up during its rectification*.

With very volatile liquids the evaporation occurring during the experiment is a source of error. This would, however, make the diamagnetism of the fluids in question too great, so that it can afford no explanation in the present case.

15. The three *acids* which I subjected to examination are not equally diamagnetic; *hydrochloric acid* is the most strongly so, *nitric acid* comes next, and *sulphuric acid* last.

* This supposition is also supported by the circumstance that ordinary alcohol burns with a blue flame, whilst in the rectified this was yellow.

46. The deportment of the *ferrocyanogen salts* is most remarkable. In the 46th paragraph of my memoir upon the action of the magnet upon vapours and liquids, I have denoted both of them as diamagnetic; which Faraday, by allowing crystals of the two salts to oscillate, had also found them to be. The fact is undoubted in the case of the *ferrocyanide*, though I must withdraw my assertion, that a saturated solution of this salt is more strongly diamagnetic than water. It was based upon the observed motion which this solution, contained in a watch-glass and placed upon the approximated poles of a magnet, assumes on closing the circuit, an indefinite mode of estimation, which, probably on account of the slight transparence, preponderated in favour of the solution of the ferrocyanide. But the case is quite different as regards the assertion that the *ferridcyanide* is diamagnetic; on the contrary, it is *decidedly magnetic*. My former statement refers to a period at which I was unacquainted with the results contained in my memoir on the repulsion of the optic axes of crystals by the poles of a magnet, and arose from giving a false interpretation to a correct observation.

In the first series of experiments, a tolerably concentrated solution of the ferridcyanide, obtained from a chemist's shop, was found to be decidedly magnetic. Its *magnetic attraction* amounted to 164, placing the *diamagnetic repulsion* of water at 100. To control this result, in the second series of experiments I examined crystals of the ferridcyanide, procured from a chemical manufactory, these were finely powdered, and the watch-glass then filled with them. They were strongly magnetic; for the same weight, they were 7·4 times as strongly magnetically attracted as water was diamagnetically repulsed. I immediately supposed that the contrary assertion might have arisen from a magnetic axial action. To decide this point, I selected two crystals, one a small one, which Prof. Bergemann gave me as chemically pure; and a larger one, which I had long had in my possession, and which came from the Schonebeck manufactory. Both crystals, when suspended so as to oscillate horizontally between the *approximated* apices of the poles, on closing the circuit flew to the nearest of the apices of the poles; this occurred even when the current was excited by a single cell instead of ten. But when the crystals, by shortening or elongating the silkworm-thread by which they were suspended, were slightly raised above

or depressed below the line of the apices of the poles, they decidedly assumed an equatorial position, as a strongly diamagnetic uncrystalline body of the same form would have done. I shall again recur to this subject on some future occasion; for the present I must leave it. It is however so far certain that the ferridcyanide derived from different sources is magnetic.

17. The magnetism of the ferridcyanide, in opposition to the diamagnetism of the ferrocyanide, is the more remarkable, as the latter ($FeCy+2KCy$) is a compound of the protocyanide of iron, and the former (Fe_2Cy^3+3KCy) of the percyanide of iron with cyanide of potassium; whilst the percyanide is a form of combination in which the amount of iron compared with the cyanogen diminishes in a greater proportion than in the protocyanide. In a certain sense, the different reaction of the proto- and perchloride of iron, in which case (in solution) the latter is certainly not *magnetic*, but less *diamagnetic* than the former, forms an analogy to this.

The magnetism of the ferridcyanide appears to be too great to suppose, as occurred to me for a moment as probable, that it might be ascribed to an admixture of protochloride of iron, the amount of which would then be too great.

18. Finally, if we glance at the last column of the table in the 11st paragraph, which, *for equal weights*, gives the diamagnetism of the different substances, it is evident that this diamagnetism in all the substances enumerated, which are not mixtures in indefinite proportions, may be expressed within the limits of error of observation by perfectly simple numerical relations. The greatest deviation, one-nineteenth, occurs in sublimed sulphur and chloride of sodium; but even in these cases the probable error is greatest, on account of the want of uniformity in pressing in the powdered substance. The simple numerical relations alluded to are:—

Phosphorus, water, sulphuret of carbon and hydrochloric acid	1
Sulphuric ether and oil of turpentine	[illegible]
Sublimed sulphur and chloride of sodium	$\frac{3}{4}$
Nitric acid	[illegible]
Oxide of bismuth and sulphuric acid	$\frac{1}{3}$
Mercury	$\frac{1}{4}$

Are these relations accidental, or will they be generally confirmed? We must wait and see whether the latter occurs.

§ 4. *On the Comparison of the Intensities of magnetic attraction and diamagnetic repulsion.*

49. In my memoir on the relation of magnetism to diamagnetism, I have shown, that when magnetic and diamagnetic substances are mixed, and hence magnetic and diamagnetic forces exist together, the former forces decrease less in proportion to the *increase of the distance* than the latter; hence that the *same body* may at one time react like a *magnetic*, at another like a *diamagnetic* body. It thus follows that it is impossible to express generally by numbers the relative intensities of magnetic and diamagnetic forces, for how could this be possible when the same body, according to its distance, is at one time attracted, at another repelled by the electro-magnet, so that in the case of the same body the active force may not only diminish, but also change its sign? In a later memoir upon diamagnetic polarity, I have shown that in the phænomena above mentioned the distance comes into consideration, not as such, but merely inasmuch as *the force of the electro-magnet diminishes* with the distance from the poles, that, at least, the same body may be diamagnetically repelled by a *powerful* electro-magnet and attracted under the same conditions by a *weaker* one; that *when the force of the electro-magnet increases, the diamagnetism increases in a greater proportion than the magnetism.* The law deduced in the former memoir hence holds good, merely requiring a different theoretical interpretation and becoming extended. But now, according to my view, the expression by absolute numbers of the quotients of the magnetic attraction of one body and the diamagnetic repulsion of another, must not be attempted, this quotient is a function of the strength of the electro-magnet.

I shall next describe a series of observations, which at first sight appear very surprising, but on further consideration are a necessary consequence of the laws detailed in the previous paragraph

50 In the determination of the intensity of the diamagnetic repulsion of phosphorus, as described above, the watch-glass filled with this substance was suspended from one end of the beam, and balanced so as to be kept oscillating close above the two halves of the keeper. On exciting the magnetism by ten Grove's cells, it was attracted, and a weight of about 0·25 grm.

in the scale hanging from the other end of the beam was requisite to separate the watch-glass from the two halves of the keeper; but after it had been withdrawn, the watch-glass at a certain distance (about 50 millim.) from the electro-magnet, *was held just by the latter* in such a manner that when further removed from the magnet it was attracted, and when *more approximated* to it, it was repelled. On opening the circuit, the watch-glass containing the phosphorus separated further from the electro-magnet.

When only two cells were used the phænomenon described above was still better observed: even a less excess of weight separated the watch-glass, and the latter when in greater proximity to the electro-magnet assumed the repose of *stable* equilibrium. When this had ensued and the magnetism was then withdrawn, the watch-glass separated completely from the electro-magnet.

The same phænomenon was very well seen with ten cells, when the phosphorus was removed from the watch-glass and placed immediately in the brass ring. The position of equilibrium in this case existed at a distance of 4 millim. to 5 millim. from the two halves of the keeper.

51. Finally, when mercury was placed in the watch-glass instead of the phosphorus, its withdrawal then ensuing with the slight excess of weight 0·05 grm., the watch-glass assumed a stable position of equilibrium at a very small distance (about 1 millim.) from the halves of the keeper, so that at a little distance it appeared still to adhere to the halves of the keeper. A considerable excess of weight was requisite to remove the watch-glass further from the halves of the keeper; this separation occurred immediately when the magnetism was withdrawn. In another experiment 120 grms. of mercury were poured into a porcelain cup, spherically rounded at the bottom, and this was suspended in a brass ring. As the attraction of the empty cup with the ring was too feeble, an iron rod (arranged axially) was fixed by means of wax to the corresponding beam; on using ten cells the attraction then rose to 1·20 grm. The cup with mercury was then withdrawn by a load of 0·80 grm., so that we get 0·40 grm. as the diamagnetic repulsion of the mercury. The stable position of equilibrium occurred at an elevation of 1 millim. to 2 millims.; on opening the circuit, the cup was raised more than 100 millims.

52. The experiments in the two preceding paragraphs possess demonstrative power only when the position of equilibrium with closed circuit is quite in the vicinity of the poles, so that the tendency of the balance to return to the oblique position of equilibrium before the closing of the circuit, acting in the same direction as the diamagnetic repulsion, may not increase the appearance observed, and under certain circumstances alone produce it. It is therefore desirable to possess some modification of the preceding experiments, in which no such disturbing action occurs, which might so easily lead to false conclusions.

I again suspended a watch-glass, in which was placed a rounded piece of bismuth, in the usual manner over the approximated poles, to one arm of my great balance, and then by counterpoids brought it into the horizontal position of equilibrium. At the same time there was an arrangement which allowed of the watch-glass being placed at different distances above the poles, by raising or lowering the balance without the equilibrium being disturbed. After this had been determined, the electro-magnetism was produced successively from a different number of Grove's cells. The magnetism and diamagnetism must then show their presence by the attraction and repulsion of the watch-glass. The magnitude of this attraction and repulsion, measured by the raising and lowering of the watch-glass, is given in the following series of experiments, so that an approximate value may be made of the preponderating magnetic or diamagnetic force:—

I. The watch-glass in contact with the armature:—

			millims.
Number of cells	8	Repulsion	5·0
... ...	2	...	0·5 (scarcely)
... ...	1	No perceptible effect	

II. The watch-glass raised 1·5 millim.:—

Number of cells	8	Repulsion	3·5
... ...	4	...	2·25
... ...	3	...	1·5
... ...	2	...	0·5
... ...	1	Attraction	1·0

III. The watch-glass raised 3·5 millims.:—

Number of cells	8	Repulsion	1·0
... ...	4	Attraction	1·0
... ...	1	...	3·0

IV. The watch glass raised 5·5 millims.—

			millgr.
Number of cells 8		Attraction	3·0
	4		3·2

V. The watch glass raised 8·5 millims.—

Number of cells 8		Attraction	5·0
	4		5·0

By merely comparing a couple of the data from the preceding observations, it is seen how, with the same suspension of the watch glass containing the piece of bismuth, the entire mass, at a distance of 3·5 above the poles, is *diamagnetically repelled* and *magnetically attracted* with nearly equal force, according as the current is excited by *eight* or *four* cells; further, that the magnetic attraction increases considerably when the electro-magnets are weakened by using only *a single cell* instead of *four cells*. It is also distinctly evident how a greater distance from the poles, corresponding to a diminution of the power of the electro-magnets, produces the same effect. When magnetic attraction exists with a given strength of current, we do not obtain the greatest effect nearest to the poles; on the contrary, this greatest effect takes place *at a considerable distance* from them; it *decreases even to evanescence* by approximation to the poles of the electro-magnet if the latter is sufficiently powerful, and then by continuing the approximation *diamagnetic repulsion* is apparent, which constantly *increases* till the glass is in contact with the poles. On employing *eight* cells, the point of indifference is situated at a distance of about 4 millims. from the poles, and with *four* cells from 1 millim. to 2 millims. nearer to the poles. The maximum magnetic effect appears in both cases, at least with eight cells, not to be attained with a distance of 8 millims.

53. In performing the preceding experiments, we constantly observe, that even in the case of the most decided diamagnetic effect, at the moment of closing, no *repulsion*, but rather a very evident *attraction*, occurs, and that this is converted into repulsion only after the lapse of some time. The explanation of this phenomenon must be sought for in the fact, that when the circuit is closed the power of the magnet attains its entire strength only after a certain time, and not instantly. The observation in question is consequently a fresh confirmation of our law.

54. All the phænomena described, with all their modifications,

are perfectly explained by the fact that the electro-magnet acts throughout upon a combination of magnetic and diamagnetic substances, and by the magnetism diminishing less in proportion than the diamagnetism with the force of the electro-magnet, and thus also with the distance from it. This is in fact what I had previously stated, even in the first of the two above-mentioned memoirs, without being aware of the phænomena which have been described, in the following words:—"It appears moreover to be a necessary consequence of the results obtained, that the same body, perhaps in the form of a sphere, at a greater or less distance from one of the poles of the magnet, may at one time be repelled throughout its entire mass, at another may be attracted."

55. Thus if we could increase the power of the electro-magnet to such an extent, that not only the magnetic attraction, but also the diamagnetic repulsion, exceeded the force of gravity, we should have the remarkable phænomenon, *that a body formed from a proper mixture of magnetic and diamagnetic substances, and oscillating freely in the air above the poles of the magnet, would be retained by the latter.* The experiment might be performed even now, if we were to invert the poles of the magnet, when the force of gravity might be balanced by the magnetic attraction acting in an opposite direction, instead of by the counter-weight, and then the excess of the magnetic attraction alone over the gravity and the diamagnetic repulsion would remain active.

56. That the diamagnetism increases more rapidly than the magnetism when the strength of the electro-magnet increases, may be confirmed by direct weighings.

To show this provisionally, I took a hollow hemisphere of sheet brass, and suspended it in the same manner as the watch-glass. About 115 grms. of bismuth were then fused in it, and allowed again to cool. The solidified mass could be taken out, and again replaced. On using in succession *two, three* and *ten* cells, the attractions of the empty brass cup amounted respectively to—

grm.	grm.	grm.
0·69	1·13	2·15,

and that of the brass cup containing the bismuth to—

0·53	0·71	0·48,

whence we have, as the diamagnetism of the bismuth,

0·14	0·42	1·67

If in the above determinations, a weight of 0·60 grm. had been placed in that scale-pan which receives the weights for the separation, the brass cup containing the bismuth would be *repelled* by the electro-magnet in the first and third determinations, but *attracted* in the second.

57. In the sixth paragraph of my memoir of the 8th of September, I have drawn the conclusion on theoretical grounds, that *by the admixture of two substances, one of which is magnetic, the other diamagnetic, we cannot procure a body which is absolutely indifferent to the magnet*. I was then only enabled to attribute this to a body which is indifferent at a given distance becoming diamagnetic on the diminution of the distance, and magnetic on its increase. This may now be extended to the effect that the same body may under exactly the same general circumstances become magnetic under the influence of feeble magnetism; but when the magnetic force is increased, passing through the indifferent state, it may become diamagnetic. A direct confirmation of this appeared to me desirable. I therefore took a gramme of the crystals of protosulphate of iron which had recently formed; these I carefully dried, dissolved in 50 grms. of distilled water, and filled the watch-glass which had been used in the determinations of intensity with the solution. When the electro-magnetism was excited by two cells, it was found, by the method made use of in the former determinations, that the watch-glass with its contents was somewhat more strongly attracted than the empty watch-glass. But when *ten* cells were set in action, the reverse occurred. Thus the solution reacted *magnetically* in the first case, and *diamagnetically* in the second.

The watch-glass, with its cover and ring, was still too strongly magnetic to give these kinds of determinations all the accuracy of which they are susceptible. The above proximate determination of the point of indifference is in general more accurate than the corresponding one in the 22nd paragraph of my memoir of the 22nd of January 1848, because in the latter the evaporation of the uncovered fluid, arising from its small quantity and great extent of surface, interferes with the result.

Faraday states that 18·6 grs. of crystals of the protosulphate of iron are insufficient for the removal of the diamagnetism of 10 cubic inches of water. From this it is evident, placing the weight of the English cubic inch of water at 252·46 grs., that the proportion is as 1 : ...

58. We have already (23.) found the magnetism of *protosulphate of iron dissolved in water* to be equal to

125,

the magnetism of the iron for the same weight being placed at 100,000. If in the same space which was previously filled with the unit of weight of the green vitriol, we now diffuse merely the fiftieth part, the magnetism becomes reduced to

2·5.

Its intensity would thus amount to only the 40,000th part of that which occurs with a unit of weight of iron.

If we assume the same number as the measure of the diamagnetic repulsion of the water, a gramme of water uniformly diffused within the watch-glass, on using *six* Grove's cells, would experience a repulsion of about $\frac{10}{10000} = \frac{1}{800}$ grm. (18.). Hence the water filling the watch-glass, which weighs about 11·5 grms., would suffer a repulse of

0·14 grm.

This number is somewhat greater than that found by observation; it was obtained with the use of *ten* cells.

From this it appears, that if we regard the removal of the magnetism of the green vitriol as a compensating power, *a large amount of magnetism is requisite to neutralize a small amount of diamagnetism.*

This result, if it is all generally applicable, is evidently connected with the fact, that magnetism increases less in proportion to the increase of the force than diamagnetism, which appears to lead to the conclusion that a greater coercive force is opposed to the excitation of the latter.*

If, in the preceding development—but evidently however with less reason—instead of the magnetism of the green vitriol in an aqueous solution, we took the magnetism of the solid green vitriol, we should have

0·08 grm.

as the diamagnetic repulsion of the water in the watch-glass, which would be too little, for this was found to be the repulsion when *four* cells only were used.

§ 5. *On the Influence of Heat upon the Intensity of Magnetism and Diamagnetism.*

59. The influence which heat exerts upon magnetism has been a subject of numerous investigations. However, the influence

* Compare my Memoir of the 21st February 1848.

which it exerts upon permanent steel magnets has alone been examined with accuracy. This permanent magnetism is destroyed by a white heat; it was also found that at this temperature iron ceased to be attracted by a magnet. The observations of M. Pouillet refer to this point: he found that cobalt, even at the highest temperature, remains magnetic; that as the heat increases, chromium ceases to be magnetic a little below a red heat, nickel at 662° F., and manganese at 68° to 77° F. below zero. After the discovery of diamagnetism it occurred to me whether the magnetic condition of the body at these limits might not have passed into the diamagnetic state. But Faraday found that white-hot iron was always distinctly magnetic, although but slightly so. He was never able to observe a transition into the diamagnetic state, nor has he observed any influence upon the diamagnetism of solid and fluid bodies. He merely imagined, quite recently, after having observed that warm air is more strongly diamagnetic than cold air, that heat might increase the diamagnetism of all bodies. The method adopted in our determinations of the intensity gives here also the most certain explanation.

60. A hollow hemispherical cup of sheet brass, 56 millims. in diameter, was filled with white sand, and a small piece of sheet iron placed horizontally in such a manner that the sand formed a layer 6 millims. to 8 millims. in thickness above it. Three thin silvered copper wires, which converged superiorly, were fixed to this cup, and supporting the scale-pan, could be suspended to the beam of the balance. The cup with the sand was heated over a coal fire, suspended to the balance, brought into equilibrium, and placed as usual above the approximated round halves of the keeper; the magnetism was then excited by the current of a single Grove's cell, and the weight which was requisite to pull off the cup was determined. These determinations of weights were repeated constantly during the gradual cooling. To avoid the loss of time, this was effected by gradually placing first shot, and then fine sand, both of which were subsequently weighed, into the other pan of the balance until the separation ensued. To determine about the temperature, which at the first separation might be 572° F., the time at which the cup was withdrawn in each case is given in the following table, together with the weight requisite to produce the separation. I may remark here that after the fourth separation the cup still hissed when touched

externally with the wet finger. The current was not interrupted during these determinations of weight. The weights are those directly required for the separation, correction appears unnecessary, because the cup with the sand only is scarcely at all affected by the electro-magnet.—

Time h ′ ″	Weights for separation grms
9 50	153 70
9 52 30	158·25
9 54	159·80
9 56	161·70
9 57 30	162 40
9 59 15	163·16
10 15	166·75

We thus see how the magnetism of sheet iron, on cooling to the temperature of the room, *continually increases.* The difference amounts to 8 per cent. of the intensity determined at this temperature.

61. *Peroxide of iron* was next examined. For this purpose, instead of the brass cup, I took a somewhat smaller one of porcelain, which was suspended to the balance in a brass ring. The peroxide, about 25 grms., was heated to 752° F. at least; but its temperature at the first weighing had sunk to about 572° F. Nine weighings, immediately following each other, were taken; and after the ninth weighing the temperature of the peroxide was 84° F. The current was excited by three of Grove's cells. The attraction of the empty porcelain cup, which amounted to 0·14 grm. only, has been deducted in each case:—

	grms
1. Weighing	52·01
2. „	56 97
3 „	59·54
4. „	63·04
5. „	65·96
6. „	67·61
7 „	67·91
8. „	68·67
9. „	69·61

The magnetism of the peroxide of iron, at the highest temperature observed, is thus *rather more than 25 per cent. less than at the temperature of 74° F.* For the same temperatures it dimi-

nishes in the case of the peroxide of iron *more rapidly* than with iron

62 Lastly that protoxide of nickel the magnetism of which was previously determined was submitted to experiment in exactly the same manner as the peroxide of iron above The heat required was about the same, but six Grove's cells were used to excite the current The following are the corrected magnetic attractions —

		grm
1	Weighing	0 963
2		0 963
3		1 082
4		1 150
5		1 206
6		1 325
7		1 182

After the fourth weighing the porcelain cup still could not be held in the hand without pain, but this could be done easily after the fifth After the last weighing but one the temperature of the cup was below that of the blood Hitherto the weight required for the separation was determined by placing in the proper scale pan pieces of paper the size of which was constantly diminished and the total weight of which was subsequently determined From this time the constant increase of the magnetism was observed directly for six to eight minutes because more weights could constantly be added gradually to the last but one In this way the last weight was found

The above results are extremely remarkable because *a considerable change* in the original elevated temperature *did not produce any change in the intensity of the magnetism of the protoxide of nickel* The two first weighings agreed perfectly The subsequent ones show at least, *that the magnetism at a lower temperature increases simultaneously with the latter in a more accelerated degree than at a higher temperature*

The deportment of the oxide of nickel is probably in close connexion with the observation of M Pouillet above mentioned, that nickel ceases to be magnetic at 660° F, which, in accordance with the more recent investigations, signifies merely that the magnetism is reduced to a minimum

Is this limit, at which the magnetism vanishes, the same in the case of the protoxide of nickel also?

63 To determine the influence which heat exerts upon the diamagnetism of bodies, I first took bismuth In the brass cup mentioned in paragraph 60, I fused 116 grms of this metal, it was heated to beyond the point of fusion, and then placed above the pole of the magnet The magnetism was excited by eight Grove's cells (the nitric acid had been used once previously) After the attraction of the empty cup had been found to be

1 97 grm,

the attraction of the cup containing the fused bismuth was determined in the manner described above, and during the gradual cooling and solidification of the bismuth, these determinations were continued, without opening the circuit, until the mass had again acquired the temperature of the room

In the first experiment, the weights requisite to produce the separations successively were—

0 95, 0 76, 0 41, 0 37, 0 35, 0 235, 0 19, 0 35, 0 38, 0 35, 0 35 grm

The first experiment proved beyond a doubt, in opposition to the expectation which I had based upon Faraday's opinion, that the intensity of the diamagnetism diminishes at more elevated temperatures The relative measure of these for the limits of temperature in the experiment was

1 02 and 1 62 grm

A glance at the results of the weighings gives rise to the supposition that during the sixth and seventh weighing the balance was not in perfect order This being premised, the diamagnetism ceases to alter when the bismuth has cooled down to a certain temperature The third before the last only would then contain a small error, which is easily explained by the nature of the process being one in which haste is unavoidable

64 The same experiment was repeated, with every precaution, in the same manner, except that ten Grove's cells (with nitric acid which had been once previously used) were applied, and the fused bismuth was of a higher temperature The attraction of the empty brass cup amounted to

2 15 grm

The weighings, which were then uninterruptedly continued in succession, and in which counterpoise was effected by means of paper and fine sand, yielded the following results —

	Attraction of the cup containing the bismuth	Diamagnetism of the bismuth
1	1.87	0.28
2	1.49	0.66
3	1.11	1.04
4	0.91	1.21
5	0.79	1.36
6	0.68	1.47
7	0.64	1.51
8	0.62	1.53
9	0.57	1.58
10	0.42 (uncertain*)	
11	0.49	1.66
12	0.48	1.67

To give some idea of the temperature of the bismuth, I may mention, that in the fourth weighing the metal in a fluid state escaped from the interior through the solidifying upper crust, that the temperature after the tenth weighing was judged to be at 158° F. to 176° F.; after the eleventh at about 131° F., and after the last at about 104° to 113° F. After this last weighing I satisfied myself merely that the attraction remained constant, or at least did not vary ·5 milligrms.

65. Hence it is indisputably certain, *that the diamagnetism of bismuth diminishes as the temperature increases*. This diminution is considerable. During the experiment described the intensity of the diamagnetism, which on the contrary increases as the temperature diminishes, was augmented *sixfold*.

If we suppose that during its cooling the bismuth becomes oxidized, and thus increases in weight, or that some magnetic body (iron), the magnetism of which increases as it cools, remains mixed with it, both of these causes would affect the result obtained, so as to augment the increase of the intensity of the diamagnetism of the bismuth.

The above result is especially remarkable, because by it the hypothesis that magnetism and diamagnetism when once called into action are an identical excitement of matter, is sup-

* The cup moved from the halves of the keeper when weights had not been placed in immediately before. When the attraction was diminishing, an observation of this kind would be most accurate, if the supposition that an unobserved concussion might have caused the separation had not arisen.

ported *by both being modified in the same manner by heat, this view has already been shown to be borne out by both exhibiting polarity.*

66. A series of important questions is connected with the above experiment, which was also subsequently repeated in a porcelain cup, and the same result obtained.

Is there a limit to diamagnetism, so that at a certain degree of temperature it entirely disappears or is reduced to a minimum, as is the case with the magnetism of iron, or other magnetic metals? In the case of bismuth this limit would lie between 572° and 752° F.

67 During the last experiment the state of aggregation of the bismuth became changed. It occurred to me to investigate by experiments with other diamagnetic substances, whether the transition from one state of aggregation to another exerts any influence upon the intensity of the diamagnetism; such indeed does not appear to be indicated by the last weighings.

I first selected *stearine.* This was fused exactly as in the experiment with bismuth, in the same brass cup, and heated considerably above the boiling-point. On applying a current of the same intensity, it proved to be always *equally diamagnetic,* even during its solidification, and until it acquired the temperature of the room, at least the difference in the attraction of the cup filled with stearine did not amount to 5 milligrms.

68. 7 5 grms. of sublimed sulphur were then taken, fused in the hemispherical porcelain cup, and heated above its point of fusion. The cup was 45 millims in breadth at the top, but as it was not sufficiently magnetic to overcome by its attraction the repulsion of the substance within it, a rod of iron 60 millims. in length and 4 millims in diameter was axially directed, and fixed by means of wax to the corresponding end of the beam, as in a former experiment. On using ten cells and fresh nitric acid, a weight of

1·200 grm.

was required to separate the empty porcelain cup. On instituting the experiment as before, and counterpoising again after each experiment, the following weights were found requisite to separate the cup containing the sulphur, which was at first in a state of fusion, but after the third weighing began to solidify, from the halves of the keepers—

		grm.			grm.
1.	Separation	0.956	4.	Separation	0.956
2.		0.968	5.		0.956
3.		0.968	6.		0.956

Diamagnetism = 0.211 grm.

It is hence evident, that the temperature within the limits of the experiment exerts *no* influence upon the diamagnetism of the sulphur, or at least one which is scarcely perceptible.

69. Lastly, 120 grms. of mercury by weight were subjected to examination in the same cup and with the same adjustment. In this case, after each separation, the temperature of the mercury could be determined without disturbing the experiment, by the immersion of a thermometer. The temperature determined in this manner is however merely approximative, and somewhat less than that corresponding to the moment of the separation:—

		Attraction.	Temperature.
1.	Weighing	0.791 grm.	260 F.
2.		0.788	180
3.		0.806	144
4.		0.791	124
5.		0.806	114
6.		0.806	100
7.		0.806	91

Diamagnetism = 0.400 grm.

Hence, in the case of mercury also, the intensity of the diamagnetism at different temperatures is *invariably the same*.

The different weighings here control each other, and thus also afford a measure of the accuracy of the series of experiments.

70. I shall now subjoin a final experiment, which was made before I was in possession of the proper porcelain dish, with 144 grms. of *impure* mercury, in the same brass cup in which the bismuth was examined:—

Attraction of the empty cup, 2.15.

	Attraction of the cup and mercury.	Temperature after the separation.	Diamagnetic repulsion of the mercury.	Magnetic attraction of the mercury.
1.	1.84	230 F.	0.31 grm.	
2.	1.82	167	0.33	
3.	2.28	136		0.13
4.	2.28	113		0.13

Thus the impure mercury used was diamagnetic at a higher and magnetic at a lower temperature. This appears to arise from the magnetic substances mixed with it, the magnetism of which diminishes as the temperature increases.

In conclusion, I am about to have an apparatus constructed for the purpose of making accurate admeasurements, in which a thermometer is fixed in the porcelain cup used for the separation, and is separated simultaneously with it; this indicates the temperature at each moment in which the intensity of the magnetism or diamagnetism of the substance to be examined is determined. I shall then be in a condition to subject metallic nickel to experiment.

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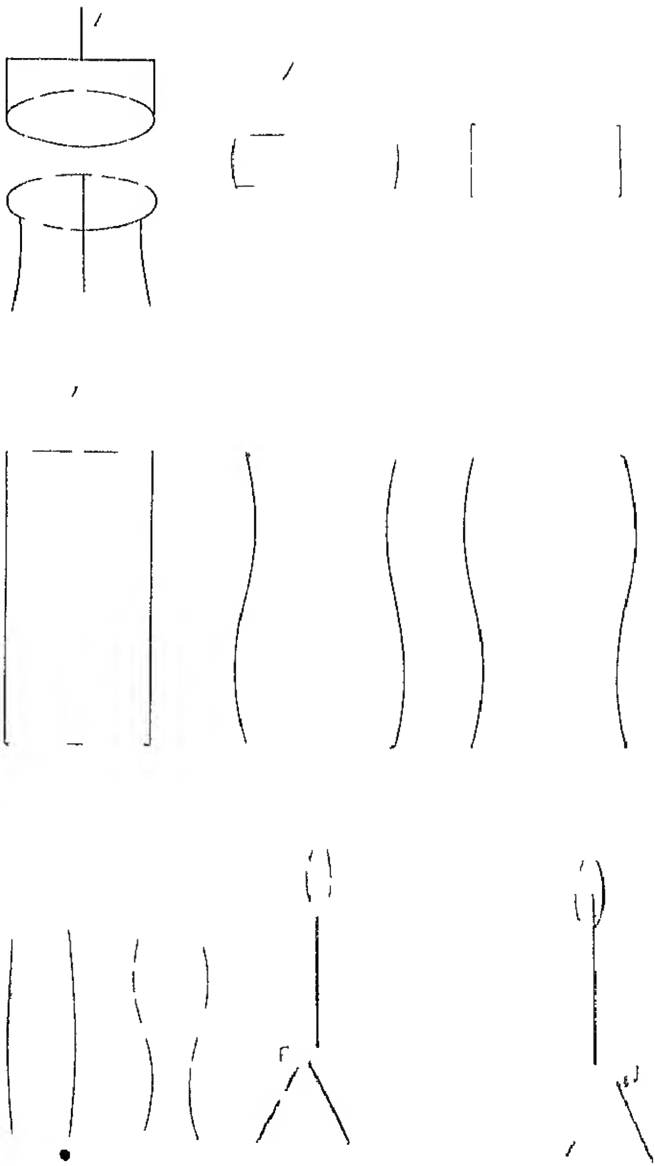
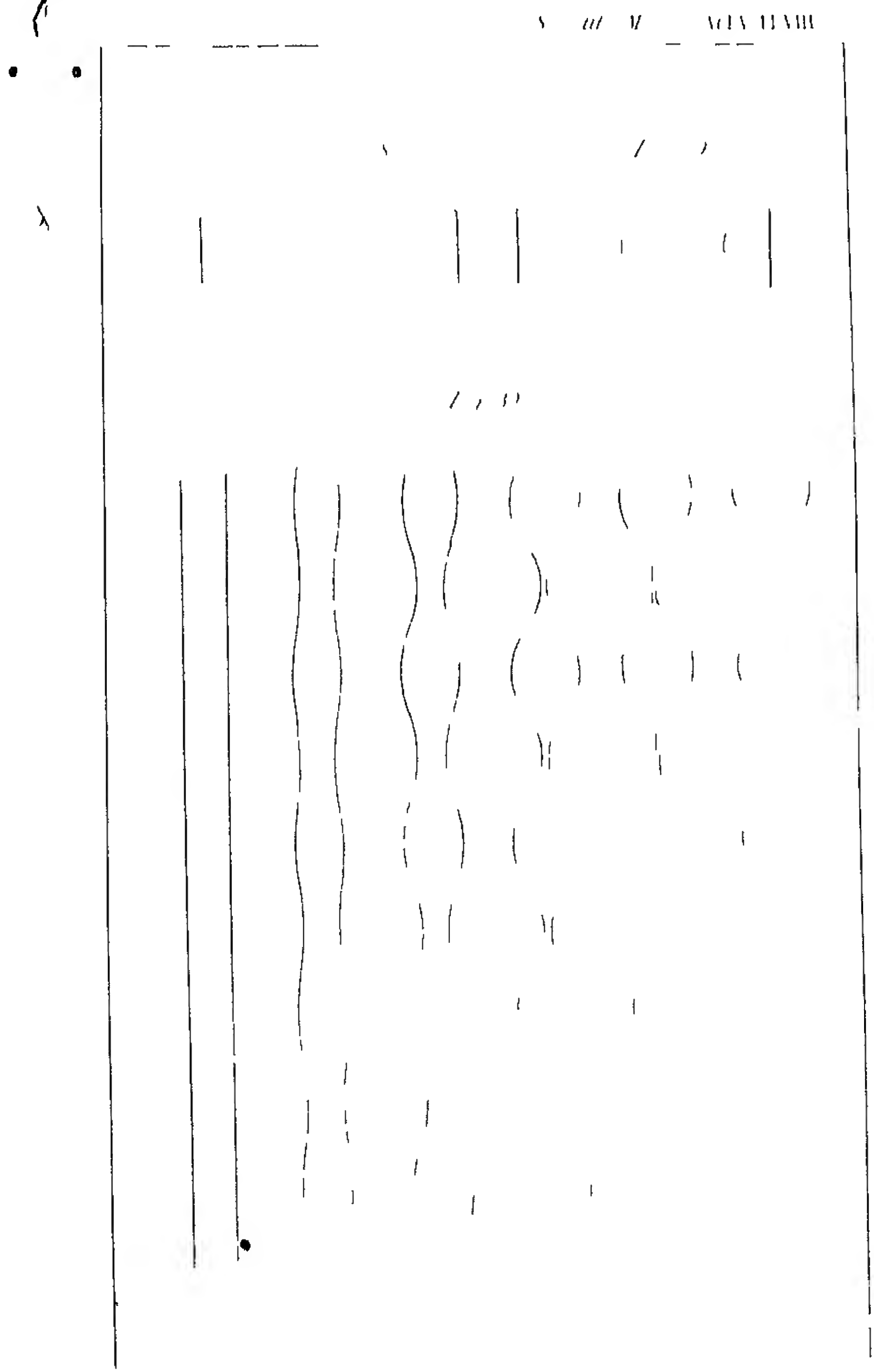




suspended by the use of two springs *y y* in the shape of a Y, which with their two arms compass both cylinders at the same time, the one touching wood whilst the other touches metal, and thus upon the principle of the commutator transform alternating currents into currents of a like direction. The points of contact of the one spring are situated diametrically opposite to those of the other, the one *y* passing from the higher support 10), slides upon the lower surface of both cylinders, the other *y* passing from 2) slides upon the upper surface. This arrangement, applied for the purposes of chemical decomposition, eliminates the gases separately, and moreover in double the quantity they are produced by the usual arrangement, in which the opposing current is not reversed, but is suspended by interrupting the connexion.

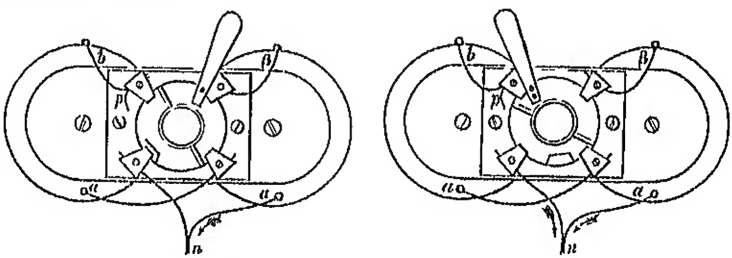
The different combinations of the springs are accordingly the following:—In common experiments without the insertion of a spiral for the production of the extra current, 9) and 3) slide upon the cylinder w_2, as is depicted in fig 7; upon the cylinder w_1, however, instead of the spring proceeding from 5), one that proceeds from the clamp 1), and moreover 1) and 9) continuously, 3) on the contrary intermittently. Alternating currents are however obtained when that which has hitherto been a secondary connexion becomes a chief connexion; currents in a like direction, when it is inclined obliquely, and slides on the once interrupted edge. The galvanometer, the apparatus for producing incandescence in platinum and charcoal, as also the human body, are inserted between 4) and 8). For uninterrupted currents in the same direction, *y y* alone are used. The arrangement with an inserted spiral for alternating currents is represented at fig. 7. When the sparks of the secondary current are not to be examined, the springs 13) and 14) are left out. With currents of a like direction, the springs *y y* are inserted alone in the clamps, whilst the apparatus for measuring the currents is inserted between I and III instead of between I and II. If the current is to be interrupted often during one revolution of the keeper, the spring 3) is made to slide upon the cylinder w_3.

The weight of the covered wire is 1220 grammes, the thickness of the uncovered wire is about $\frac{1}{3}'''$, its length 880$'$. The height of the cylindrical rolls of wire is $1\frac{1}{4}$ inch, their diameter $14'''$, that of the outer coil $2\frac{1}{2}''$. The front iron plate of the

keeper is $5''$ long, $2''$ broad, and $\frac{1}{2}''$ in thickness. Each of the four cylinders w has a diameter of $16'''$, the magnet, consisting of four lamellæ, is $10''$ long, the height of the four pieces together is $22''$. The internal distance between the poles is $1''$, the external $4\frac{3}{4}''$. The rotating wheel is at the side, and revolves obliquely to prevent the abrasion of the crossed cord; it can be drawn out from the base of the machine, by which means the requisite amount of tension can be given to the cord. At each turn of the wheel the keeper revolves $8\frac{1}{2}$ times. The support extending from 8 to 11 on the left side is $5''$ high; the supports on the right hand are only $2''$ high, by which means the side view of the apparatus is better seen. The distance of the rotating keeper from the magnet is regulated by the screws between which the axis turns. The two wire coils surrounding the limbs of the keeper can be connected in a twofold manner, either so that the one forms a continuation of the other, or that both are connected at their two extremities so as to form a so called parallel connexion $110'$ in length. The changes in the intensity of the resulting current which are produced when the wire is coiled in a particular manner, have lately been shown by M. Lenz*. For if L represents the resistance to conduction of one of the coils, Λ the resistance to conduction of the apparatus inserted for measuring the current, then with a parallel connexion there are two ways presented to the current induced in the wire coil at its exit, namely the apparatus for measuring the current and the other wire coil, between which it divides itself in an inverse ratio to their resistance to conduction. If A therefore represent the electromotive force of a coil of wire, then with a parallel connexion a current of the intensity $\frac{2A}{2\Lambda + L}$ will circulate through the measuring apparatus; if on the contrary, the connexion is continuous, a current will pass of the intensity $\frac{2A}{\Lambda + 2L}$. If therefore the apparatus for measuring the current offers as great a resistance to conduction as one of the electromotive coils of wire, *i. e.* if $\Lambda = L$, then the parallel connexion is quite as advantageous as the continuous, and there is no occasion in this case for any arrangement to effect both connexions. As however the same machine has to be used with different kinds of apparatus for measuring the current, and it is not convenient to

* *Bulletin Scientifique de l'Académie de St. Pétersbourg*, ix. p. 78.

have a keeper specially coiled for each, it will be found advantageous to use the second combination for such modes of measurement as offer great resistance to conduction, and the first for such as offer but little resistance. Now, as the human body offers the greatest resistance of all the modes of measuring the current that have been applied for the purposes of *physiological* experiment, the successive connexion is to be preferred to the parallel; on the contrary, the parallel connexion will be found more applicable for producing incandescence in platina wires and charcoal points, for the sparks which accompany the interruption of a short connexion, and for the magnetization of soft iron which is enclosed in a connecting spiral. Which of the two combinations is preferable for chemical decomposition will depend, for a given thickness of wire, upon the distance between the electrodes of the voltameter, and upon the resistance to conduction offered by the electrolyte. The parallel combination of the wire coils may therefore be called with as much right the *physical*, as the successive connexion is called the *physiological*. The apparatus which effects both kinds of connexion by turning a hand, and which may be called a pachytrope, is not represented on the square piece of wood attached to the keeper in fig. 7, in order not to complicate the drawing, but it is depicted by itself in the following woodcut, and in the first position of the moveable hand on the right side for continuous connexion, *i. e.* for physiological action, in the second, for parallel connexion, *i. e.* for physical action.



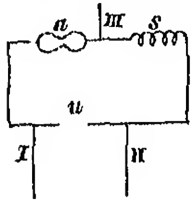
The disc of copper, which is exactly divided into two halves, is fixed upon a wooden support, and can be turned by means of the handle under the four plates of copper, so that the handle is alternately in contact with the left or the right upper plate. a b are the ends of the one wire coil, $\alpha\,\beta$ those of the other, p is a connecting wire on the plate, under which b is

clamped, to the axis of the keeper, and by means of that to the cylinder w_2 (fig 7), n is a connecting wire from the left hand plate passing underneath to the cylinder w_1 with which the end a is directly connected The portion cut out of the copper disc is supplied by ivory In the position of the handle to the right in the first figure, this plate of ivory is exactly below the copper plate under which n is clamped, consequently the commencement only of the right hand wire coil α is in connexion with the cylinder w_1, the end β by means of the right half of the copper disc is connected with the commencement of the second spiral a, whose end b is brought into conducting connexion with the cylinder w_2 by means of p This connexion is therefore $n\,\alpha\,\beta\,a\,b\,p$, *i e* both coils are connected one behind the other In the second figure, on the contrary, α is connected with a by means of the left half of the copper disc through the wires which cross each other without touching β by means of the right half is connected with b, the connexion is therefore $n\left\{\begin{matrix} a\,b \\ \alpha\,\beta \end{matrix}\right\}p$, the currents of both coils are therefore united with each other as well on entering as on leaving them

69 Suppose in fig 7 the pachytrope fixed to the base of the keeper, the cylinders w_3 and w_4 removed as also the supports E, I, G and likewise the springs proceeding from them and the extra spiral, we have then Saxton's machine with the improvements of M Oertling For the purpose of making the following experiments I have added the parts just mentioned In this form the apparatus can be recommended as a very convenient instrument for demonstrating the action of the extra current at the commencement and end of a primary current The support L is intended to effect the insertion of the spiral for producing the extra current The cylinder w_1 is connected with the spiral through the spring proceeding to 5) by means of the wire S clamped at 6) The connexion then proceeds further through 1) and 3) by means of the intermittent spring to the cylinder w_2 The two other supports F and G, as well as the cylinder w_4 are only used for the purpose of showing the sparks of the secondary current and will be noticed hereafter, § 83 The cylinder w_3 constructed upon the principle of a lightning wheel or mutator, presents eighteen interruptions to the intermittent spring 3) It is used for the purpose of rendering perceptible the increase and decrease of the physiological action during one whole revolution of the keeper The cylinders w_1 and w_4 are fixed

in an insulated manner upon the common axis of rotation; the cylinders w_0 and w_2, on the contrary, are directly fixed upon it, and are therefore in conducting connexion with it. The spiral S S, which may be called the extra spiral, was composed, when it is not otherwise expressly noticed, of two coils of well varnished insulated copper wire, each 400′ in length, of which only one is represented in fig. 7. The thickness of the wire is half a line, the internal diameter of the coil from 2½″ to 1¾″. These two spirals can be connected in a uniform manner or crossways; as it is well known that this has no influence upon the extra current, this arrangement affords us a simple mode of ascertaining whether we have really to do with this current or not. Into the three supports I, II, III, wires are screwed, either two of which may be connected by means of handles* through the body, or by the voltameter or galvanometer, as has already been mentioned.

The apparatus is therefore arranged in the manner represented below, where *a* represents the rotating keeper with its coils, *s* the extra spiral, *u* the interruption by means of the intermittent spring (3) upon the cylinder w_2, and lastly, I, II, III the wires leading to the apparatus for measuring the current. These last admit of three different modes of connexion, namely, I with II, I with III, and II with III. In the first mode the keeper and the extra spiral are in the circuit, in the second the keeper only, and in the last only the spiral.



70. After this detailed description of the apparatus, it will be easy to account for that which occurs when the keeper revolves. During the rotation of the keeper from 0° to 90°, *i. e.* from its horizontal position before the poles of the magnet to the vertical position at right angles to the line connecting the poles, the surrounding wire of the keeper is throughout in metallic connexion, for the spring (3) is always in contact with metal upon the cylinder w_2. The increasing intensity of the primary current in the wire *p* excites in the spiral S an extra current A circulating in an op

* The so called gold strings intertwined with metal, which are used to fasten the handles in the common Saxton's machine, must never be used when the intensity of the physiological action is to be determined; for the intensity of the shock depends essentially with these upon the amount of force with which the strings are stretched. Spirally coiled copper wires firmly clamped with screws, which are sufficiently elastic and always effect a uniform contact, are to be preferred.

posite direction, which consequently weakens the action of the primary current. At this moment the position becomes vertical, the spring 3) comes in contact with the inserted piece of wood *u* upon the cylinder w_2, the primary current of the keeper *a* ceases, and there is then excited an extra current E in the spiral S when the latter forms one continuous connected whole, which is in the same direction with, and increases the action of the primary current. If the formation of this second extra current in the same direction with the primary current is to be prevented, then, the instant connexion is broken at *u*, the extra spiral S must be removed from the closing connexion. This is effected when I and III are united. If, on the contrary, I and II are connected, we then obtain the primary current *p* weakened by the influence of the incipient extra current A, which, circulating in an opposite direction, is produced during the rotation of the keeper from 0° to 90°, and augmented by the action of the final current F, which, circulating in the same direction with the primary current, is excited when connexion is broken at *u*. In which direction the final action is excited, *i. e.* whether $p - A + E$ is greater or less than *p*, can be ascertained by inserting in place of the spiral S a length of wire not forming a spiral, but offering an equal amount of resistance to conduction*. The connexion of I with II gives therefore the action of the primary current alone. If lastly II and III are connected, we obtain when S is an uncoiled wire no physiological action; when S is a spiral, on the contrary, a current in the same direction with the primary current, *i. e.* the action of the final extra current by itself.

71. With a straight wire inserted we obtain therefore for physiological tests—

With the connexion I and II the current *p*
I and III the current *p*
II and III no current.

With an inserted extra spiral, on the contrary,—

For the connexion I with II the current $p-A+E$
I with III the current $p-A$
II with III the current E.

That no physiological action is obtained with the connexion II

* The most convenient for this purpose is a thin German silver wire bent up and down as the lines in the letter N, or the resistance measurer recommended by Wheatstone, consisting of a wooden and a metallic screw upon which the wire coils and uncoils.

and III, when the inserted wire is straight, is naturally only then the case, when the intensity of the magnet is not very great. If however the magnet used in the machine is a very powerful one, then the influence of the human body as a secondary connexion upon the principal current is no longer unimportant. Very sensible shocks are perceived when a powerful magnet is used even with continuous sliding springs without an inserted spiral. This however was not the case with the machine here described, for however powerful the shocks were with an intermittent spring (quite unbearable when the rotation was rapid), yet none were perceptible with one that slid uninterruptedly. The influence of the body as long as it forms a secondary connexion (0° to 90° and 180° to 270°) may therefore be here disregarded. This facilitates very much the examination of the complicated phænomena in this department, for it follows directly from the absence of physiological action, when a straight wire is inserted and connexion made between II and III, that the powerful action obtained with the spirally coiled wire is solely to be ascribed to the final extra current E. For other rheoscopical tests however, when connexion is made between II and III, the current p takes a greater or lesser part in the results which are obtained.

1. *Physiological action.*

72. Without insertion of the spiral more powerful shocks are obtained, as well with one and twofold interruption (90° or 90° and 270°), when the hand of the pachytrope is arranged for physiological than when it is arranged for physical effects. The whole of the following phænomena, on the contrary, are much more clearly perceived when the hand is arranged for physical effects, in which case the primary current possesses the property of magnetizing soft iron more powerfully*. If I and III ($p-A$)

* If the magnet of the Saxton's machine is removed, and instead of the extra spiral between S and S a galvanic battery is inserted, we obtain from the handles I and II and I and III, when the keeper is rotated, the incipient current of the galvanic battery, for the keeper is by this arrangement converted into a connecting electro-magnet to the galvanic battery, the magnetism of which becomes evanescent as soon as the intermittent spring comes into contact with the inserted piece of wood, and thus induces the extra current in the coils of wire. This induced current passes through the battery and the body when connexion is made between I and II, and only through the body when I and III are connected. The connexion II and III produces no shock, for the electro-magnet is then excluded, and the battery alone remains in the circuit. This shock was more powerful with a keeper used in the machine when the hand is arranged for physical than when it is arranged for physiological effects.

are connected by means of handles through the human body, the shocks are weaker with inserted spirals than without them, the reason of which is obvious. But this physiological action is weakened still more by the insertion of unenclosed bundles of iron wire and tubes of sheet iron into the spirals; it is not so much weakened by the insertion of iron bundles of wire in entire tubes, solid rods of soft iron, of soft and hard steel, of cast iron and nickel; the action remains nearly the same as with inserted empty spirals when the inserted rods are composed of copper, zinc, tin, brass, bismuth, antimony or of the so called unmagnetic metals in general. All these phenomena remain unaltered when the spirals are connected in a like or in an alternating manner. The facts here adduced therefore, indicate the existence of an extra current in an *opposite direction* to the primary current: and moreover, no difference is perceptible in this action whether the primary current is continuous in the same direction, or whether it is alternating.

73. If the weakening of the physiological action here observed is to be attributed to an extra current circulating in an opposite direction, then this weakening influence must be very much diminished when the extra spiral is allowed to exert an inducing action upon a secondary wire placed parallel to it. To test this, a narrower extra spiral having 100′ of wire was inserted between the clamps 1) and 6) and this spiral itself was inserted into a spiral which may be called the secondary spiral likewise consisting of 100′ of wire. When a bundle of iron wires or a solid iron rod was now placed into the extra spiral, the shocks from the handles I and III were very inconsiderable as long as the outer secondary spiral was not closed, that is, as long as no secondary current could be produced in it. As soon however as this secondary spiral was closed and as soon as the secondary current could be made manifest in it by any of the modes of testing, then the shocks from the handles I and III again became powerful. Two extra spirals each 100′ in length were now inserted between the clamps 1) and 6) and upon each of these again a secondary spiral, likewise 100′ in length. By a transverse connexion of these secondary spirals, the induced secondary currents neutralized each other, not however when the connexion was in a like direction. The shocks in the handles I and III were in the first case much more powerful than in the

last, because the transversely connected secondary spirals performed the part of an unconnected spiral

74. The weakening effect produced by the insertion of the extra spiral filled with iron has a threefold cause. The current circulating in the wire coils of the surrounded keeper before connexion is broken by the spring, traverses also the coils of the extra spiral, by which means it experiences a greater resistance to conduction. If the inserted spiral has the same length of wire as the coil of the keeper, which is here the case, then the resistance is five times as great when the hand is arranged for physical effects The shock which is produced on breaking the closed circuit is therefore perceptibly diminished even when the wire of the spiral is stretched out straight. But the coils of this spiral now exert an inducing action upon each other, as does also the incipient magnetism in the inserted iron. The incipient extra current thus produced in the wire of the extra spiral increases therefore the action of the augmented resistance to conduction, and it is evident that, as these causes act in the same direction, an addition to the number of the extra spirals must constantly increase this action. This occurs indeed in so palpable a manner, that, when five spirals having 2000 feet length of wire were inserted together, and iron was placed within them, the shocks at last almost entirely disappeared.

75. If connexion is made by II and III (E), in which case the empty extra spiral alone remains in the circuit, then more powerful shocks are obtained when the hand of the pachytrope is arranged for *physical*, than when it is arranged for *physiological* effects. The insertion of unenclosed bundles of wire and tubes of sheet iron very much increases the shock. This increase is less with iron bundles of wire in entire tubes, with solid iron, steel, cast iron and nickel. With the unmagnetic metals the change was too slight to enable us to say in which direction it occurred If the extra spiral, enclosing a bundle of iron wire, is surrounded with the secondary spiral mentioned at 73), then the very powerful shocks obtained from the handles II and III with an unconnected secondary spiral are very much weakened as soon as this secondary spiral is closed by means of metal When two extra spirals were inserted into two secondary spirals, with a transverse connexion of the former, the shocks from the handles II and III were powerful, but they were weakened by a

like connexion in which the secondary current does not compensate itself It follows, from the united results detailed at 72) and 75) that the incipient extra current is increased by the same means in its negative action, as is the final extra current in its positive action and that in both cases bundles of wire exert a more powerful physiological action than solid iron

76 In the fourth section the remarkable phænomenon has been discussed, that the physiological action of the secondary current of the Leyden jar is weakened by the insertion of solid iron into the connecting spiral and increased on the contrary by the insertion of bundles of iron wire This was explained in the following manner —The phænomena produced simultaneously in iron by the action of the connecting wire namely, magnetic polarity and electric currents act here in such a manner that the retarding influence of the electric currents overpowers the increasing action of the magnetic polarity, and hence the final result with solid iron has a weakening tendency, whilst with more lasting currents, *e g* the galvanic current, in which the magnetism has time to develope itself, its action overpowers that of the electric currents and hence an augmenting action also results from solid iron This view, that the secondary current of the Leyden jar is only different from other induced currents in consequence of the shortness of the primary current producing it, which want of duration is without effect upon the electric currents induced in iron but prevents the complete development of its magnetism, gains in probability if it can be shown that the same phænomenon may be produced by other means than frictional electricity, now this can be done in the following manner

77 If the extra spiral is surrounded with a secondary spiral, its physiological action will be diminished, as we have seen at 73) The surface of a solid iron cylinder exerts the same influence as a secondary spiral An increase in the length of wire of the extra spiral weakens the primary current If the number of inserted extra spirals is augmented, and a solid iron cylinder placed within each, then the primary current weakened by the lengthening of the wire will only be capable of exciting a slight degree of magnetism in these iron cylinders If the intensity of the electric currents excited by the coils of the extra spiral on the surface of the iron cylinder does not diminish in the same but in a less degree than the magnetism simultaneously excited in the

mass of iron by these currents, then with a certain number of spirals we must obtain by means of solid iron a decrease instead of an increase of power. This was directly observed to be the case when five spirals each 100′ in length were added to the keeper of the machine, whilst bundles of iron wire as decidedly increased the action. This phænomenon is therefore quite identical with that observed for induction by machine electricity.*

78. When I and II ($p-A+E$) are connected, in which case the keeper and the extra spiral are included in the circuit, we obtain phænomena which are a combination of those observed with the connexion I and III ($p-A$), and of those with the connexion II and III (E). When only one or two empty spirals are inserted, the shocks are very powerful even with a slow revolution. This great intensity of the shocks renders it difficult to examine the action of inserted iron. Now we have seen at 74), that, with the connexion I and III ($p-A$), when the length of the inserted wire was increased, and particularly when this was in the form of successive spirals, the resulting current was always weaker, and that at last with five spirals it was almost imperceptible. When therefore $p-A$ is nearly equal to zero, then $p-A+E$ will assume more and more the form of E. But it has been again shown at 77), that, with the connexion II and III (E) on the insertion of *one* spiral, solid iron increases the action; on the contrary, when five spirals were employed, and each contained a rod of iron, the action was weakened, whilst bundles of iron wires even in the last case decidedly render the action more powerful. If therefore with the connexion I and II ($p-A+E$), and the insertion of one spiral, the increased action of the current p through E by means of iron is greater than the decreased action of this current p through A by means of the same iron, we shall obtain eventually an increased action. By the use of many spirals the action of the negative A is more and more increased, but then E also comports itself as a negative quantity, as the electrical currents induced in soft iron weaken the physiological action of the extra spiral more than the magnetism excited in the iron augments it. The resulting current must there-

* It appears to me not improbable, although I have made no direct experiments upon the subject, that similar experiments might be instituted with the extra current of the galvanic battery. With a sufficient number of spirals we ought to obtain, when solid iron is inserted, a weakening action upon the shock on breaking contact; when bundles of iron wires are inserted, an increasing action.

fore also be weakened by the addition of solid iron, and the increased action with one spiral must pass through a stadium of inactivity into a diminishing action when the number of spirals is gradually increased.

By a remarkable coincidence the conditions for inactivity were exactly supplied by the two spirals, which when it is not otherwise particularly noticed, always constituted the extra spiral in the experiments with Saxton's machine. I obtained namely a weakening action with certainty only, when solid iron was inserted; with other kinds of iron the intensity of the shock remained unaltered. I concluded from this therefore that both extra currents A and E almost completely compensated each other, and that the insertion of iron increases to a nearly equal degree two magnitudes forming a difference. When instead of the two wide spirals I chose one that closely fitted the iron cylinder and covered it throughout its whole length, the physiological action was then decidedly increased by soft iron, and still more by bundles of iron wires. When this extra spiral was inserted into a secondary spiral, the shocks from the handles I and II on closing the secondary spiral were perceptibly diminished. This increase of power by means of iron even took place when two narrow extra spirals were used. For when these were inserted into two secondary spirals connected in a like or in an alternating direction, in the first case the shocks were weaker than in the last, which all tends to confirm the view, that the positive action of E overpowers the negative action of A. If five spirals were inserted an increase in the power of the shock was obtained, as with the connexion II and III when solid iron was contained in the spirals, a diminution however was observed when the spirals contained bundles of wires.

79. Corresponding results to those which have here been adduced were obtained when the break was effected by means of the intermittent spring 3) not at an azimuth of 90° but of 15°, or with alternating currents at an azimuth of 15 and 215*. But

* The cylinder w_2 upon which the intermittent spring 3) slides can be turned as can also w_3 and w_4 so that this spring can break the connexion at any azimuth that is required. This turning is easily effected by filling up the spaces between the cylinders w_1 w_2 w_3 w_4 with wooden rings round the axis and pressing them altogether against the first cylinder which is fixed by means of a spring at B. This spring is not represented at fig 7 nor are the wooden rings in order that it might be more distinctly seen which cylinders are insulated and which immediately attached to the axis. If the cylinders cannot be turned then a spring of a determinate length becomes necessary for every azimuth.

the remarkable phænomenon was observed with the connexions I and II and II and III, in both of which cases the final extra current was active, that the shocks, which were perceptible on the insertion of iron into the extra spiral consisting of the two commonly used spirals, disappeared when the keeper was rapidly rotated, and with a still more rapid rotation a physiological action again appeared. This may possibly be explained upon the supposition, that when the rotation is slow the current excited in the extra spiral by the action of the coils of wire upon each other is formed simultaneously with the current induced in these wire coils by the evanescent magnetism of the inserted iron, so that then the maxima of intensity of both currents coincide. With a more rapid revolution, on the contrary, this latter current remains behind the former, so that with a certain velocity of rotation its maxima coincided with the minima of the former. In this case a current of unchanged intensity would traverse the body, which giving rise to a perfectly uniform sensation would not be perceived; similar phænomena have been observed in relation to this with preparations of the frog. With a yet more rapid rotation the maxima again coincide, and produce inequalities of intensity which make themselves perceptible. Thus it would also be explained why these physiological phænomena of interference can only occur to the full extent, namely, to the point of complete evanescence, with a determinate mass of iron, and why we are able to obtain them with the greatest ease by using iron wires, the number of which can be regulated accordingly*.

2. *Sparks.*

80. As the extra spiral and the keeper are included in the circuit of the current when complete metallic connexion is made, we obtain immediately at the cylinder w_2 the same case as was determined in the physiological experiments by the connexion of I and II, namely, $p - A + E$. But as, during the rotation of the keeper from 0° to 90°, the iron enclosed in the extra spiral becomes magnetized, which magnetism cannot become quite evanes-

* In all determinations of the increase or decrease of a physiological action mentioned in this memoir, I have never trusted to my own judgement alone but have always called in the assistance of others. In the course of the experimental series, which were often very extensive and required generally two observers, I have had to thank for their assistance the Messrs. Von Wyss, Kopp, Du Bois and Karsten.

cent at the moment connexion is broken the presence of the iron A will augment the action more than E, and the sparks will consequently be diminished*. Now this occurs in so remarkable a manner, that on the insertion of iron cylinders into the spiral the brilliant spark which was previously observed at the point of interruption u on the cylinder w_2 almost completely vanishes. That this diminution of the spark is caused by an extra current produced by the spiral is evident; for if II and III are connected by metal, the spark on breaking connexion at u reassumes its full splendour, whilst with the connexions I and II and I and III every spark at u is naturally prevented. If, lastly, the extra spiral is inserted into a secondary spiral, the spark at u is very much increased by its connexion.

81. As it has been shown at (78) that an obvious diminution of the shocks was only perceived with solid iron rods where bundles of wires exhibited an indisputable augmentation, so is the diminution of the spark much more considerable when solid iron rods are inserted than when the same mass of iron in the form of insulated bundles of wires is used, and still more marked when the bundle of wires is enclosed in a conducting case (a brass tube) than when unenclosed. Everything that favours an augmentation of the extra currents tends to postpone, as regards time, the maxima of their intensity. By the same means also the action of the incipient extra current is increased, that of the final extra current on the contrary diminished. The insertion

* Many observations render it exceedingly probable that a galvanic current by means of which iron is magnetized attains the maximum of its intensity rather than the iron attains the maximum of its magnetic polarity. It is therefore not impossible that if the electric current is interrupted during the time that the magnetism is on the increase in the iron, the magnetic intensity should continue to increase for a short time *after* the interruption of the electric current. If we regard magnetization in conformity with the theory of Ampère as an influence directing the electric currents already existing and surrounding the individual molecules, then it would amount to this, that the elementary currents which are actually circulating in a rotatory manner do not immediately resume their original state the moment the motive power ceases, but continue to move in the direction which has been given to them for a short space of time. According to the view of Coulomb elementary magnets would have to be substituted for elementary currents. Now as long as the magnetism increases A is induced in the connecting wire and not E. The occurrence of E is therefore still further postponed by the presence of the iron, and consequently the duration of the discharge altogether increased. All these phenomena however will not occur when the primary current has already continued for such a length of time that it itself as well as the magnetism produced by it have already attained their maxima *before* the interruption, and have consequently become stationary.

of unmagnetic metals into the extra spiral appeared to cause no diminution of the spark, even when this was composed of five connected spirals

82 These facts tend therefore also to explain why a spring which breaks connexion in azimuth 135° on the insertion of iron into the spiral, produces a diminution, although but slight, in the vividness of the spark, and why the physiological action also, with the connexion I and II through the body, appears somewhat diminished, although, without the insertion of the spiral in this position of the keeper, the primary current would have already exceeded its maximum As a general conclusion, therefore, in whatever part of the second quadrant the interruption is effected, the first extra current will always have been increased for a longer time by the inserted iron than the second, the primary current will therefore have lost more in the first quadrant by means of the extra current than it will gain up to the point of interruption in the second by the insertion of iron It also appears that the intensity of the primary current in the second quadrant decreases much more slowly than it increases in the first quadrant, for the sparks and shocks are much more intense when the spring breaks connexion in azimuth 135° than when that is effected in azimuth 45° The reason why this occurs when no spiral is inserted is, because the coils of the keeper itself, with the nucleus of iron which they enclose, may in a certain sense be considered as its own extra spiral

83 If spark experiments are to be instituted, corresponding to the physiological experiments in which the body closed the circuit either by I and III or II and III, then an arrangement must be made to break the metallic connexions I and III and II and III at the instant the spring breaks connexion at u This was effected by the addition of a fourth cylinder w_4 (fig 7), identical with the cylinder w_2, on which u presses, and moreover insulated from the axis, upon which any two of the connexions I and III or II and III slide with a certain pressure, the one continuously, which is clamped at 11) in the stand G, the other intermittent, proceeding from 13) in the stand F If the clamp 7) is connected with 12) by a wire indicated by the dotted line in fig 7, and in the same manner 15) is connected with 4) by a second wire, then, at the instant the spring proceeding from 13) comes in contact with the wood, the previously existing secondary connexion II and III is

broken if, on the contrary, 7) is connected with 12) and 15) with 8) by means of wires, then when that spring touches the wood the previously existing secondary connexion I and II is broken

It must be however borne in mind, that this case, as well as that of chemical decomposition, which will be considered directly, is not perfectly comparable with the experimental arrangement in the physiological experiments for as the body presents a considerable resistance to conduction, its influence upon the primary current could be disregarded as long as it formed a secondary connexion with u closed This is by no means the case when as here with u closed either I and II or II and III forms a perfect metallic secondary connexion, in which case the galvanometer shows that a great part of the primary current takes this route Now we can always picture to ourselves the extra current excited in the extra spiral under the form of a greater resistance which this spiral opposes to the primary current p produced by the keeper The insertion of iron increases this resistance, and in this case a greater portion of p will pass through the secondary connexion I and II than when no iron is present in the extra spiral and indeed the spark is then much more brilliant, namely, that on the cylinder w_1 whilst that at u on the cylinder w_2 is nearly extinguished With the connexion II and III the augmentation of the spark in the secondary connexion at w_1 is not remarkable inasmuch as E is there increased As with the adopted arrangement of the apparatus the spark on breaking connexion at u on the cylinder w_2 and that between I and III or II and III on the cylinder w_1 appear directly side by side, the growing intensity of the one corresponding to the decreasing intensity of the other when iron is inserted into the extra spiral, presents a very instructive spectacle

3 *Chemical decomposition*

81 If the current is retained constantly in the same direction by means of the forked springs $y\,y$, and if the voltameter is to be introduced immediately into the circuit of this current but not as a secondary connexion, then the connexion will be different when an extra spiral is inserted than when that is not done If the wires I and II proceeding from 1) and 8) lead to the voltameter this then forms the connexion between the two cylinders w_1 and w_2 by means of the stands C and D, as only one

arm of the spring is in metallic connexion with one cylinder, and each stand consequently only in metallic connexion with one cylinder. If, on the contrary, the voltameter is inserted between the wires I and III, the connexion then proceeds from the stand D through the extra spiral to the stand E, and from this to the voltameter. In the first case the resistance to conduction would consist of the resistance offered by the wire surrounding the keeper, and that of the fluid between the electrodes of the voltameter. By the addition of the usual extra spiral the first part of this resistance is increased about five times. Nevertheless the whole amount of gas thus obtained was only about one fifth of that obtained without the extra spiral, and the quantity was very much more lessened by the insertion of iron in the form of rods or bundles of wires. If the quantity of gas obtained is taken as the measure of the quantity of electricity which has passed through the wire in a certain time, then this is actually diminished by the action of the revolutions of the extra spiral upon each other, and by the magnetism of the iron which it encloses.

When the usual springs 3), 5), 9) are used instead of the forked springs y y, but in such a manner that 3) also is in constant metallic contact with the cylinder w_a, and the voltameter is interposed in the first instance without this spiral, and immediately in its stead between 1) and 6), afterwards in conjunction with this spiral so that it may be supposed to occupy a position in the middle of the inducing wire S, then with alternating currents analogous phænomena are obtained to those which have been observed with currents in a like direction.

85. If however the voltameter is to form a secondary connexion with uninterrupted currents, at one time to the keeper, at another to the extra spiral, then it must be interposed, the springs 3), 5), and 9) preserving a like position, at one time between I and III and then between II and III. In both cases the insertion of iron considerably accelerates the decomposition of water. The current therefore which divides itself between the primary and secondary connexion, experiences, when iron is present in the extra spiral, as well in the coils of this spiral as also in the coils of the wire surrounding the keeper, a greater amount of resistance than when the extra spiral contains no iron. If, on the contrary, the voltameter form a secondary connexion to the straight portion of the principal connexion

then an imperceptible portion only passes through it, for if u is in constant metallic connexion, and I and II are connected by the voltameter, no decomposition ensues. Similar relations are observed with I and III when connexion is broken at u in azimuth 90°, or 90° and 270°, *i.e.* when the voltameter after having formed for some time a secondary connexion, now becomes the chief connexion. Gas is then produced in the voltameter between I and II, with one break only in azimuth 90°, therefore with a current in a like direction, and moreover much more gas with empty spirals than when iron is contained in them.

The chemical effects therefore correspond with the phænomena observed with the sparks. Here also the phænomena dependent upon the extra current are more distinct, when the hand of the pachytrope is arranged for physical than when it is arranged for physiological effects.

86. Lastly, it may be asked, what phænomena will occur when the current flowing at the commencement in a metallic circuit without any secondary connexion is on the breaking of this circuit closed by the voltameter? In the drawing, the moment connexion is broken at u, I and III or II and III must first be connected by the voltameter. This is effected in the following manner:—Suppose the primary current to be only once interrupted in azimuth 90°, and that the voltameter is inserted between 5) and 15). The springs 13) and 14) are inclined so much towards the left upon the cylinder w_1*, that when the spring 3) comes in contact with wood, the spring 14) on w_1 touches metal and *vice versâ*, whilst 13) is constantly in connexion with metal. Besides this, 7) is connected with 12) by a cross wire. As long as 3) is in contact with metal on w_2, the connexion is made from w_1 through 5), 6), the spiral, and 1), 3) to w_2, whilst the voltameter, in consequence of the interruption on w_4, forms no secondary connexion. When however 3) reaches the inserted wood, the connexion from w_1 to w_2 is then effected through 5), 7), 12), 13), w_1, 15), the voltameter, and 8), 9), the spiral is therefore excluded. It is obvious however that the time ar

* The inserted plate of wood on w_4 which is next to the cylinder w_3 is intended for alternating currents and c v is therefore only one sixth of the circumference and again occurs in a diametrically opposite position. The inserted piece of wood nearest to the end of the axis B occupies on the contrary one half of the circumference of the cylinder.

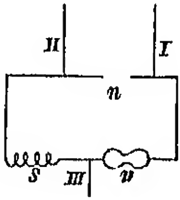
tion of p—A is not here obtained, because, during the rotation of the keeper through the nearest semicircle, the voltameter, as in § 81, is directly interposed in the circuit of the current, as the keeper also remains in the connexion. This is not the case when the voltameter is inserted between II and III instead of being placed between I and III. Here the spiral is in the connexion, and the keeper remains excluded. If the commutation here happens a little too late, we therefore obtain no action, and with I and III, according as the previous current is active or not, a variable or invariable quantity of gas when iron is inserted into the spiral. A further prosecution of this investigation therefore did not appear advisable, as the slightest alteration in the point of contact of the spring upon the cylinder exerts a considerable influence.

4. *Galvanometer.*

87. As with continuous sliding springs alternating currents succeed each other, we obtain in this case, even when the galvanometer[*] effects a secondary connexion, the phænomena of the so called deviation in a twofold direction, according to which the needle is moved in the same direction to that which it already occupied towards the coils of the galvanometer, before being acted upon by any current whatever. Something similar naturally happens, when with a spring that breaks connexion twice (in azimuth 90° and 270°), the wire of the galvanometer, during the rotation of the keeper through the second and fourth quadrant, does not form a secondary but the chief connexion. In this latter case therefore the phænomena are more prominent.

88. If, on the contrary, the intermittent spring (3) breaks connexion only once in azimuth 90°, then in like manner the galvanometer is traversed by alternating currents, but only when

* In galvanometric experiments the extra spiral must be considerably removed from the Saxton's machine. For as soon as the rotating keeper is brought from its horizontal position before the poles of the magnet into a vertical position, the magnetism in the magnet becoming free exerts an inducing influence upon the iron which is contained in the extra spiral. The current thence induced in the spiral is perceptible at considerable distances when the galvanometer needle is truly astatic. To ascertain the distance at which this disturbing influence ceases, we have only to connect the ends of the extra spiral in the first instance alone with the galvanometer, and then to turn the keeper. If no action then ensues, the spiral must be at the proper distance from the machine.



the galvanometer is inserted between I and III do we obtain deviation in a twofold direction, with the connexions I and II or II and III, on the contrary, a normal current, which, particularly when the rotation is rapid, overpowers the alternating current. If this current flows from I to II in the galvanometer which is inserted between II and I, then the galvanometer inserted between II and III indicates a deflection in the direction from III to II, with this peculiarity that the deviation which becomes stationary precedes a current in an opposite direction on the first revolution, which is likewise obtained when the keeper is turned half a revolution without being rotated.

It must however here be remarked, that in this manner, on the whole, no analogous phænomena are obtained to those which have been found with other modes of trial. The production of the final extra current in its full energy requires that the current previously circulating in a closed metallic conductor be suddenly very much diminished in intensity, either, as in the case of sparks, by an actual break in the circuit, or for physiological and chemical actions by the insertion into the interrupted circuit of a substance (the voltameter or the human body) offering a much greater resistance to conduction. If the want of continuity produced at u is supplied by a galvanometer (as in the connexion I and II), or if the current excited in the rotating keeper remains closed by the galvanometer between I and III on breaking connexion at u, then no interruption whatever occurs, and the zero of the current is transferred to azimuth 180°. The circuit is only broken in reality with the connexion II and III when I is able to form. The same applies when, to avoid alternating currents, the keeper is only turned round half a revolution.

89. Now, by means of the springs 13), 14) the galvanometer was first inserted at that moment, when the spring 3) at u broke connexion once in azimuth 90°, and the keeper was kept in continual rotation. The results were as follows:—

1. Galvanometer between 8) and 15). At the break the spiral is excluded from, and the galvanometer enters into the connexion. This corresponds therefore with the connexion III

and I. The current proceeded from I to III, in the figure therefore from *s* to *a*.

2. Galvanometer between 8) and 4). First it forms a secondary, then a chief connexion. The keeper as well as the extra spiral remain in the connexion. The current is from I to II, in the figure therefore also from *s* to *a*.

3. Galvanometer between 4) and 15). At the break the keeper leaves the connexion, whilst the voltameter with the extra spiral come into the connexion. The current proceeds from 15) to 4). This corresponds in the drawing with III to II. Here too therefore the current is in the direction from *s* to *a*.

It was further ascertained, that with forked springs the constant direction of the current, determined by the galvanometer immediately included in the circuit, is the same with an inserted spiral as without it, therefore p is in a like direction with $p-A+E$.

5 *Experiments with the empty wire keeper.*

90 Although it is probable *à priori* that primary currents, which are excited by the magnetization of soft iron, are identical in their action with currents which are induced by a magnet in motion, it appeared nevertheless desirable to prove this empirically. Instead of the iron keeper surrounded with wire, the empty keeper described at § 40 was employed. The insertion of the spiral, even without a nucleus of iron, produced with this hollow wire keeper exactly the same modifications in the physiological action as those described with the iron keeper at § 72. The result is important, because it upsets the opinion that the presence of iron is essential in causing A to overpower E, and because it will now be seen that we were justified in having neglected in the foregoing researches the reaction of the extra current upon the magnetism of the rotating keeper. Nor has the form of the iron exerting the inducing action any influence, for the same phænomena are obtained when the keeper of the machine is composed of bundles of iron wires.

6 *Immediate production of sparks on interrupting conduction.*

91. Lastly, a phænomenon observed during the researches with Saxton's machine deserves a short notice, because it affords means for answering the question, whether the spark observed

on breaking connexion in a wire through which an electric current is circulating occurs at the moment connexion is broken, or at a measurable time after this interruption. This occurs in Saxton's machine, when the sliding spring leaving the metal comes in contact with wood, which happens in a certain position of the keeper. If the spark is perceptible at the moment the interruption is effected, the keeper must have this position; if it appears at a later period, the position of the keeper must correspond with a later stadium of rotation. The difference between the two positions will be greater the quicker the keeper is rotated. Now, when the machine is caused to rotate slowly or quickly in the dark, the keeper, illuminated by the sparks, appears perfectly stationary in that first position, even when viewed by a telescope provided with cross threads directed towards a certain mark upon the keeper. No measurable space of time therefore transpires between the interruption to conduction and the appearance of the spark, although by this means a lesser magnitude than the $\frac{1}{1000}$ of a second could be measured.

92. By the researches adduced in this section it has therefore been proved, that the presence of iron modifies the negative effects of the incipient extra current in the same manner that it modifies the positive effects of the final extra current, and that both are closely related to the secondary currents as regards all their properties which are susceptible of proof. It is true that these researches only extend to the particular case in which the primary current is a magneto-electric current. But these currents appear at present to offer the only attainable means for instituting such investigations. Besides, without them the fact observed for induction with frictional electricity, that an increased physiological action is effected by bundles of iron wires, and a decreased action by solid iron, would be altogether without analogy. In § 77 of this section, I have succeeded in showing the same phenomenon by means of Saxton's machine. This appears to indicate, that as the modifications in the action of iron upon the currents induced by it, according as it is used in the form of solid rods or bundles of iron wires, may be traced to a change in the duration of these currents, so likewise the secondary current of a Leyden jar differs only from the currents induced by other sources of electricity in the instan-

taneousness of the primary current which produces it, and in its corresponding shortness of duration.

93. From the experiments which Faraday has detailed in his fifteenth series*, upon the electricity of the Gymnotus, and from those instituted by Matteucci upon the Torpedo†, it may be presumed that it will be possible to obtain secondary currents of sufficient strength from the primary currents of those animals, capable of being submitted to similar modes of trial to those which have here been put in practice with secondary currents from other sources of electricity. If the fish could be enclosed by means of collectors in the inner spirals of a differential inductor, we should then be able to ascertain, by allowing a bundle of wires in the one spiral to oppose a solid iron cylinder in the other, what influence the breaking up of a mass of iron into a bundle of wires exerts upon these currents. We should thus be able more accurately to determine what position these currents would occupy in the series in relation to those produced from other sources of electricity. If these were arranged according to the time which a given quantity of electricity requires to be neutralized, they would form a series somewhat like the following:—

1. The current of a discharging Leyden jar, extra current from the same source, secondary currents of the first and higher orders, and lastly, currents induced by bundles of iron wires, which have been electro-magnetized by an electric battery. These currents exerting a powerful physiological action without a retardation of the current, do not affect the galvanometer needle.

2. Currents induced by solid iron, magnetized by means of frictional electricity.

3. Currents of higher orders, induced by electro-magnetized bundles of iron wires, when the primary current is of galvanic or of magneto-electric origin. The lower the order of the current, the more distinct is its galvanometric action.

4. Currents of the first order induced by bundles of wires when the magnetizing current is of galvanic, thermo-electric or magneto-electric origin.

5. The same currents induced by solid iron.

* Philosophical Transactions for 1839, Part I.

† *Essai sur les Phénomènes Electriques des Animaux*, Paris, 1840, 8.

6 The current of Saxton's machine with an empty wire keeper

7 The current of the same machine with a surrounded iron keeper

8 The current of the closed thermo or hydro circuit

In the history of the science, the first and last members of this series were the first to be discovered Such a wide gap lay between them that doubts were entertained as to their identity Now that a number of intermediate members have been supplied by the phænomena of induction, the gradation has become so gentle, that any endeavour to draw a line of demarcation between them must be perfectly arbitrary

Article V.

Investigations on Radiant Heat. By H. Knoblauch*.

[From Poggendorff's *Annalen der Physik*, &c. for January and March 1847.]

Description of the Apparatus.

In my investigations I made use of a *thermo-multiplier*, an instrument which has been brought to such wonderful perfection by Becquerel, Nobili and Melloni, that in experiments on radiation an unqualified preference must be conceded to it beyond all other thermoscopes.

The accuracy of this apparatus, upon which, with its great delicacy, its peculiar value depends, is owing to the following circumstances:—

1. That, on account of the coating of the pile with lamp-black, it is equally susceptible of every kind of calorific rays.

2. That, after reduction of the deviations of the needle of the galvanometer to degrees of electric force, its indications may be regarded as measures of the heat received by radiation, because the intensity of the electric current excited in the pile by the difference of temperature, is proportionate, within the limits of these experiments, to this difference of temperature.

The truth of the first position has been placed beyond doubt by the investigations of Melloni, and that of the second by those of both Becquerel and Melloni.

The Thermal Pile which I made use of consists of twenty-five pairs of bars of bismuth and antimony, each of which is 35·5 millim. long, 2·3 millim. broad, and 1·5 millim. thick. They are carefully isolated as far as the part where they are soldered, and cemented into a brass ring, beyond which they project 5·5 millim., forming five series, each consisting of five pairs. Their extremities are sloped off, so that the anterior surface of each pair forms a right angle, its sides being 2·1 millim. and 1·0 millim. in length. The surfaces of both sides of the pile are exactly the same, and are coated with lamp-black of uniform thickness.

The side next the source of heat is furnished with a polished metallic cylinder 30 millim. in diameter and 60·9 millim. in

* Translated by J. W. Griffith, M.D.

length, and the opposite side with a cylinder of the same diameter, but only 19 millim. in length. Both by these and by screens properly placed, the pile is protected from all extraneous rays, so that, excepting the temperature of the surrounding air, which acts equally on all sides, it is only exposed to the influence of the source of heat.

The conducting wires which connect it with the multiplier are fastened, by means of binding screws, to copper sockets, in which the poles of the pile terminate.

The Multiplier is constructed on Nobili's principle, which appeared to correspond to the desired object better than any other recently proposed arrangements, and its accuracy has been completely verified in the series of experiments about to be detailed.

One advantage in my instrument consists in the wire being composed of electrotype copper; hence the disturbing influence of the iron* contained in the ordinary conducting wire was avoided. Certainly the wire was drawn through steel, instead of which a ruby might have been used; however, there is no fear of the wire becoming contaminated with iron from this source, because the aperture through which it passed was completely covered with copper, and moreover it was carefully washed in dilute acid before being covered with the silk. How fully the object was attained in this way is evident, among other circumstances, from the double needle with the purified copper wire remaining only 1°·5 on either side of zero on the scale; whilst in the coils of an ordinary conducting wire it could not be approximated more than to 20°. The slight deviation of 1°·5, which arose from the magnetism of the copper†, might certainly have been avoided by closing the fissure between the coils‡; but other circumstances appeared to render this unadvisable. The length of the copper wire which surrounds one needle of the astatic arrangement in 160 turns, is 31·5 millim.; its thickness is 1·1 millim. The mean length of the coils of wire is 9 centim. 2·5 millim.; their breadth, 4 centim. 6·5 millim.; their mean distance from each other, 15·0 millim. The two equal portions of which the galvanometer wire consists, and which being wound around each other form the coils, may be combined so as either to allow the current to pass through them successively, and thus singly the whole length of the wire, or so that it simultaneously

* Moser, Dove's *Repert.* vol. i p. 261.

† H. Schroder, Pogg. *Annal.* vol. liv. p. 59, and vol. lvi. p. 339.

‡ Péclet, *Ann. de Chim. et de Phys.*, ser. 3, vol. ii. p. 103.

enters and passes from both portions, and thus passes double through half the length.

The combined magnetic needles are 7·0 centim. in length, 1·1 millim. diameter in the centre, and 17·0 millim. distant from each other. The little ivory column which supports them is suspended from the finest possible silk-worm thread 30 centim. in length. Thus they form a system, which completes a simple oscillation in sixteen seconds, and assumes an almost constant position, determined by the torsion of the thread and the combined action of the magnetism of the needles*, of about 45° towards the magnetic meridian. The upper needle vibrates above a circular disc of copper precipitated by galvanism, of 8 centim. 5·0 millim. in diameter, and cut through at an angle of 90°, and upon the silvered margin of which the graduated division of the circle is engraved.

A cylindrical glass case, 6 centim. high and 14 centim. in diameter, surrounds the whole; and its upper plate being only 1 centim. from the copper disc, admits of our reading off to half a degree. In its centre a glass tube 32·5 centim. high and 22·4 millim. in diameter is placed; this surrounds the silk thread, which is fixed to a metallic rod in its upper part, 14 centim. in length, capable of moving in a vertical direction, and serving at the same time to stop the motion of the needles.

To protect the instrument from vibrations, it was placed upon a bracket, which was fastened to the wall by brass nails. The wires which united it to the thermal pile are not screwed immediately into the galvanometer, but to separate solid copper sockets, which are in firm union with the extremities of the coils.

The thermal pile and multiplier were made by M. Kleiner, one of the most skilful mechanics in Berlin. I convinced myself by experiment, that on using a pile of twenty-five pairs it would be more advantageous to pass the current through the entire length of the galvanometer wire, than to conduct it in two portions through half the length; for the same source of heat which in the first case caused a deviation of 28° in the magnetic needle, in the second combination of the wire caused a deflection of only 26°·5; or in the first case of 37°, in the second of 35°; or in the first of 51°, and the second of 48°.

The former mode of closing the circuit has therefore been used throughout the series of experiments†.

* Moser, Dove's *Repert.* vol. i. p. 260.

† That exactly the reverse should occur with a single pair, in which the

I shall pass over the manifold difficulties which impeded my observations, and which entailed a long series of fruitless experiments, by which I ultimately succeeded in ascertaining the entire range of disturbing influences, and, as I think, in overcoming them; for every one who engages in these investigations has to learn the effect of local influences from his own experience; and an opportunity will hereafter be taken of detailing the minor conditions which must be taken into consideration in the critical examination of the results.

I. *On the Passage of Radiant Heat through Diathermanous Bodies, with especial regard to the Temperature of the Source of Heat.*

The results to which such investigations as have hitherto been made on the immediate passage of radiant heat through certain bodies have led, may be briefly summed up in the following positions:—

1. Heat passes through certain (diathermanous) substances, and this in an immeasurably small space of time.

2. In one and the same body the quantity of heat transmitted is proportionate to the smoothness of its surface.

3. The loss which heat suffers on radiating through a substance is less in proportion as it has already penetrated through thicker layers of this substance.

4. Radiant heat passes through different bodies in different proportions; however the property of bodies to transmit it has no relation to their transparency.

5. Rays from one and the same source of heat, which are transmitted in succession through different diathermanous substances, experience from this, losses which vary according to the nature of the bodies, and are always greater than those which they experience when transmitted through homogeneous bodies.

6. Rays of heat, from different sources, which directly produce similar elevations of temperature, pass through one and the same substance in dissimilar proportions.

resistance to the conduction of the electromotive elements was comparatively small to that of the wire closing the circle, was a simple consequence of Ohm's law. (The Galvanic Circuit, considered Mathematically, by Dr. G. S. Ohm.—Scientific Memoirs, Parts VII. and VIII.)

In this case, experiment produced a deflection of 26° for a certain intensity of thermo-electrical excitation, when the current ran simply through the length of the wire of the multiplier; but of 37° when it simultaneously passed through both portions of the coils; and in another experiment, in the first case, 36°; and in the second, 50°.

The experiments of Delaroche and Melloni, which were made with regard to this point, for directly comparing the transmission of heat from different sources through diathermanous bodies*, appear to indicate that the power of heat to radiate through these bodies increases in proportion to the temperature of its source.

Thus Delaroche found that a constant number of 10 rays of heat, passing through a glass screen,

with a source of heat of	357°	was contained in	263 rays,
... ...	650°	...	139 ...
... ...	800°	...	75 ...
... ...	1760°	...	34 ...
in an Argand lamp burning freely		. .	29 ...
and in one furnished with a glass chimney		.	18 ...

And Melloni observed—to give a single example only from among the numerous ones which he has adduced—that

of 100 rays of heat,	which copper emits at 212° . .	33
... ...	which copper emits at 730° . .	42
... ...	which red-hot platinum emits .	69
... ...	from a Locatelli's lamp	78

pass through a plate of fluor spar 2·6 millim. in thickness. Two observations only form an exception to the position advanced; for pure rock salt, according to Melloni's investigations, is penetrated by rays of heat from every source in a uniform manner; and prepared rock salt, according to Melloni and Forbes, is penetrated by heat in a degree which increases in proportion to the diminution in the temperature of its source.

Melloni withdraws a former observation, according to which the heat of red-hot platinum passes through black glass better than that of an Argand lamp; and shows that the instances, besides the example adduced, in which Forbes considered that he had observed the better transmission of heat of a lower temperature, had not afforded pure results of radiation, and therefore could not enter into consideration in this point of view.

The two instances specified therefore stand alone in opposition to an apparently general law. One only directly contradicts it, and relates to a substance which differs from other diathermanous bodies in numerous respects. Hence the supposition of an influence of temperature on the transmission of heat through

* [On this point the reader should perhaps be referred also to Professor Powell's paper, Phil. Trans. 1825.—Ed.]

diathermanous media did not appear to me sufficiently disproved, and I therefore endeavoured experimentally to decide the question,

Whether the power possessed by rays of heat, of passing through certain bodies, has any perceptible relation to the temperature of the sources from which they are derived.

After the extended investigations made by Melloni on the most dissimilar bodies with such extreme care, I could not expect to discover new substances which (for the sources of heat used by him) might be classified with prepared rock salt as regards the transmission of heat. I therefore preferred changing the sources of heat instead of the diathermanous media.

1. In the first series of experiments I made use of red-hot platinum, the flame of alcohol, an Argand lamp, and the flame of hydrogen. The former was kept at a red heat without flame (according to Davy's method*), by being placed on the wick of a spirit-lamp, which it surrounded spirally. The alcohol-flame had a uniformly trimmed wick, which never carbonized, and dipped in the fluid contained in a glass vessel. The Argand lamp, which was at a constant level, with a double current of air, had a cylindrical wick without a chimney. The flame of hydrogen issued from the tube of a gasometer constructed for the purpose, and which allowed the gas to escape under a constant pressure. The constancy of these sources of heat during the experiment was tested most carefully. They were, of course, only allowed to act upon the thermoscope to such an extent as would allow of their being submitted to comparison, being protected from the rays of parts accidentally heated with them by polished metallic screens.

However uncertain determinations of temperature may be with regard to this point, nevertheless all natural philosophers are agreed that the degree of heat of a red-hot spiral of platinum wire is less than that of a flame of alcohol, which is able to raise the wire to a yellow heat; and less than that of an Argand lamp, in which carbon is raised to a white heat. Moreover, all would agree that the hydrogen flame† has the highest temperature among the sources of heat we have mentioned.

The next question is, whether, in correspondence with the position advanced by Delaroche, the heat of the alcohol flame and the Argand lamp would pass through diathermanous bodies

* Communicated to the Royal Society of London, Jan. 23, 1817.

† Mitscherlich, *Lehrbuch der Chemie*, 3rd edit., part i. p. 289–290.

comparatively better than that of red-hot platinum, and the heat of the hydrogen flame more freely than that of the three other sources.

On this point experiment has decided as follows:—When the red-hot platinum had emitted rays upon the above-described thermal pile to such an extent that the needle when it was combined with the multiplier deviated to 20°, it went back to 12° when a plate of colourless glass 1·3 millim. in thickness was introduced between the source of heat mentioned and the thermal pile. These 12° corresponded to the heat which passed through the glass. But when the alcohol flame, by its immediate action upon the thermoscope, had produced a similar deviation of 20°, the needle receded to 11° when the same glass plate was inserted at the same spot; consequently the heat of the alcohol flame passed by radiation through the glass plate to a less extent than that of the red-hot platinum. The heat of the Argand lamp, which had also directly caused a deviation in the needle to 20°, on inserting the glass produced a deviation of 15°. Finally, when the hydrogen flame radiated upon the thermal pile so as to deflect the needle to 20°, on inserting the glass screen it returned to 12°. Hence it is evident that the heat of the hydrogen flame and the red-hot platinum, notwithstanding the great *difference in their temperature*, is capable of passing through a glass plate 1·3 millim. in thickness to an equal extent, but that the heat of the alcohol flame possesses this power in a *less* degree than that of the red-hot platinum, although its temperature is higher than that of the latter, and the heat of the Argand lamp in a much *greater* degree than that of the hydrogen flame, notwithstanding its temperature is decidedly *lower**.

When, with the same direct action of the sources of heat, the glass screen was exchanged for a plate of alum 1·4 millim. in thickness, with red-hot platinum the magnetic needle receded to 8°·25; with the flame of alcohol, to 7°·5; the Argand lamp, to 10°·5; and with the flame of hydrogen, to 7°·75.

Thus the heat of the hydrogen and alcohol flame, with *great difference in the temperature*, passes through the plate of alum to the *same* extent; and that of the Argand lamp, and even that of the red-hot platinum, pass through this *more copiously* than the heat of the hydrogen flame, although they have a far *less* degree of heat.

* [This agrees exactly with Prof. Powell's results, Phil. Trans. 1825; and according to that author's views the reason is obvious.—Ed.]

Radiation through gypsum exhibited similar phænomena. The heat of the hydrogen flame certainly passes through potash- and magnesian mica less freely than that of the three other sources; an observation to which I wish especially to draw attention, because it is opposed to the expectations which we should make after the experiment instituted by Melloni with mica.

The following table contains the results which have been obtained on the transmission of heat through the diathermanous bodies above mentioned, as well as some others, for various direct deflections.

Table I.

Thickness in millimetres.	Substances inserted.	Deflection by direct radiation.	Deflection after insertion with			
			Red-hot platinum.	Flame of alcohol.	Flame of Argand lamp.	Flame of hydrogen.
1·5	Red glass	20°	11·25	10·75	14·25	12·00
1·4	Blue glass	......	10·75	10·75	11 75	11·00
1·4	Alum	......	8·25	7·50	10·50	7·75
0·2	White mica ...	20°	17·50	18·25	19·00	15·25
0·1	Green mica ...	......	17·75	18·25	17·75	16 25
1·3	White glass ...	......	12·00	11·00	15·00	12·00
4 4	Rock salt	20°	16 50	15·50	17·00	15 75
3·7	Calcareous spar	......	8·25	8·00	12·50	8·50
1 4	Gypsum.........	......	7·75	6·25	10·25	6·25
0·2	Glass-paper ...	20°	11·75	11 50	14·25	11·50
1·5	Red glass	22°	12·50	12·00	15·75	13·25
1·4	Blue glass	......	12·25	12·00	13·50	12·00
1·4	Alum	......	8·25	8 00	10·50	8·00
0·2	White mica ...	21°	18·25	18 75	19·25	16 50
0·1	Green mica ...	......	18 75	19·50	19·00	17 50
1·3	White glass ...	......	12·75	11·00	15·50	13·00
4·4	Rock salt	21°	16·75	15·25	17 75	16·00
3·7	Calcareous spar	......	9·00	8·50	14 00	9·50
1·4	Gypsum.	......	8·25	6·75	11 25	6·50
0·2	Glass-paper ...	20°	11·50	11·75	14·25	11·50
1·5	Red glass	29°	15·75	14·75	19·75	15 50
1·4	Blue glass	......	13 75	13·50	15 25	13 50
1·4	Alum	......	9·75	8·75	12·25	8·50
0 2	White mica ...	25°	20·00	21·00	22·50	18 25
0·1	Green mica ...	......	20·50	21·75	20 75	19·00
1·3	White glass ...	......	14·25	13·50	17·75	14·25
4·4	Rock salt	24°	20·75	18·75	21·75	20·50
3·7	Calcareous spar	......	10·75	8·75	16·00	10·50
1·4	Gypsum.........	......	9·75	7·50	11 50	7·50
0·2	Glass-paper ...	24°	12·50	12·50	16·00	12·25

TABLE I. (*continued*).

Thickness in millimetres.	Substances inserted.	Deflection by direct radiation.	Deflection after insertion with			
			Red-hot platinum.	Flame of alcohol.	Flame of Argand lamp.	Flame of hydrogen.
1·5	Red glass	35°	19·25	18·50	24·75	20·50
1·4	Blue glass	......	18·75	18·50	21·25	18·50
1·4	Alum	......	13·75	13·25	17·00	13·50
0·2	White mica ...	32°	26·25	28·75	29·75	25·75
0·1	Green mica ...	......	27·75	29·50	28·00	26·50
1·3	White glass ...	......	19·00	16·25	22·00	19·00
4·4	Rock salt	29°	24·25	21·50	25·25	23·50
3·7	Calcareous spar	......	12·00	11·50	20·25	12·50
1·4	Gypsum........	......	10·00	8·50	12·75	8·50
0·2	Glass-paper ...	28°	14·25	14·25	18·50	14·00*

It is thus evident that the radiation of heat through diathermanous bodies does not stand in relation to the temperature of the source of heat in any one of the instances which occur here.

2. To render the experiment as clear as possible, I also observed the transition of the heat emitted by radiation from *one and the same* body at different temperatures.

(1.) For this purpose, *with low degrees of heat*, I made use of a Leslie's cube†, the sides of which were 8 centim. in length; in this I heated water to ebullition, and then allowed it to cool gradually. The cooling took place so slowly, that the temperature of the cube, during the short period of the insertion of a diathermanous substance, was considered as constant.

The following phænomenon occurred:—When, by the approximation of the cooling cube before each insertion, a constant deflection of 35° was produced, the needle each time receded to 11° when the colourless glass 1·3 millim. in thickness was introduced between the source of heat and the thermal pile, even when the temperature of the former was between 100° and 212° F. Thus the heat was capable of passing through the glass plate

* It may perhaps appear remarkable that I have not produced the same direct deflections of the thermoscope for all diathermanous bodies, for the sake of greater uniformity. The reason is, that I was compelled to be as sparing as possible with the hydrogen which formed one of the flames, because each reproduction of it interrupted the proper series of experiments for a considerable time, and disturbed the comparison of the results. I therefore always started from that deflection which the radiation of the hydrogen-flame produced without continued regulation, by arranging that of the other sources of heat according to it. I might certainly have reduced the various observations by calculation to a common one; however, I have omitted this tedious process, because not the slightest object would be gained by it except the more elegant form.

The above table shows that the rock salt which I used did not allow the rays from all sources of heat to pass through it in the same manner as was observed by Melloni with his.

† J. Leslie, An Experimental Inquiry into the Nature and Propagation of Heat. Lond. 1804, p. 6.

to the same extent whatever the degree of heat of the radiating body was, within the limits to which this investigation extended. In this experiment it was a matter of indifference whether the radiating surface of the Leslie's cube consisted of metal or glass, or whether it was coated with lamp-black, wool, or other substances. The same was found to be the case with all other diathermanous bodies. Thus the needle receded each time to 18° when, with a constant direct deflection of 35°, white mica 0·2 millim. in thickness was inserted between the Leslie's cube and the thermoscope; and each time to 20°, when this was exchanged for green mica 0·1 millim. in thickness. The following table will exhibit this still more distinctly.

TABLE II.

Radiating surface of the Leslie's cube.	Distance of the latter from the thermal pile in inches.	Temperature of the Leslie's cube according to F.	Deflection on inserting Red glass, 1·5 millim.	Blue glass, 1·4 millim.	Alum, 1·4 millim.
1. Lamp-black. Deflection by direct radiation 35°.	8·5	100°	9 50	8 50	3·50
	10 0	113	9·50	8 75	3·50
	12·0	124	9·50	8 50	3·75
	13·5	147	9 75	8 50	3 50
	16·0	212	9 50	8 50	3·50
2. White glass. Direct deflection 35°.	5·50	82°	9·50	8 50	3 50
	7·00	104	9·50	8 50	3 50
	9·00	122	9 50	8·50	3 50
	12 00	158	9·75	8 75	3·50
	13 75	212	9·50	8 50	3·50
3. Black paper. Direct deflection 35°.	7 0	95°	9·50	8 25	3·50
	9 5	120	9 75	8·50	3 50
	10·5	156	9 50	8 50	3·50
	11·5	169	9 50	8·50	3 50
	15 5	212	9·50	8 50	3 50

Radiating surface of the Leslie's cube.	Distance of the latter from the thermal pile in inches.	Temperature of the Leslie's cube according to F.	Deflection on inserting White mica, 0·2 millim.	Green mica, 0·1 millim.	White glass, 1·3 millim.
1. Lamp-black. Deflection by direct radiation 35°.	7·0	95°	18 00	20 25	11·00
	8·5	109	17·75	20 25	10 75
	13 5	145	18·00	20 00	10 75
	15·0	182	18·00	20·50	10·75
	16·5	212	18 00	20·25	11 00
2. White glass. Direct deflection 35°.	8·0	100°	17 75	20·25	11·00
	10·5	127	18·00	20·00	11·00
	12·0	143	18·00	20·00	11·00
	14·5	165	17·75	20·25	11 00
	16·5	212	18 00	20 25	11·00
3. Red wool. Direct deflection 35°.	5 5	93°	17 75	20·25	10 75
	10·0	127	18·00	20·50	10·50
	11·5	145	18·00	20·25	10 75
	13·5	160	18·00	20·25	10 75
	15·5	212	17·75	20·25	10·75

TABLE II. (*continued*).

Radiating surface of the Leslie's cube.	Distance of the latter from the thermal pile in inches.	Temperature of the Leslie's cube according to F.	Deflection after inserting			
			Rock salt, 4·4 millim.	Calcareous spar, 3·7 millim.	Gypsum, 1·4 millim.	Glass-paper, 0·2 millim.
1. Lamp-black. Deflection by direct radiation 35°.	9·5	113°	20·00	7·50	8·75	14·00
	10·5	122	19·75	7·50	8·50	14·25
	12·0	135	20·00	7·25	8·75	14·00
	14·0	160	20·00	7·50	8·75	14·00
	16·0	212	20·00	7·50	8·75	14·00
2. White glass. Direct deflection 35°.	6·50	95°	19·75	7·25	8·75	14·25
	9·50	120	19·75	7·25	8·50	14·25
	10·00	133	20·00	7·50	8·75	14·25
	11·00	156	20·00	7·50	8·50	14·25
	13·75	212	20·00	7·50	8·75	14·00
3. Black silk. Direct deflection 35°.	6·5	93°	20·00	7·50	9·00	14·25
	8·0	104	20·00	7·50	9·00	14·00
	10·0	122	20·00	7·75	9·00	14·00
	12·0	140	19·75	7·75	9·00	14·25
	18·0	212	20·00	7·50	8·75	14·25

Thus it is proved that the temperature of one and the same source of heat within the limits of these experiments, *i. e.* between 88° and 212° F., has not the slightest influence upon the transmission of the heat radiating from it through diathermanous bodies.

I must here again refer to Melloni's experiment, which has been alluded to above (p. 195), and appeared to show that the property of heat to pass through mica increases with the temperature of the source of heat even between 122° and 212° F. As my experiments, just now detailed, did not agree with this statement, I have repeated them so very frequently that I have no doubt of their accuracy. So far as I have repeated Melloni's experiments, this is the only case in which my results differ from those of this distinguished philosopher, for whom I entertain the most profound respect and admiration.

(2.) The next question was, how heat at *temperatures above* 212° F., radiating from one and the same body, would behave as regards transmission through diathermanous bodies.

For the purpose of investigating this, I placed a cylinder of blackened sheet iron, copper or brass, 17 centim. in height and 3 centim. in diameter above the flame of an Argand lamp, by which I was enabled to heat it to different and sufficiently constant degrees of temperature. I certainly had no means of determining these during the experiment in ordinary thermometric degrees,

however the pile itself indicated elevations and depressions of temperature with the utmost accuracy, which was perfectly sufficient for the decision of the present question.

On transmission by radiation, it was evident that the heat emitted by the metallic cylinder at the increased temperature passed through some substances comparatively better; through others in the same proportion as on the application of a lower degree of heat.

Thus the galvanometer needle, which by direct radiation upon the pile was deflected to 35°, receded to 11° on inserting the colourless glass, the cylinder being nine inches from the thermoscope; but only to 13° when it had so high a temperature as to require removal to a distance of thirty-six inches to produce a similar deviation of 35°.

With green mica, in the first instance a recession of the needle to 20°·25 was obtained; in the latter to 26°; whilst under all circumstances the needle receded to 3°·5 when the plate of alum 1·4 millim. in thickness was introduced between the source of heat and the thermal pile, and to 8°·5 when gypsum of a similar thickness was used.

The following table contains the details of the observations:—

Table III.

Radiating metallic cylinder.	Distance of the latter from the thermal pile in inches.	Approximative determination of its temperature according to F.	Deflection after the insertion of		
			Red glass, 1·5 millim.	Blue glass, 1·4 millim.	Alum, 1·4 millim.
Iron cylinder. Deflection by direct radiation 35°.	7·0	Below 234°	9·50	8·75	3 50
	12·5	—	9 75	8·75	3·75
	14·5	—	9·50	8·50	3·50
	24·0	Above 234°	10·25	9·25	3·50
	33·5	—	10 25	9·25	3·50
Copper cylinder. Direct deflection 35°.	10·0	Below 234°	9·50	8·50	3·50
	15·0	—	9·25	8·50	3·50
	20·0	—	9·50	8·50	3 75
	30 0	Above 234°	10·00	9·25	3·50
	38·0	—	10·50	9·50	3·50
Iron cylinder. Direct deflection 40°.	10·0	Below 234°	15·00	14·50	12·50
	30·0	Above 234°	15·75	15·25	12·25

TABLE III. (*continued*).

Radiating metallic cylinder.	Distance of the latter from the thermal pile in inches.	Approximative determination of its temperature according to F.	Deflection after the insertion of		
			White mica, 0·2 millim.	Green mica, 0·1 millim.	White glass, 1·3 millim.
Iron cylinder. Deflection by direct radiation 35°.	8·0	Below 234°	18·00	20·25	11·00
	10·0	—	18·00	20·50	11·00
	14·5	—	18·75	21·50	11·00
	24·0	Above 234°	21·00	24·00	11·50
	38·5	—	24·00	26·50	12·50
Brass cylinder. Direct deflection 35°.	9·0	Below 234°	18·00	20·25	11·00
	11·0	—	18·00	20·25	11·00
	15·0	—	20·25	22·50	11·50
	24·0	Above 234°	20·75	23·50	11·50
	36·0	—	23·50	26·00	13·00
Iron cylinder. Direct deflection 40°.	10·0	Below 234°	22·50	25·00	13·00
	31·0	Above 234°	28·00	31·50	15·00

Radiating metallic cylinder.	Distance of the latter from the thermal pile in inches.	Approximative determination of its temperature according to F.	Deflection after the insertion of			
			Rock salt, 4·4 millim.	Calcareous spar, 3·7 millim.	Gypsum, 1·4 millim.	Glass-paper, 0·2 millim.
Iron cylinder. Deflection by direct radiation 35°.	9·5	Below 234°	20·25	7·50	8·75	14·25
	16·0	—	20·25	7·50	8·75	14·25
	24·0	—	22·25	7·50	9·00	14·25
	26·0	Above 234°	22·25	7·50	8·75	14·25
	34·0	—	23·75	8·00	8·75	14·25
Copper cylinder. Direct deflection 35°.	14·5	Below 234°	20·00	7·50	8·75	14·25
	16·0	—	20·25	7·50	8·75	14·25
	20·0	—	20·25	7·50	8·50	14·25
	31·5	Above 234°	21·75	7·75	8·50	14·25
	39·5	—	22·75	7·75	8·50	14·25
Iron cylinder. Direct deflection 40°.	9·0	Below 234°	28·75	9·50	11·25	19·50
	35·5	Above 234°	32·50	9·50	11·00	19·25

The distances of the heated cylinder from the thermoscope given in this table clearly show the great increase of the heat. At the moment when the change in the transmission of the heat observed in some diathermanous bodies occurred, its temperature amounted to about 234° F. Even at its greatest heat no trace of redness was visible even in the dark.

3. The transmission of heat, radiating from one and the same body *at different stages of a red heat*, still remained to be examined. For this purpose I heated a spiral of platinum wire to

a red, yellow and white heat over the chimney of a Berzelius's lamp. The visible portion of the flame of the alcohol was never allowed to rise above the metallic cylinder of the lamp, and the thermal pile was protected from its rays by polished screens of tinned iron.

Experiment showed that when the direct radiation of heat from each of the sources mentioned thrown upon the thermal pile had deflected the needle to 35°, on transmission through colourless glass the heat from platinum at dark redness produced a deflection of 10°·5 ; at a red heat, of 17°·25 ; at a yellow heat, of 17°·25 ; and at a partial white heat, of 21°·12. Thus the rays from platinum at a red and yellow heat, having a great difference in temperature, are transmitted by colourless glass in exactly the same proportion. When the glass was replaced by the plate of alum we have so frequently mentioned, a deflection of 10°·2 in the needle was produced by the platinum at dark redness; of 11°·4 for platinum at a red heat; of 9°·1 for that at a yellow heat; and of 12°·4 for that at a partial white heat. Thus the heat of the platinum at a yellow heat passes through the plate of alum to a less extent than that of the platinum at a red heat, nay even than that of platinum at a dark red heat, notwithstanding its far higher temperature.

The same applies to gypsum. The heat of platinum at dark redness passes most imperfectly through mica; that of platinum at a red heat comparatively better; that at a yellow heat still better; and that at a white heat best of all. Hence we find every possible case, independent of the temperature of the source of heat. The following table contains the observations on this point:—

TABLE IV.

Thickness in millim.	Substances inserted.	Deflection by direct radiation.	Deflection after the insertion, by Platinum at a dark red heat.	Platinum at an evident red heat.	Platinum at a yellow heat.	Platinum partly at a white heat.
1·5	Red glass	20°	7·75	9·50	9·00	11·00
1·4	Blue glass......	......	7·37	8·70	7·87	9·50
1·4	Alum...........	......	6·37	6·50	4·50	7·50
0·2	White mica ...	20°	12·75	16·50	17·25	17·75
0·1	Green mica ...	......	13·75	16·87	17·50	17·75
1·3	White glass ...	......	6·12	10·37	10·37	12·50
4·4	Rock salt	20°	13·50	16·62	15·50	16·88
3·7	Calcareous spar	......	5·75	8·50	7·00	10·37
1·4	Gypsum.........	......	6·20	6·50	3·12	7·00
0·2	Glass paper ...	20°	9·62	10·50	11·00	12·00
1·5	Red glass	35°	12·50	18·40	17·31	21·31
1·4	Blue glass......	......	11·75	16·44	15·20	18·40
1·4	Alum...........	......	10·20	11·40	9·10	12·40
0·2	White mica ...	35°	20·25	27·44	29·50	30·60
0·1	Green mica ...	......	23·10	28·50	29·94	30·81
1·3	White glass ...	......	10·50	17·25	17·25	21·12
4·4	Rock salt	35°	24·26	29·60	28·95	30·25
3·7	Calcareous spar	......	9·07	14·15	12·55	17·00
1·4	Gypsum.........	......	9·81	11·80	9·50	12·70
0·2	Glass paper ...	35°	15·12	17·25	18·12	19·12
1·5	Red glass	40°	13·00	19·00	18·00	23·12
1·4	Blue glass......	......	12·25	17·12	16·50	19·62
1·4	Alum...........	......	11·25	11·50	10·00	12·50
0·2	White mica ...	40°	24·50	32·50	34·75	35·62
0·1	Green mica ...	......	27·50	33·62	35·00	35·75
1·3	White glass ...	......	12·25	20·75	20·88	24·88
4·4	Rock salt	40°	30·00	35·25	34·25	35·75
3·7	Calcareous spar	......	10·50	15·75	15·00	20·00
1·4	Gypsum.........	......	12·00	12·87	11·75	15·12
0·2	Glass paper ...	40°	18·25	20·62	21·75	23·25

The numbers which refer to the direct deviation of 20° are each the arithmetic mean of two observations, those for 35° the mean of four, and those for 40° also of two experiments. The results of the individual series thus combined agreed so perfectly with each other, that the above numbers may be regarded as accurate to within half a degree.

Hence it is experimentally placed beyond all doubt, *that the passage of radiant heat through diathermanous bodies is not in immediate connexion with the temperature of its source, as was probable from previous experiments, but is alone dependent upon the structure of the diathermanous substance, which is penetrated*

*by certain rays of heat in a greater degree than by others, whether this occurs at a lower or higher temperature**.

That free *radiant heat* is really the agent concerned in the cases we have detailed, follows with certainty from the following observations :—

1. When, after the insertion of one of the above bodies between the source of heat and the thermal pile, the needle of the multiplier has arrived at a certain deflection, if the source of heat be removed whilst the inserted substance retains a fixed position as regards the pile, the needle at the same time returns to the same point, whatever may have been the extent of the amount of deflection. Hence this does not arise from the inserted body itself becoming heated.

2. If the thermal pile be removed from the field of rectilinear radiation of the source of heat, whilst it preserves a constant distance from the inserted substance, which remains exposed to the heating rays, the needle immediately returns to the same point which it attains on the removal of the source of heat; a further proof that the deviation observed cannot be ascribed to the heat absorbed by the former substance.

3. In almost all the bodies experimented upon, the indication of the thermoscope is diminished when it is coated on both sides with lamp-black; and when thus its power of absorption and radiation is increased at the expense of the transmission.

* I obtained the following results when radiant heat from a Leslie's cube at 212° F., red-hot platinum, the flame of alcohol and an Argand lamp was transmitted through *prepared rock salt* :—

Thickness in millim	Substance inserted.	Deflection by direct radiation.	Deflection after the insertion in the case of			
			Source of heat at 212° F.	Red-hot platinum.	Flame of alcohol.	Argand lamp.
2·9	Rock salt coated with lamp-black	20°	13·00	11·75	11·75	10·00
2·9	Rock salt coated with lamp-black	25°	14·25	13·75	13·50	12·00
2·9	Rock salt coated with lamp-black	30°	16·75	16·50	16·25	13·75
2·9	Rock salt coated with lamp-black	35°	20·25	19·50	19·25	16·50

Thus the heat of the red-hot platinum and the flame of alcohol, the temperatures of which are undoubtedly different, radiate through the prepared rock-salt in the same manner, whilst the heat from the source at 212° F., conformably with Melloni's discovery, passes through it better than that of the Argand lamp.

4. When the substances are inserted in their ordinary state before the thermal pile, the needle recedes to a point which it does not leave during the observation; whilst whenever they are coated with lamp-black, and thus become more heated by absorption, an increase of this deflection is perceived.

Moreover, my results agree perfectly, as far as they are comparable, with those of Melloni, who during his observations has convinced himself partly in the same way, that the heat emitted by the screens bears no proportion to that transmitted by them. A *diffuse* transmission cannot have occurred in the cases mentioned, as the diathermanous bodies were all polished as highly as possible.

Their thickness was a matter of no importance, since they were not to be compared with each other, but merely served for the investigation as regards the sources of heat.

That the *different form and size of the sources of heat which have been compared* did not induce any differences in the transmission, has been proved by direct experiments; for it was found that when the needle of the multiplier was deflected to 50° by direct radiation upon the pile, on introducing the red glass it receded to 21°·25—21°, when either a Leslie's cube the sides of which were 4 centim., 8 centim., or 15 centim. 7 millim., or a cylinder 17 centim. high and 3 centim. in diameter was used as a source of heat at 212° F. The direct deflection being 50°, the same result of a return of the needle to 31°·5 was obtained, whether a small glass spirit-lamp or a large Berzelius's lamp was used; and the same deviation of 37°·5, both for the small flame of a wax-light and for the large one of an Argand lamp.

The following summary which contains the arithmetical means of every three observations, shows that this was also the case with other diathermanous bodies:—

TABLE V.

Thickness in millim.	Substances inserted.	Deflection by direct radiation.	Deflection after the insertion by A Leslie's cube of 4 centim.	8 centim.	15 centim. 7 millim.	A cylinder 17 centim. high and 3 centim. in diameter.
1·5	Red glass	50°	21·00	21·25	21·00	21·00
1·4	Blue glass......	......	20·50	20·25	20·25	20·50
1·4	Alum	......	14·00	14·25	14·00	14·25

TABLE V. (*continued*).

Thickness in millim.	Substances inserted	Deflection by direct radiation.	Deflection after the insertion by A small flame of alcohol.	The large flame of a Berzelius's lamp.	The small flame of a wax-candle.	The large flame of an Argand lamp.
1·5	Red glass	50°	31·50	31·50	37·50	37·50
1·4	Blue glass	......	30·00	30·00	33·00	33·00
1·4	Alum	......	15·25	15·50	24·00	24·00
1·5	Red glass	60°	39·42	39 70	45·20	45·30
1·4	Blue glass......	......	37 80	37·70	41·00	41·00
1·4	Alum	......	23·92	23·42	32·92	33·00

The deflection of the galvanometer-needle, produced by direct radiation, upon the constancy of which the accuracy of all the comparisons detailed depends, had been previously tested before the insertion of each diathermanous substance.

Three only of the latter were used in succession in the investigation with regard to the various sources of heat; so that the observations relating to them, each of which required a minute and a half or two minutes, never extended beyond a time during which all the conditions of the experiment could be considered as sufficiently constant*.

To preserve this uniformity as much as possible, the position of the thermal pile remained unchanged, whilst the source of heat was more or less approximated to it, until the constant direct deflection used for comparison was produced. The diathermanous bodies were always inserted at the same spot behind a diaphragm, and at a constant distance from the thermoscope.

II. *On the Heating of Bodies by Radiant Heat.*

It is a fact, which has been long known and proved by an extended series of observations—

1. That different substances are heated to a different extent by the radiation of heat from one and the same source.

2. That in every instance the extent of the increase of heat depends upon the structure of the surface.

More recent experiments by Baden Powell and Melloni have shown—

* For this reason the numbers in these, as well as in all the subsequent instances, should only be compared so far as I have brought them into relation with one another.

3. That one and the same body is not uniformly heated by rays of heat emanating from different sources, which exert the same direct action upon a thermoscope coated with lamp-black (see above, p. 188). I shall only select two very characteristic observations from those which I have made in regard to this point.

When I coated a metallic disc on one side with carmine, on the other with lamp-black, and exposed it immediately before the thermal pile to the rays of an Argand lamp, in such a manner that the carmine side was towards the source of heat and the blackened one next the pile, it was found that when the direct deflection of the multiplier-needle by the Argand lamp amounted to 35°, that produced by the above arrangement amounted to 9°·5. Under the same circumstances, however, I obtained a deflection of 10°·17, when, instead of a flame, I produced radiation upon the carmine-surface from a metallic cylinder at a heat below redness, which gave a direct deflection of 35°.

When the metallic disc was covered with black paper instead of carmine, the magnetic needle was deflected by the heat from the plate placed before the thermal pile, in the first instance to 10°·75, in the second to 10°·12.

Thus the carmine-surface is comparatively less heated by the rays from the Argand lamp than by those of a cylinder heated to 212° F., whilst with black paper exactly the contrary occurs. The following numbers, which refer to a greater direct deviation (each being the arithmetical mean of two observations), will show this still more decidedly :—

TABLE VI.

Surface inserted and becoming heated.	Deflection by the direct radiation of the source of heat.	Deflection after the insertion with		Deflection by the direct radiation of the source of heat.	Deflection after the insertion with	
		The Argand lamp.	The heated cylinder.		The Argand lamp.	The heated cylinder.
Carmine	35°	9·50	10·87	50°	13·75	15·62
Black paper ...	35°	10·75	10·12	50°	15·25	14·00

Whilst from former observations the rays of heat emitted by sources having a lower temperature appear almost invariably to be more capable of heating bodies than those at a higher temperature, it is clearly shown by the experiments we have just

detailed, *that this heating, the intensity of the heat received by radiation being the same, is perfectly independent of the temperature of its source, but is occasioned by the nature of those absorbing substances, which are more susceptible of some rays than of others.*

The influence of the *thickness* of the bodies exposed to the rays of heat upon their becoming heated has scarcely hitherto been investigated. Leslie remarked that metals of different thickness became heated to the same extent; but that wooden screens, which he placed before a heated cube, were less heated in proportion to their thickness. Thus his thermoscope indicated—

20° behind a plate of pine-wood			$\frac{1}{8}$th inch in thickness,	
15°	...	...	$\frac{3}{8}$th	...
9°	...	...	1	...

Melloni also found, by means of the thermo-multiplier, that thick paper became less heated than thin.

However, these experiments did not appear to me to decide the question—

In what relation the heating of a body stands to its thickness; which has therefore formed the object of the following investigation.

Whilst in making the observations in the previous section I endeavoured to ascertain the influence exerted by the media placed between the source of heat and the thermal pile when heated, upon the latter; in this case I took special care to make it as conspicuous as possible, and so that it acted exclusively upon the thermoscope. I therefore placed the bodies to be heated immediately before the latter, and furnished them on that side next the pile with a coating impervious to direct rays.

The substances which I used in these experiments were colourless transparent varnish, black, opake, but diathermanous lac, and white lead, which is usually regarded as adiathermanous. I placed these in *layers of different thickness* upon thin metallic discs in every respect alike. To improve the dispersion from the latter after it has become heated, I coated them on the sides next the pile with paper. Lamp-black would certainly have been more effective for this purpose; but it is scarcely possible to lay it upon several plates in exactly the same manner, which would have been indispensable, because, as Melloni has shown, the dispersion varies with the thickness of the layers of lamp-

black. I therefore preferred the above coating, so as not to increase the delicacy of the apparatus at the expense of the accuracy of the comparison.

For the sake of completeness, I examined the heating of the substances mentioned, of unequal thickness by different sources of heat, applying for this purpose those which had always shown the greatest difference, viz. an Argand lamp and a metallic cylinder heated to about 212° F.

Since besides lamp-black (p. 188) only metals (p. 206) absorb all kinds of rays of heat to the same extent, the substances mentioned for the present object could only be placed upon metal if it was required to observe their action after having become heated by different sources of heat, without the secondary influence of the surfaces beneath, which in every other instance would have been disproportionately heated.

Experiment yielded the following result:—When by direct radiation from an Argand lamp upon the thermal pile, a deflection of 60° was produced in the needle of the multiplier, if a metallic plate was placed immediately before the thermoscope, with its polished surface turned towards the source of heat, whilst next the pile it was coated with paper, in a short time a constant deviation of 10°·5 was produced, which arose from the metallic plate becoming heated.

Thus the coated plate became more heated when the number of layers of varnish covering it was increased.

When the flame was exchanged for the cylinder at a dark red heat, which produced the same direct deviation of 60°, and the uncoated metallic screen was again inserted at the spot already mentioned, the needle as before moved to 10°·5. However, it was deflected to 17°·5 when the metallic plate, covered with one layer of varnish, was exposed to the rays from the heated cylinder, and to 20°·75 when with eight layers of varnish.

Thus in the latter case the heating was greater for each individual plate than with the first; and increased from that coated with a single layer of varnish to that with eight layers in a greater degree than in the experiment with the Argand lamp.

The same phænomenon occurred with black lac and white lead. Thus, under exactly the same conditions, the heat communicated from a metallic sheath coated with a thin layer of lac, when acted on by the rays of the Argand lamp, produced a deflection of 14°·5; that from the same, covered with a thicker

layer, 18°·12; and by the rays of the cylinder at a dark red heat, the first a deviation of 18°·62, the latter of 22°·12. It must here be taken into consideration, that the power which deflects the galvanometer-needle a certain number of degrees *higher* is *greater* than that which causes it to deviate the same number of degrees *lower*.

The numbers found with the coating of white lead are given in the subjoined table, which also contains values for other thicknesses of the bodies spoken of, and a different direct deflection from that mentioned (in each case the arithmetic mean of two observations):—

Table VII.

Deflection on inserting a metallic plate										
Not coated.	Coated with varnish.				Coated with black lac.				Coated with white lead.	
	1 coat.	2 coats.	4 coats.	8 coats.	Thinnest coat possible.	Thicker coat.	Still thicker coat.	Very thick coat	Thin coat.	Thick coat
Argand lamp. Deflection from direct radiation 35°.										
6·50	8 25	8·25	8·25	8·25	7·12	8·25	8·62	9·50	7·25	8·00
Metallic cylinder at a dark red heat										
6 50	9·00	9 25	9·50	9·50	9·12	9 87	11·62	12 00	8·75	9·62
Argand lamp. Deflection from direct radiation 60°.										
10·50	14 50	15·12	15·62	15 75	14·50	16 25	17 37	18·12	16·12	18 50
Metallic cylinder at a dark red heat.										
10·50	17 50	18·12	20 12	20 75	18 62	20 25	21·37	22 12	17·00	19·50

The differences which are thus rendered evident could only arise from the heat which is absorbed by the substances applied as coatings, and in this manner communicated to the metallic plates. It is thus shown, *that the substances employed become heated, within the limits of this experiment, to a degree the extent of which is proportionate to their thickness.*

This observation is directly opposed to the experiments made by Leslie and Melloni upon other substances. Nevertheless, both are correct. The cause of this difference is, that in my experiments I have not yet attained the limit beyond which these earlier experimenters had already passed. For the connexion of the phænomena is as follows:—If we expose a body to rays emanating from a source of heat, those which are not reflected from its surface penetrate it, and impart heat to one layer after another, as long as they pass through it without

hindrance. Each of these layers then communicates its caloric by conduction and radiation to the neighbouring ones. The amount of heat imparted in this manner to any body therefore increases in proportion as the number of the absorbing layers increases; but it attains its maximum as soon as these acquire a thickness beyond which the heat cannot penetrate either by radiation or conduction.

In the series of experiments which has been detailed, the thickness was never so great but that every coat laid on became heated, and that the heat of all could act upon the metallic surface, which emitted the rays against the thermal pile by means of the paper covering; but in the experiments of Leslie and Melloni, the screens inserted (which were only diathermanous in the thinnest layers) were so thick, that a small portion only of the heat from their anterior surfaces reached the side next the thermoscope; and hence its action upon the latter must have been diminished to the same extent as this portion was weakened by increasing the thickness.

The limit at which the heating of a body ceases to increase in proportion to the diminished thickness, is determined for one and the same source of heat by the substance, and for one and the same substance by the nature of the source of heat. We have reserved the more minute examination of this point in certain cases for a future investigation.

Melloni considered that it is impossible to detect the elevation of temperature which thin diathermanous plates experience from radiant heat, and therefore concluded indirectly regarding their becoming heated. In the experiments detailed, I succeeded in proving it by the direct method, and in experimentally confirming Melloni's conjectures in a palpable manner, *that the temperature of a body, when the thickness increases, is more raised the less it is diathermanous to the rays transmitted to it.* Thus, as has been already mentioned, the observations contained in the table at p. 209 always show, in the case of the same body, with the cylinder at a dark red heat, a greater increase in the heat required in proportion to the thickness than in the Argand lamp; whilst direct transmission has shown that the heat of the former is transmitted in a less degree than that of the latter by white varnish and black lac.

That diathermanous bodies, as has hitherto been only supposed, *in reality become most heated by those rays which pene-*

trate them least, can also be proved by observation in colourless glass, which, as is known, is also less perfectly penetrated by the heat of the metallic cylinder than that of the Argand lamp; for a glass mirror 1·5 millim. in thickness, the rough metallic surface of which was turned towards the thermal pile, when acted upon by the rays of the former, produced a deflection of the galvanometer-needle to 12°·25; by those of the lamp to 11°, when direct incidence from both these sources of heat had deflected the needle to 45°.

It scarcely requires to be again mentioned, that this proof of diathermanous bodies becoming heated by radiation does not affect the results detailed in the first section, as it has been experimentally shown (p. 203 and 204), that under the circumstances in which the experiments on the transmission were instituted, it had no perceptible share in the production of the results.

III. *On the Property of radiating heat in Bodies.*

It is already known that different substances radiate heat at the same temperature in an unequal degree, and that this property, in one and the same substance, is dependent,—1st, upon the structure of its surface; 2nd, upon its thickness.

1. Although Leslie had previously expressed the view, that the hardness of bodies influenced their radiating power, Melloni first endeavoured to prove that the changes produced in the radiation of one and the same body by scratching its surface, could only be ascribed to the modifications of its hardness produced at the spots concerned.

He obtained the following deflections of the thermo-multiplier; by radiation from

A silver plate, beaten out and polished . . .	10°·0
A silver plate, beaten out and scratched . .	18°·0
A silver plate, cast and polished	13°·7
A silver plate, cast and scratched	11°·3;

and found that in agate, ivory and marble, the degree of roughness did not produce any alteration in the radiation, a remark which had previously been made by Leslie with regard to glass, paper and lamp-black. Hence Melloni drew the conclusion, that more heat is always emitted when the scratching exposes softer parts of the radiating substance; less when it produces a

condensation of it; but that no change ensued in this respect when the hardness and elasticity of the surface were not modified by the scratching.

For the sake of convincing myself of the truth of this interesting law, which has not hitherto been further examined, I made the following experiment:—

I first caused a Leslie's cube of 8 centim., which consisted of *two cast* and *two rolled plates of lead*, and was retained at a temperature of 212° by boiling water, to radiate its heat against the thermal pile placed at a constant distance from the heated surfaces. The surfaces of the two pairs were too different for this experiment to have allowed of our concluding upon the connexion of the radiation with the hardness and density. I could not succeed in proportioning the two leaden plates of each pair so as to produce the same deflection of the thermo-multiplier by their radiation. At a certain distance of the heated surfaces from the pile, the radiation of one of the cast plates produced a deflection of the galvanometer-needle to 48°·25, that of the other to 49°; and the radiation from one rolled plate a deflection of 51°, that of the other of 50°·5.

There was no question that lead becomes condensed at the shining mark made by drawing a steel instrument across it. In conformity with Melloni's conclusion, therefore, the scratching must diminish the radiation of the surfaces of the lead, and this to a greater extent in that cast than rolled. This was confirmed by experiment. When that *cast* plate which had produced the greater deflection of 49° was scratched, its radiating power was diminished so that it became equal to that of the other surface which radiated less perfectly. They now both produced a deflection of the needle of 48°·25 when at an equal distance from the thermoscope. When the longitudinally scratched leaden plate was covered with transverse stripes, its radiating power became still less. Retaining the same position to the pile, it deflected the galvanometer-needle to 47°·25 only.

Of the *rolled* plates, that which produced the deviation of 50°·5 was scratched. The radiation was also diminished in this case, for it only caused the needle to deviate to 48°·5. When the surface was scratched in both directions, the radiating power was increased to the extent of producing a deflection of 49°·75; which might arise from the lead being condensed as before at the very portions scratched, but rendered less dense at those

points at which the raised margins of the furrows meet. This constituted a difference between the rolled and the cast plate, in which the transverse stripes also diminished the radiation.

The following table contains the numbers which have been obtained in different experiments, each being the arithmetic mean of two observations:—

TABLE VIII.

Cast leaden plate at 212° F.	I.	II.	I.	II.	I.	II.	I.	II.
	Smooth.	Smooth.	As before, but nearer the pile.		Smooth.	Scratched.	As before, but nearer the pile.	
Deflection by direct radiation...	34·62	34·87	**48·25**	**49·00**	41·00	41·00	**48·25**	**48·25**

Cast leaden plate at 212° F.	I.	II.	I.	II.	I.	II.	I.	II.
	Smooth.	Closely scratched.	As before, but nearer the pile.		Smooth	Scratched in both directions.	As before, but nearer the pile.	
Deflection by direct radiation...	37·50	36 25	**48·25**	**47·50**	40·50	40·00	**48·25**	**47 25**

Rolled leaden plate at 212° F.	1.	2.	1.	2.	1.	2.	1.	2.
	Smooth.	Smooth.	As before, but nearer the pile.		Smooth.	Scratched.	As before, but nearer the pile.	
Deflection by direct radiation...	35·25	34·62	**51·00**	**50·50**	42·25	41 00	**51·00**	**48·50**

Rolled leaden plate at 212° F.	1.	2.	1.	2.	1.	2.	1.	2.
	Smooth.	Closely scratched.	As before, but nearer the pile.		Smooth.	Scratched in both directions.	As before, but nearer the pile.	
Deflection by direct radiation...	41·00	39·00	**51·00**	**48·00**	42·50	40·00	**51·00**	**49·75**

With the same object as that just stated, I made a second experiment.

Melloni ascribes the increase which the radiation from a *copper plate* undergoes when the latter is scratched, to the circumstance, that by this proceeding less dense portions would become

exposed. Were this the case, the difference in the radiation from polished surfaces and those scratched in both directions should diminish when, without altering their inequalities, these plates are coated with layers of the same metal of equal thickness.

This may be effected by precipitating copper upon them by galvanism. To procure this coating as uniform as possible by the same electric current upon one polished and three roughened plates which I wished to examine, I had them so soldered together as to form the lateral walls of a cube, the scratched surfaces being turned inwards. This was filled with the cupreous solution, from which the precipitate was formed as usual after the galvanic process was commenced. When it had attained a sufficient thickness, the cube was soldered in such a manner that the surfaces, which were furrowed in both directions and had now become coated, were turned outwards.

The next question was, whether the difference in their radiation was now less than in those plates which were not coated and had the same inequalities.

This has been experimentally proved most distinctly. Thus, whilst the polished surface of the cube composed of ordinary rolled sheet copper, which before being coated was exactly the same as that already described, at a temperature of 212° F. produced a deflection of 29°, and one of the sides of the same cube which was scratched in both directions a deflection of 47°·75 in the thermo-multiplier, the smooth surface which had been coated by galvanism at the same temperature and distance from the pile, deflected the needle to 49°·25; that scratched, to 51°·5. In the first instance the difference amounted to 18°·75; in the latter to 2°·25. Certainly the difference between the *amount of heat* emitted by the smooth and the scratched plate might be equal in both experiments, and yet the difference in the *deflections* produced by them be less the second time, when observed at higher degrees, than the first time. However, the great diminution in this difference we have mentioned (from 18°·75 to 2°·25), could not be ascribed to this inequality in the indications of the instrument; for when the coated cube was removed so far from the pile that the radiation from its smooth surface deflected the needle to 33°, the radiation from the scratched plate at the same place produced an indication of 35°·5 in the multiplier. Thus the difference amounted to only 2°·5, which is less than the deflections comprised in the first observation. Hence

there is no doubt that the difference between the amount of heat emitted by a polished and a scratched plate of copper in reality diminishes when these surfaces are coated with a uniform layer of this metal.

The following numbers (each consisting of the arithmetic mean of two observations, afforded by the radiation of differently engraved plates in the cases which have been considered) will show this still more clearly :—

TABLE IX.

Radiating surfaces.	Plates of rolled copper-plate at 212° F.	Plates of rolled copper, having the same inequalities, but coated by galvanism, at 212° F.		
	Deflection by direct radiation.			
Smooth	29 00	49 25	37·50	33·00
Scratched longitudinally	40·00	50 25	39·00	34·00
Scratched circularly.......... ...	42·50	50·87	39·50	35·50
Scratched in both directions ...	47 75	51·50	39·50	35·50
	Distance of the plates from the pile in Rhenish inches.			
	3·25	3·25	8·00	9·00

That the differences of the radiation by the galvanic precipitate were not perfectly equal is explicable; for it could not be expected that the copper would be deposited with exact uniformity upon spots of unequal thickness.

The experiments described have thus confirmed the position advanced by Melloni, that the scratching of the surface influences the radiating property of bodies merely so far as it modifies their density and hardness, and that it increases or diminishes this according as it loosens or condenses the parts concerned.

Moreover, the increase which the emission from the metallic surfaces acquires from the copper precipitated by galvanism may also be ascribed to the slight density of this coating in comparison with that of rolled copper. The amount of this is shown by the above table, especially in that example in which the heat emitted by the smooth rolled copper plate produced a deflection of 29°; that coated by galvanism, at the same temperature and the same distance from the thermoscope, a deviation of the needle of 49°·25.

Oxidation of the metal, which as we know also increases the radiation, did not come into play in this phænomenon, because the copper cube was used for experiment immediately after being coated and whilst the deposit had a bright metallic surface.

Although it is shown that the density and hardness exert an influence upon the radiation of heat under the circumstances pointed out, of course it must not be understood that it is caused by them alone. In different bodies, in which various other relations are simultaneously called into action, the property of emitting heat cannot, as Leslie has attempted, be referred to the hardness alone.

2. With reference to the increase of the radiation in proportion to the *thickness* of the bodies laid upon a heated cube, I shall communicate two series of experiments, merely because they were made upon those substances which had yielded a greater heat with an increase in the thickness (see p. 209). They were colourless, transparent varnish, and black, opake diathermanous asphalt-lac. After painting them in layers in various numbers or of unequal thickness upon a Leslie's cube, which during the experiment had been retained at a temperature of 212°, their radiation upon the thermal pile produced the deflections of the multiplier contained in the following table:—

TABLE X.

Leslie's cube at 212° F.	Distance of it from the thermal pile in Rhenish inches.	Coated with varnish.				Coated with black lac.			
		1 layer.	2 layers.	4 layers.	8 layers.	Very thin layer.	Thicker layer.	Still thicker layer.	Thickest layer.
Deflection by direct radiation.	(12)	17·00	20·00	21·75	29·00	19·50	23·25	27·25	29·00
	(8)	24·00	28·00	30·50	39·00	29·00	31·25	37·00	39·00
	(7)	26·50	30·25	33·00	40 50	32·75	37·25	41·00	43·00
	(6)	28·50	33·25	35·75	42·75	40·50	43·25	46·75	48·75

These differences are too considerable to render it necessary to bring forward other examples. This phænomenon has already been correctly explained by Count Rumford, by assuming that the heat radiates from a certain depth beneath the surface; and Melloni, on the same principle, has given a satisfactory account of the whole process.

If this increase in the radiation of bodies be compared with the increase in heating power *under increasing thickness* of the layers in action, which has been pointed out in the previous section (pp. 207 to 209), we find in it *a new cause for the agreement of the radiation and absorption of heat.*

On comparing these functions, however, it must not be over-

looked to what extent this is strictly admissible. It holds good unconditionally in the case of *one* and *the same body*; *i. e.* all agents which increase or diminish its radiation, also increase or diminish its absorption, and *vice versâ*. Thus scratching the surface of a substance increases both its radiating and absorbing power when it exposes softer parts of it, diminishes both when it condenses the same parts, and has no influence upon it when the hardness of the body is left unchanged. As we have seen, also, the radiating and absorbing power is increased to a certain extent by increasing the thickness of bodies, but is diminished by diminishing it.

The comparison of the two phænomena does not however apply generally to *different substances, i. e.* a body which, at a definite heating power, for instance, exhibits a higher radiating power than another, does not therefore possess a better absorptive power; for the proportion of the amounts of heat absorbed by it changes with the nature of the rays of heat which reach it; and also the quantities of heat emitted by them appear under different conditions to alter in a different manner.

Melloni maintains that a substance which at a given temperature (212° F.) emits more heat than another, always in the same proportion absorbs more than it when exposed to the same temperature (212° F.). However, it is a question whether this conclusion is satisfactorily proved by the experiments of this distinguished philosopher, because they merely refer to six bodies, two of which moreover (lamp-black and metal) absorb equally all kinds of calorific rays, and hence cannot enter into consideration in a question of the unequal absorption of heat emitted from different sources.

The observations of Rumford and Leslie on this point are not conclusive, because they do not allow of an accurate comparison of different bodies in regard to the radiation and absorption of heat; nor do those of Ritchie, which appear to prove the absolute similarity of the two phænomena, but in fact, as far as they are published, merely extend to lamp-black and metal; and we have already explained why in this case they cannot lead to a general conclusion.

3. Those experiments which have hitherto been made do not give any explanation of a question the solution of which does not appear to me void of interest; it is this:—

Does the radiating power of one and the same body vary ac-

cording as it is heated to a given degree by rays from different sources of heat?

To decide this point, I coated thin paper, stretched upon a metallic frame, on both sides with lamp-black so thickly that direct transmission of the heat was impossible. When this was exposed immediately before the thermal pile to the rays of an Argand lamp or a metallic cylinder at 212° F., which produced the same direct deflection of the thermoscope, certain deflections of the multiplier were obtained, which were occasioned by the layers of lamp-black becoming heated on one side and their radiation upon the other. The effect of absorption was found in former experiments (pp. 188 and 189) to be the same in both cases; hence if these deflections corresponded with each other, the radiating power must also be the same in both cases. This was really the case. The needle deviated to 9°·87, whether the rays of the Argand lamp or of the heated cylinder acted upon the blackened surface, provided that each of these sources of heat had produced the same direct deflection of 35° in the pile.

The same result was obtained when carmine or black paper was caused to radiate upon the thermoscope; for when the former, with the blackened side next the source of heat, was exposed to the pile, an indication of 10°·5 was obtained in the multiplier, whether the heat was imparted by the flame of the Argand lamp or the dark source of heat; and when these surfaces were replaced by black paper, which was also coated with lamp-black on the side next the heating rays, each time a deviation of 10°, or nearly so, was observed in the needle when the direct deflection amounted to 35°.

It is thus shown that the radiating power of the bodies examined is the same, be the calorific rays by which they are heated of ever so different kinds.

The subjoined table contains the details of the observations (arithmetic means of every two numbers):—

TABLE XI.

Surface inserted, and becoming heated for radiation.	Deflection by direct radiation from the source of heat	Deflection after the insertion, produced by		Deflection by direct radiation from the source of heat.	Deflection after the insertion, produced by	
		The Argand lamp.	The heated cylinder		The Argand lamp	The heated cylinder
Paper covered on each side with lamp-black	35°	9 87	9·87	50°	11 50	14·62
Carmine on the side next the source of heat	35°	10 50	10·50	50°	15·50	15 50
Black paper coated with lamp-black on the side next the source of heat	35°	10·00	9·87	50°	14·12	14·25

To make this experiment in a more direct manner, I heated the radiating bodies also immediately by the different rays.

For this purpose I substituted a plate of charcoal for the paper coated on both sides with lamp-black, and obtained the same result as before. I placed carmine upon some wire-gauze, which had the effect of keeping its separate parts together. When it was exposed in this manner before the thermal pile to the rays of an Argand lamp or the metallic cylinder at 212° F., which produced the same direct deflection, in both cases different indications were given by the instrument.

The question was, whether this difference was only to be ascribed to the unequal absorption of the different rays by the carmine-surface, or whether its radiating power was also concerned in it.

I tried to convince myself of this by the following experiment:—If the carmine-surface next the pile is coated with lamp-black, whilst upon the other side the sources of heat mentioned act upon it, the difference observed in the galvanometer arises solely from the carmine-surface absorbing the heat of the heated metallic cylinder to a greater extent than that of the Argand lamp; for, as we are aware, the radiation from the carbon-surface does not in this case produce any difference. But if the coating of lamp-black be removed, so that the carmine

is turned towards both the thermal pile and the source of heat, two things may happen. Either the above difference in the deflections remains constant in proportion to them: the radiating power, even with the carmine, then has no share in it; or it is altered: when the unequal radiation of the carmine-surface is proved to occur in both cases. If this, *e.g.* is increased, it would be a proof that the carmine-surface radiates comparatively better when heated by the metallic cylinder than when heated by the Argand lamp, for the same reason, perhaps, because it absorbs the former better than the latter.

Experiment has decided in the first case; for the blackened carmine-surface placed next the pile produced a deflection of 9°·5 when exposed to the rays of the Argand lamp, and of 10°·87 when exposed to those of the heated cylinder. The difference thus amounted to 1°·37. The free radiating carmine-surface in the first case caused the same deflection in the needle of 9°·5, in the second of 10°·5. The difference of 1°·0 thus found with the same intensity of the deflections, the first being 1°·37, is not greater than comes within the limits of error of observation.

The same occurred on using black paper. Thus when it was coated with lamp-black on the side next the pile, a deflection of 10°·75 was produced by the rays of the Argand lamp, and of 10°·12 by those of the heated cylinder; when the radiation was free, in the case of the former a deflection of 10°·62, of the latter of 9°·87. In the first experiment the difference was 0°·63, in the second 0°·75. Both may be considered identical; and hence we must conclude that the radiating power of the black paper is independent of the nature of the heat absorbed.

The following table, in addition to the observations detailed, which refer to a direct deflection of 35°, contains others for a greater intensity of the sources of heat; these also yield the same results. (The numbers are each the arithmetic mean of two observations.)

TABLE XII.

Surface of substance inserted, and becoming heated for radiation.	Deflection by the direct radiation of the source of heat.	Deflection after the insertion, produced by		Difference between the deflections	Deflection by the direct radiation of the source of heat	Deflection after the insertion, produced by		Difference between the deflections.
		The Argand lamp.	The heated cylinder			The Argand lamp.	The heated cylinder	
Carmine blackened next the pile	35°	9 50	10·87	1·37	50°	13·75	15 62	1 87
Carmine not blackened	35°	9 50	10 50	1 00	50°	13·62	15·12	1 50
Black paper coated with lamp-black next the pile	35°	10 75	10·12	0·63	50°	15 25	14 00	1 25
Black paper not coated	35°	10·62	9 87	0·75	50°	15 50	14·12	1 38

Thus, under those circumstances in which the same bodies exhibit an unequal absorptive power, their radiating power is one and the same; and those differences which have hitherto been observed when they are not heated to the same extent, are therefore pure functions of the former, and independent of the latter.

In all cases therefore in which the elective absorption of certain substances is to be determined from the amount of heat which they transmit to the thermoscope, it is a matter of indifference whether they are coated with lamp-black (see p. 206), paper (p. 207), or any other substance, for the purpose of increasing their radiation upon the pile. Within the limits of the experiments detailed, there will therefore be no fear of disturbing the above differences by any foreign influence exerted by them.

IV. *Comparison of the Heat radiated from different bodies within a certain range of Temperature.*

All former observations upon radiation have only related to the *quantities of heat* emitted by different substances at certain temperatures. The object of the present investigation is to ascertain—

Whether the heat which radiates from certain bodies, at one and the same temperature or within certain limits of temperature, is of a different kind, according as it is emitted by different bodies, or is excited in them in a different way.

We possess two means of judging of the dissimilarity or similarity of rays of heat, transmission and absorption. Thus we know that different kinds of heat permeate one and the same diathermanous substance differently (p. 191 and 203), or one and the same substance to an unequal extent (p. 206 to 207); whilst the same kind of heat does not admit of our recognising any differences in either the one or the other case.

When there is a choice between the two means, that by transmission deserves unqualified preference, because it is a more delicate test-method than absorption. I at least have always found that the differences yielded by the transmission of different rays of heat through the same diathermanous media are always greater than those found by the absorptive method, and could often very readily perceive small shadows with the transmission when they were imperceptible by the absorptive process.

Hence I have also endeavoured to decide the present question by observing whether the heat emitted in the different cases radiated through the same diathermanous bodies in a different or always in the same proportion.

1. A number of *adiathermanous substances* were first heated to 212° F. by *conduction*, by placing them upon metallic cubes, which were kept at this temperature by boiling water. When it is required to examine the heat radiated from different surfaces as regards its transmission through diathermanous bodies, the same deflection of the thermo-multiplier must first be produced by each of them before the insertion of the diathermanous media about to be used for experiment between the source of heat and the thermal pile. This object was attained by approximating or removing the former to such a position that the requisite deflection of the galvanometer-needle was produced. Experiment led to the following result:—When the heat from the bare metallic surface had passed through a diaphragm, and acted upon the pile so as to deflect the needle of the multiplier to 35°, this was found to recede to 10°·25 when red glass 1·5 millim. in thickness was introduced behind the perforated screen by the side of the thermoscope. The deviation of 10°·25 was produced by the heat which passed through the glass. The same deflection was obtained when, instead of the metallic surface, wood, porcelain, paper, lamp-black, white lead, or any other substance, had radiated upon the instrument.

The same occurred in all other diathermanous substances.

Thus a deflection of 7°·17 was observed each time a plate of calcareous spar 3·7 millim. in thickness was substituted for the red glass, whether the direct radiation which deflected the needle to 35° was produced by heated metal, wood, porcelain, paper, or any other substance.

The appended table (which contains the arithmetic means in every case of three observations) shows how great the agreement was when the heat emitted by eleven adiathermanous substances was examined by means of red and blue glass, rock salt, calcareous spar and gypsum:—

TABLE XIII.

Thickness in millimetres.	Substances inserted.	Deflection produced by direct radiation.	Deflection produced by the insertion of the substance radiating at 212° F.					
			Metal.	Wood	Porcelain.	Leather.	Cloth.	Pasteboard.
1·5	Red glass	35°	10 25	10·17	10·17	10 17	10·17	10·25
1·4	Blue glass	. ...	9 17	9·08	9 25	9·17	9·17	9 25
1 4	Alum.	......	3 92	4·00	3·83	3·83	3 92	3·92
4·4	Rock salt	......	20·58	20 66	20·66	20·58	20·66	20 75
3·7	Calcareous spar...	......	7 17	7·17	7·08	7·17	7·25	7·25
1·4	Sulphate of lime..	35°	8 80	8·66	8·75	8 66	8 80	8·75

Thickness in millimetres.	Substances inserted.	Deflection produced by direct radiation	Deflection produced by the insertion of the substance radiating at 212° F.				
			Black paper.	Carmine (thick layer).	Lampblack.	White lead.	Red Venetian lac.
1 5	Red glass	35°	10 25	10·08	10 08	10·17	10 25
1 4	Blue glass ...	. . .	9 17	9 17	9·17	9·17	9·17
1 4	Alum	......	4 00	3 92	3 83	3 92	3 75
4·4	Rock salt	... ..	20·66	20 58	20 50	20 66	20 58
3·7	Calcareous spar...	....	7·25	7 17	7 33	7 17	7 17
1·4	Sulphate of lime..	35°	8 75	8·75	8·66	8 75	8·75

Neither, as will be evident from the following table, could any difference be perceived in the transmission when the surface of the radiating bodies was modified by being scratched, although this had the most decided influence upon the quantity of heat emitted:—

Table XIV.

Thickness in millimetres.	Substances inserted.	Deflection produced by the following bodies radiating at 212° F., after the insertion					
		Plates of tin.				Plates of copper.	
		Bright.	Scratched longitudinally.	Scratched in both directions.	"Clouded" by scratching.	Smooth.	Engraved in both directions.
		Deflection by direct radiation 35°.					
1·5	Red glass	10·00	10·25	10·00	10·25	10·08	10·17
1·4	Blue glass	9·25	9·25	9·25	9·25	9·17	9·17
1·4	Alum	3·75	4·00	3·92	3·75	3·92	3·83
4·4	Rock salt	20·50	20·50	20·75	20·75	20·66	20·58
3·7	Calcareous spar	7·25	7·00	7·00	7·25	7·33	7·17
1·4	Sulphate of lime	8 75	8·75	8·50	8·75	8·70	8·58

Thickness in millimetres.	Substances inserted.	Deflection produced by the following bodies radiating at 212° F., after the insertion					
		Plates of lead.		Discs of wood.			
		Smooth.	Scratched in both directions.	Smooth.	Scratched.	Rough.	More rough.
		Deflection by direct radiation 35°.					
1·5	Red glass	10·25	10·17	10·25	10·25	10·25	10·50
1·4	Blue glass	9·17	9·17	9·00	9·00	9·00	9·25
1·4	Alum	3·83	3·92	3·75	3·50	3·50	3·75
4·4	Rock salt	20·66	20·75	20·50	20·75	20·75	20·50
3·7	Calcareous spar	7·17	7·17	7·25	7·25	7·00	7·25
1·4	Sulphate of lime	8·70	8·80	8·75	8·50	8·50	8·75

If we connect with this the result obtained above (p. 196 to 202), according to which the proportion of the heat permeating diathermanous media is constant whatever the temperature of the radiating body may be (between 88° and 234° F.), it is evident *that the heat which, within this range of temperature, is emitted by the most different adiathermanous substances, the structure of the surface of which is not uniform, when heated by conduction permeates in the same manner the diathermanous substances used to test them.*

2. The next question was, how the heat radiated by the bodies would react as regards its transmission through diathermanous media, when heated, not by conduction, as in the first series of observations, but by the *radiation of heat from different sources.*

To ascertain this, I first exposed the substances used to radiate

the heat, to the rays of an Argand lamp (which, as before, was used without the chimney). The size of the screen which they formed was sufficient to protect the direct rays of the flame from the thermal pile, so that the latter was reached by those only which the heated bodies themselves emitted to it through the diaphragm. By withdrawing the latter or the lamp, it was easy to produce the constant direct deflection of 35° in the multiplier, which remained constant as soon as the screens had acquired a maximum temperature corresponding to the conditions, the proper temperature. Of course this point must be waited for, before the diathermanous substances are inserted on the side of the diaphragm next the pile.

The radiating adiathermanous bodies, all of which were in discs 11 centim. in diameter, were in general the same as in the first experiments—metal, wood, porcelain, leather, cloth, &c. The black paper, as also that covered by carmine, were coated on one side with lamp-black. A third piece was blackened on both sides. A net-work of metal was coated with white lead and red Venetian lac. The transmission of the heat emitted by them gave the same result as before. On this occasion a recession of the needle from 35° to 10°–10°·33 constantly occurred when the red glass, and from 35° to 7°·08–7°·17 when the plate of calcareous spar was inserted between the heated screen and the thermoscope, of whatever the former was composed.

The following table shows how perfectly the observations in this series of experiments agree with those of the first (p. 223):—

TABLE XV.

Thickness in millimetres.	Substances inserted.	Deflection by direct radiation.	Deflection produced by the following bodies heated by the Argand lamp to produce the radiation					
			Metal.	Wood	Porcelain	Leather	Cloth	Pasteboard
1·5	Red glass	35°	10 17	10·17	10·17	10 25	10 33	10 08
1·4	Blue glass........	.	9 08	9·17	9 25	9·08	9 08	9·17
1·4	Alum..	......	3·92	3·70	4·00	3 92	4 00	3·92
4·4	Rock salt	.. ...	20 66	20 75	20 66	20 58	20 66	20 66
3 7	Calcareous spar	......	7·17	7·17	7·17	7·17	7·17	7·17
1·4	Sulphate of lime ...	35°	8 75	8 75	8 66	8 75	8·83	8 75

TABLE XV. (*continued*).

Thickness in millimetres.	Substances inserted.	Deflection by direct radiation.	Deflection produced by the following bodies heated by the Argand lamp to produce the radiation.					
			Lamp-black on the side next the source of heat, black paper on that next the pile, or *vice versâ*.	Lamp-black on the side next the source of heat, carmine on that next the pile, or *vice versâ*.	Lamp-black on both sides.	White lead.	Red Venetian lac.	Deflection produced after the insertion by the Argand lamp.
1·5	Red glass	35°	10·25	10·25	10·25	10·00	10·08	22·00
1·4	Blue glass	......	9·25	9·25	9 25	9·17	9·08	17·50
1·4	Alum	......	3·92	3·92	3·92	3·92	4·00	8·50
4·4	Rock-salt	......	20·58	20·58	20·75	20·58	20·58	28·00
3·7	Calcareous spar	......	7·08	7·17	7·17	7·08	7·08	20·00
1·4	Sulphate of lime	35°	8·83	8·66	8·75	8·75	8·66	15·00

These numbers likewise remain unaltered when the adiathermanous bodies above mentioned are heated by the *metallic cylinder at* 212° instead of the Argand lamp, although according to the former experiments (pp. 204, 205, 206) the rays from these two sources of heat are essentially different from one another.

The subjoined table (the numbers in which, as in the former, are each the arithmetic means of two observations) will prove this beyond a doubt:—

TABLE XVI.

Thickness in millimetres.	Substances inserted.	Deflection by direct radiation.	Deflection produced after the insertion by the following bodies heated for radiation by the hot metallic cylinder.					
			Metal.	Wood.	Porcelain.	Leather.	Cloth.	Pasteboard.
1·5	Red glass	35°	10·25	10·00	10·25	10·00	10·17	10·17
1·4	Blue glass............	......	9·17	9·25	9·17	9·17	9·08	9·17
1·4	Alum..................	......	4·00	3·92	4·00	3·92	4·00	3·83
4·4	Rock salt	......	20·66	20·66	20·66	20·66	20·66	20·75
3·7	Calcareous spar......	......	7·17	7·08	7·17	7·25	7·25	7·17
1·4	Sulphate of lime ...	35°	8·66	8·83	8·83	8·75	8·66	8·66

Thickness in millimetres.	Substances inserted.	Deflection by direct radiation.	Deflection produced after the insertion by the following bodies heated for radiation by the hot metallic cylinder.					
			Lamp-black on the side next the source of heat, black paper on that next the pile, or *vice versâ*.	Lamp-black on the side next the source of heat, carmine on that next the pile, or *vice versâ*.	Lamp-black on both sides.	White lead.	Red Venetian lac.	Deflection after insertion produced by the rays of the metallic cylinder.
1·5	Red glass	35°	10·25	10 17	10·25	10·08	10·08	10·16
1·4	Blue glass	......	9·17	9·25	9·25	9·17	9·08	9·18
1·4	Alum	......	3·92	4 00	4 00	4·00	3·83	3·95
4·4	Rock salt	......	20·66	20 58	20·75	20·58	20·66	20·66
3·7	Calcareous spar	......	7·08	7·25	7·25	7·25	7·33	7·20
1·4	Sulphate of lime	35°	8 83	8·66	8·75	8·75	8·83	8·75

In this case, as before (p. 196 to 202 and 224), it was a matter of indifference whether the radiating substances became heated in a higher or less degree; for the portion of heat which passed through the diathermanous substances remained the same whether the direct deflection of 35° was produced by placing the disc to be heated nearer the source of heat and further from the thermoscope, or nearer the pile and at a greater distance from the source of heat.

It is thus evident from these observations, *that the heat radiated by different bodies always passes through the diathermanous media used in their investigation in the same proportion, be the rays of heat, by the absorption of which they are heated, ever so different.*

When in these researches the object was to investigate the transmission of the heat emitted from certain bodies by the substances used for examining them, diathermanous bodies of course could not be used as the heating agents in the method described; for when they were placed, as in the previous arrangement of the apparatus, between the original sources of heat, *e. g. the Argand lamp* and the thermoscope, not only the heat from the flame radiated by them, but also that passing through them reached the pile.

Experiments of this kind could not therefore aid in deciding the question especially under consideration; but they were adapted to test the accuracy of the method itself, used for ascertaining the identity or dissimilarity of certain rays of heat.

Thus in the manner pointed out, a number of rays were obtained which differed in their capability of passing through diathermanous bodies*, and the proportion of which to each other was dependent upon the nature of the body in which they appeared. Such a mixture of different kinds of heat, according as it belonged to the one or the other substance, must therefore permeate the media under investigation to an unequal extent.

The next question was, whether these differences in the transmission, which were pre-supposed by theory to exist, would also be perceptible in red and blue glass, alum, rock salt, calcareous spar and sulphate of lime, in cases in which they were only admitted to be small.

The experiment simply consisted in placing a slightly diathermanous body, *e. g.* a plate of ivory 1·7 millim. in thickness, on *that* side of a perforated screen next which the Argand lamp

* Compare in Table XV., p. 225 and 226, the deflections for the heated bodies after the insertion with those for the Argand lamp.

was placed, and approximating the latter to it until the deflection of 35° was produced by the combined action of the rays emitted from and passing through the ivory plate. As soon as the needle had rested at this point, the diathermanous substance to be tested, *e.g.* the red glass, was introduced on the opposite side of the diaphragm before the thermal pile. We were already aware that the rays from a body below 212° F., which had directly deflected the needle to 35°, after their transmission through red glass produced a deflection of 10°–10°·25, and that of the Argand lamp, with the same direct action, after their transmission through the glass, a deflection of 21°·75 (see Table XVII. and p. 229). Thus when a portion of these rays joined the former so as conjointly to produce a direct deflection of 35°, if the method was sufficiently delicate, the instrument under these circumstances should indicate a different deviation from 10°·25 when the red glass was inserted in the same spot. This was really the case. A deflection of 13°·62 was produced.

The same result was obtained in every other instance. Thus it amounted to 16°·75 when ivory was replaced by black opake lac, and even 11°·62 when a metallic plate perforated with two needle-holes was used instead of this.

The less the first diathermanous screen interrupted the heat which the second transmitted, the higher the deflection ought to be after the insertion. Thus with the same red glass the needle only receded to 27°·5 when the first screen consisted of colourless glass 1·9 millim. in thickness.

The following table represents the great differences which also occurred in the case of the other diathermanous media. (It contains the arithmetic means of every two observations.)

TABLE XVII.

Thickness in millimetres.	Substances inserted.	Deflection produced by direct radiation.	Deflection after the insertion, produced by: The rays of heat of an adiathermanous substance between 88° and 212° F.	The rays of the Argand lamp.	Deflection after the insertion when the first screen is formed by the following substances:— Metal pierced by two needle holes.	Silk cloth.	Ivory 1·7 millim. in thickness.
1·5	Red glass	35°	10·12	21·75	11·62	19·00	13·62
1·4	Blue glass	......	9·22	18·60	10·47	15·10	12·35
1·4	Alum	......	3·92	8·67	5·55	7·55	7·92
4·4	Rock salt	......	20·62	29·50	21·37	26·25	22·25
3·7	Calcareous spar	......	7·22	20·10	17·97	16·60	11·60
1·4	Sulphate of lime	35°	8·75	15·75	9·37	13·37	11·88

TABLE XVII. (*continued*).

Thickness in millimetres	Substances inserted	Deflection produced by direct radiation.	Deflection after the insertion when the first screen is formed by the following substances —				
			Letter paper 0·05 millim in thickness.	Paper coated with carmine 0·15 millim in thickness.	Black opake lac 0·5 millim. in thickness	Black opake glass 2·0 millim. in thickness.	Colourless glass 1·9 millim in thickness.
1·5	Red glass	35°	18·50	17·25	16·75	19·00	27·50
1·4	Blue glass ...	..	15·60	13·85	13·97	16·85	21·60
1·4	Alum	......	8·30	7·55	3·92	4·42	11·80
4·4	Rock salt .. .	.. ..	27·25	21·50	21·00	26·75	31·25
3·7	Calcareous spar	....	15·22	15·35	13·85	13·10	26·85
1·4	Sulphate of lime	35°	14·00	14·00	12·75	10·62	21·37

Hence no doubt remains in my mind that differences would also have been evident in the previous experiments had any existed.

If, instead of the Argand lamp, *a metallic cylinder below* 234° F. be used, all these differences should again disappear; for the rays from the substances placed before it, as also the heat emitted by the cylinder itself, pass through the diathermanous bodies in the same manner (see Table XV.); and it must therefore be a matter of indifference as regards the radiation, in what proportion they are mixed together, according as they issue at one or the other plate.

The following numbers (the arithmetic means of two observations) confirm this:—

TABLE XVIII.

Thickness in millimetres.	Substances inserted.	Deflection produced by direct radiation.	Deflection after the insertion, produced by		Deflection after the insertion when the first screen is formed by the following substances —		
			The rays of heat of an adiathermanous substance between 88° and 212° F	The rays of the Argand lamp.	Metal pierced by two needle holes.	Silk cloth.	Ivory 1·7 millim. in thickness.
1·5	Red glass	35°	10·22	10·10	9·97	10·22	9·97
1·4	Blue glass	... ..	9·20	9·20	9·32	9·32	9·20
1·4	Alum	......	3·92	4·05	3·92	3·92	3·80
4·4	Rock salt	......	20·65	20·65	20·77	20·77	20·52
3·7	Calcareous spar	.... .	7·00	7·00	7·12	7·25	7·00
1·4	Sulphate of lime	35°	8·70	8·82	8·82	8·82	8·95

TABLE XVIII. (*continued*).

Thickness in millimetres.	Substances inserted.	Deflection produced by direct radiation.	Deflection after the insertion when the first screen is formed by the following substances:— Letter paper 0·05 millim. in thickness.	Paper coated with carmine 0·15 millim. in thickness.	Black opake lac 0·5 millim. in thickness.	Black opake glass 2·0 millim. in thickness.	Colourless glass 1·9 millim. in thickness.
1·5	Red glass	35°	10·10	10·10	9·85	10·10	10·10
1·4	Blue glass	......	9·20	9·07	9·20	9·20	9·20
1·4	Alum	......	4·05	3·92	3·80	3·92	4·05
4·4	Rock salt	......	20·77	20·65	20·77	20·65	20·52
3·7	Calcareous spar	......	7·00	7·00	7·12	7·00	7·12
1·4	Sulphate of lime	35°	8·82	8·82	8·70	8·70	8·82

A constant deviation of the needle of from 9°·85 to 10°·22 is now obtained on inserting the red glass, and of from 7° to 7°·25 on introducing the calcareous spar, be the screen of whatever substance it may, provided that the direct radiation of the heat emitted by it and that permeating it has produced a deflection of 35°. The same applies to blue glass, alum, rock salt and gypsum. All these different values, however, as far as they are related, merely vary within the errors of observation.

The same result is obtained on heating the above *diathermanous bodies* by *conduction* to a temperature below or at 212°, by placing them upon a metallic cube heated to this temperature by boiling water.

The numbers found accurately agree in this, as in the previous case, with those which were formerly obtained for diathermanous substances (compare Tables XIII., XIV., XV. and XVI.). To illustrate the comparison, we have annexed one of them in the following table (it contains the arithmetic means of every three experiments):—

TABLE XIX.

Thickness in millimetres.	Substances inserted.	Deflection by direct radiation.	Deflection produced after the insertion by the following bodies radiating at 212° F. An adiathermanous subst.	Silk cloth.	Ivory 1·7 millim. thick.	Letter paper 0·05 millim.	Carmine, thin layer.	Black opake glass 2·0 millim.
1·5	Red glass	35°	10·08	10·17	10·17	10·08	10·08	10·25
1·4	Blue glass	......	9·17	9·25	9·17	9·25	9·17	9·25
1·4	Alum	......	3·83	3·92	3·83	4·00	3·92	3·83
4·4	Rock salt	......	20·50	20·58	20·66	20·66	20·58	20·66
3·7	Calcareous spar	......	7·33	7·25	7·25	7·25	7·17	7·08
1·4	Sulphate of lime	35°	8·70	8·75	8·80	8·75	8·75	8·75

TABLE XIX. (*continued*).

Thickness in millimetres.	Substances inserted.	Deflection by direct radiation.	Deflection produced after the insertion by the following bodies radiating at 212° F.					
			Black lac.		Colourless glass			
			Thin layer	Thick layer.	1·1 millim thick	1·6 millim thick.	2·2 millim. thick.	3·0 millim thick
1·5	Red glass	35°	1·05	10·25	10·08	10·17	10·17	10·17
1·4	Blue glass ...	.... ..	9·08	9·17	9·25	9·17	9·17	9·17
1·4	Alum	.. ...	3·92	3·92	4·00	4·00	4·08	3·92
4·4	Rock salt . .	.... ..	20·50	20·66	20·58	20·50	20·75	20·66
3·7	Calcareous spar	. ..	7·17	7·17	7·08	7·00	7·08	7·17
1·4	Sulphate of lime	35°	8·75	8·80	8·80	8·75	8·75	8·70

Thus the heat emitted by adiathermanous and diathermanous bodies, within the limits of these experiments, passes through media used for testing them in exactly the same manner. Hence (as is also evident from the values given) *it is a matter of indifference whether the radiating substances* (*e. g.* black lac or white glass) *are of a greater or less thickness.*

I have yet to detail my reasons for considering as *diathermanous* the bodies last examined, in which, when acted upon by the Argand lamp, I found differences in the transmission through red and blue glass, alum, rock salt, calcareous spar and gypsum (see Table XVII.).

No one will hesitate to admit that heat directly penetrates colourless glass and the pores of silk-cloth; but it might be a matter of doubt whether a transmission in its true sense, although only diffuse, could be admitted to occur in perfectly opake glass and lac, or in a layer of carmine, paper and ivory. That this is really the case, the following investigation will prove.

If any body, except soot and metal (see p. 188, 189 and 206), be placed successively before the blackened thermoscope to the rays of different sources of heat, *e.g.* an Argand lamp and a metallic cylinder heated to 212°, which directly exert an equal action upon the instrument, it indicates different degrees. This difference in the indications may either arise from the substance inserted being adiathermanous, and becoming unequally heated when under the influence of different sources of heat, or that it is diathermanous and allows of the passage of various kinds of rays to an unequal extent; or, lastly, that the effects observed upon the thermoscope are partly produced by the screens becoming heated, partly by the rays which pass through them.

In the present case it all depends upon our satisfactorily convincing ourselves of the share taken by the latter. For this purpose the substances introduced were blackened on the side next the thermoscope, and thus the transmission, if any occurred, was prevented. The differences which occurred on inserting the body under examination, could now only be ascribed to its becoming unequally heated. If it were *adiathermanous*, these differences, even when the coating of lamp-black is removed, remain the same, as we have previously (p. 220, 221) seen in a thick layer of carmine and with black paper. If however it is *diathermanous*, then by the access of the transmitted heat various kinds of changes occur, which are best illustrated by the experiments themselves.

When the needle of the galvanometer was deflected to 40° by the direct action of the source of heat, on inserting the *black glass* coated with lamp-black next the thermal pile, in consequence of its becoming heated, it stood at 12° when the Argand lamp, and at 11° when the dark cylinder radiated upon it; whilst in the first case it deviated to 16°·25, when the coating of the lamp-black was removed, the deflection in the second case remaining the same. Thus the difference in the indications with the different sources of heat, amounted in blackened glass to 1°, in that uncoated to 5°·25. This is an infallible proof, that on removing the layer of lamp-black transmission occurs, which acts in the same manner as absorption with regard to difference of the deflections.

On introducing the *black lac*, the different sources of heat produced an equal action upon the instrument as long as it was blackened upon the side next the latter, but a different one when the coating was removed. This difference however can only be attributed to the transmission of the heat which is now permitted.

When the thin *layer of carmine* coated with lamp-black was inserted, the needle receded from 40° to 17°·37 when the former was exposed to the rays of the lamp, and to 21°·25 when exposed to those of the metallic cylinder. On removing the coating of lamp-black, it remained the first time at 19°·63, and the second at 19°·87. The difference previously observed, arising from the unequal heating, now disappears by a compensation of transmission and absorption; this being less in the case of the heat of the flame than in that of the heated cylinder, the former

less for the rays of the cylinder than for those of the Argand lamp.

With *letter paper* the difference in the indications produced by the removal of the lamp-black becomes reversed. Thus, whilst by heating the paper coated next the pile with lamp-black with the rays of the flame, a deflection of 18°·37 was found, and with those of the dark cylinder 21°·13; in the uncoated one, in the first case it amounted to 22°·25, in the last to 20°·5. On this occasion transmission also occurs, which acts in opposition to the heating, and preponderates to such a degree, that it not only overcomes the influence of the unequal absorption, but even produces a change of the difference to the other side.

This also occurred with *ivory*, as may be seen from the following table, which contains the details of the observations (arithmetic means of every two):—

TABLE XX.

Source of heat	Deflection by the direct radiation of the source of heat.	Deflection after the insertion of					
		Black glass 2 0 millim. thick.		Black lac 0·5 millim thick.		Paper coated with carmine 0 15 millim. thick	
		Blackened	Not blackened	Blackened.	Not blackened	Blackened	Not blackened.
Argand lamp..	35°	10·62	14·75	12·75	15·75	14·75	16 75
Metallic cylinder at 212° F.	....	8·75	9·50	12 63	13·25	17 75	16·87
Argand lamp..	35°	12 00	16·25	14 37	18·37	17 37	19·63
Metallic cylinder at 212° F	......	11·00	11 00	14 37	14 63	21 25	19 87

Source of heat.	Deflection by the direct radiation of the source of heat.	Deflection after the insertion of			
		Letter paper 0·05 millim. thick.		Ivory 1·7 millim thick.	
		Blackened.	Not blackened.	Blackened.	Not blackened
Argand lamp..	35°	15·63	19·25	8 75	10 37
Metallic cylinder at 212° F.	......	17 63	18 25	8·87	9·25
Argand lamp..	35°	18·37	22·25	10·00	11 87
Metallic cylinder at 212° F.	......	21 13	20·50	10·87	10·50

These changes of the difference of the thermoscopic indications on removing the coating of lamp-black would not occur in adiathermanous bodies.

Hence it is proved that black glass, black asphalt-lac, a thin

layer of carmine, post paper and ivory were in fact *diathermanous* under the conditions pointed out.

Thus the great differences which occurred on transmission through red and blue glass, alum, rock salt, calcareous spar and sulphate of lime (see Table XVII.), when these substances were exposed to the rays of the Argand lamp, arose merely from the heat *being transmitted by* them, and not from the circumstance that that *emitted* by them would be transmitted by the above media in a different proportion.

3. From the following numbers it is evident that also *the heat evolved by the vital process, e. g.* that radiated from the hand, is transmitted by diathermanous media in the same manner as those previously examined*:—

TABLE XXI.

Sources of heat.	Deflection by direct radiation of the sources of heat.	Deflection after the insertion of					
		Red glass 1·5 millim. thick.	Blue glass 1·4 millim.	Alum 1·4 millim.	Rock salt 4·4 millim.	Calcareous spar 3·7 millim.	Sulphate of lime 1·4 millim.
An adiathermanous body, between 88° and 212° F. (See Table XVII.)	35°	10·12	9·22	3·92	20·62	7·22	8·75
The hand, between 84° and 96° F.	35°	10·17	9·17	3·92	20·66	7·25	8·66

If we connect with this the fact, that the radiant heat of different bodies (with the same intensity) also heats one and the same substance to the same extent as it passes through one and the same diathermanous one in the same way, the final result of these observations is, *That the heat emitted by the most different solid bodies of unequal thickness and dissimilar structure of their surface, which have as yet been examined, as far as our present means allow, has been proved to be of the same kind, in whatever way, within the limits of these experiments* (i. e. *between* 88° *and* 234° F.), *it may be excited in them.*

Taking into consideration this fact, according to which a body always emits *heat of the same kind,* be the rays which heat it ever so different (see especially p. 225 to 227), the remark made in the first section appears explicable, that its *radiating property,* which is the cause of the *quantity* of this heat, is *one and the same* under these altered circumstances (p. 217 to 221).

* This also disproves the opinion of Forbes, that the heat emitted by boiling water and the hand must be considered as different.

This result is of some interest as regards the *determination of the specific heat* of bodies. Thus, if ice in the calorimeter absorbs the heat radiated by different substances, even within the limits of temperature stated, unequally, *i. e.* a greater or less portion of a constant amount of heat, according as it emanates from one substance or the other, the quantity of ice melted would not constitute a pure quantity for the amount of heat radiated by different substances, upon the calculation of which the whole determination rests.

The results communicated moreover lead to *a new method of ascertaining whether any substance transmits rays of heat or not.*

In the first investigations on this point the diathermancy of certain substances was considered to occur when, on inserting them before a source of heat, effects upon the thermoscope were obtained which could not arise from the introduced media, either because these had been made imperceptible to the instrument, or because these effects were diminished by the means which increased the absorption (see p. 191 and pp. 203, 204).

This method however presupposes a certain intensity of the transmission, and would not have been applicable, *e. g.* in cases, as those previously considered (p. 228 to 234), in which small quantities only of radiant heat were concerned. For these I therefore made use of the method already described (p. 231 to 234), *i. e.* I examined first the effects on the inserted substances of absorption alone with different sources of heat, by impeding the direct transmission of the heat by a coat of lamp-black, then produced the transmission, and then concluded from the effects observed whether in fact it arose from absorption or not.

The new method, which yields nothing to the former in delicacy, has the advantage of not requiring the substance to be coated with lamp-black. It will be most easily illustrated by an example.

Suppose it was required to be ascertained if ivory is diathermanous or not.

To decide this, a plate of any substance known to be adiathermanous, as wood, pasteboard or charcoal, is heated by the rays of an Argand lamp in such a manner that the radiation upon the pile through a diaphragm produces a certain deflection, *e. g.* of 35°, in the multiplier. A diathermanous substance is then inserted before the thermoscope on this side of the perforated screen. The needle then recedes, *e. g.* with red glass 1·5 millim. in thickness to 10°·25. The ivory plate is subjected to

the same proceeding as the adiathermanous surface. It is placed, as regards the Argand lamp and the thermal pile, in such a manner, that, as before, a deflection of the needle to 35° is produced, which in this case may possibly arise from heat of the flame which passes through it, as well as from the ivory plate becoming heated. The question then is, if the rays of heat, which under such circumstances have deflected the needle to 35°, will permeate the diathermanous substance in the same proportion as that previously emitted by the adiathermanous surface, *i. e.* whether now also, *e. g.* on inserting the red glass, a recession of the needle to 10°·25 is obtained. If this is the case, and occurs in the same way in all other diathermanous media used for testing, *e. g.* also with blue glass, alum, rock salt, calcareous spar and gypsum, the ivory plate is adiathermanous; for then only, as we know (p. 225 and 226), do we find no difference in this respect. But if, the second time the rays of heat do not pass in the same manner as before through the diathermanous bodies, we obtain in the case of a single one only a different deflection after the insertion as before, it is a proof of the diathermancy of the plate of ivory. Experiment gave 13°·62 on inserting the red glass, and similar differences with other diathermanous substances (Table XVII.). Ivory is thus diathermanous. The criterion for deciding the question is therefore briefly this:—

"If the heat which impinges upon the plate under examination, when exposed to an Argand lamp, cannot by transmission be distinguished from the heat of any other known adiathermanous body, the plate itself is adiathermanous. If differences do occur, it is diathermanous."

This position could not be instituted until it was known that the peculiar heat of different substances did not produce the same differences.

The temperature of the bodies compared of course should not be allowed to exceed 234° F.

That it is not a matter of indifference whether in this investigation an Argand lamp or any other source of heat is used, may be seen from the experiments detailed in Table XVIII., in which adiathermanous and diathermanous bodies could not be distinguished from one another, on exchanging the Argand lamp for a metallic cylinder at 212° F., the general method of proceeding being the same.

"Or, the substance might also first be exposed to an Argand lamp, and then to a heat of 212° F., and it be ascertained whether rays of heat in both cases escaping to it permeate the diatherma-nous media in the same or a different way. In the first case it would be adiathermanous (compare Tables XV. and XVI. together), in the second diathermanous (compare together Tables XVII. and XVIII.)."

Thus we have in reality obtained *a new and certain means of deciding the question of the diathermancy of a substance,* and are in a condition for greatly increasing its delicacy, since there is no further need of protecting the thermoscope from the heat of the substance under investigation; and the source of heat may thus be allowed to act upon it to as great an extent as we please.

Article VI.

*Memoir on Double Refraction.** By M. A. Fresnel.†

[From the *Mémoires de l'Académie Royale des Sciences de l'Institut de France*, tom. vii. 1827.]

Introduction.

Huygens, guided by an hypothesis founded on the theory of waves, was the first to recognise the true laws of double refraction in uniaxal crystals. This discovery was perhaps more difficult to make than any of Newton's on the subject of light; and what seems to prove this is, that here Newton, after fruitless attempts to discover the truth, fell into error. When we consider how greatly his curiosity must have been excited by the phenomenon of double refraction, we cannot suppose that he gave less attention to it than to other optical phenomena; and one is necessarily surprised at seeing him substitute a false rule for the construction of Huygens, as accurate as it was elegant; a construction with which he was no doubt acquainted, because he quotes his treatise on Light. But what appears still more inconceivable is, that the accuracy of Huygens's law was unacknowledged for more than a hundred years, although it was supported by the experimental verifications of this great man, as remarkable perhaps for his good faith and modesty, as for his rare sagacity. If we ventured to offer an explanation of this singular trait in the history of science, we should say that the considerations drawn from the theory of waves which had guided Huygens, led probably the partisans of the emission system to suppose that he could never have arrived at the truth by a false

* The Editor is indebted to Alfred W. Hobson, B.A., St John's College, Cambridge, for the translation of this memoir.

† The three memoirs, the substance of which is comprised in the present one, were successively presented to the Institute on the 26th Nov. 1821, the 22nd Jan. 1822, and 22nd April of the same year. In uniting them, the arrangement of the matter has been changed and considerable suppressions made; but nothing essential has been added to the new facts and theoretical views which they contained. To the latter only have been given some development necessary for their comprehension; and it has been deemed useful to insert in this memoir a complete demonstration of the transversal direction of the luminous vibrations, because on this point depends the theory of polarization and of double refraction. This demonstration has already been published in the *Bulletin de la Société Philomatique* for October 1821.

hypothesis, and prevented them from reading his treatise on Light with the attention which it deserved.

Amongst modern philosophers Mr. Young is the first who suspected the law of Huygens to be correct; it was by his advice that Dr. Wollaston verified it by numerous and precise experiments. Scarcely was the result of these experiments known in France, when Malus occupied himself with the same researches, and found, as Dr. Wollaston had done, the law of Huygens in perfect numerical accordance with all the measures given by observation. M. de Laplace, considering double refraction in the emission point of view, made a skilful application of the principle of least action to the calculation of the extraordinary refraction. He found that the motion of the luminous molecules undergoing this refraction might be explained by supposing them to be repelled by a force perpendicular to the axis of the crystal, and proportionate to the square of the sine of the angle which the extraordinary ray makes with this axis; whence it follows that the difference between the squares of the velocities of the ordinary and extraordinary rays is proportional to the square of the same sine.

This result is only the translation of Huygens's law into the language of the emission system. The calculations of M. Laplace have not thrown any light on the theoretical question, for they do not show why the repulsive force emanating from the axis should vary as the square of the sine of the inclination of the extraordinary ray to this axis; and it is extremely difficult to justify this hypothesis by mechanical considerations.

In fact, the same polarized ray undergoes the ordinary or extraordinary refraction in a rhomboid of calcareous spar according as its plane of polarization is parallel or perpendicular to the principal section of the crystal; it must be then the lateral fronts of the beam, or the parallel faces of the luminous molecules composing it, which alone determine, by the difference of their properties or physical relations, the nature of the refraction; two of these fronts must be subject to the repulsive influence of the axis and the two others insensible to it. We must suppose also the same absence of action on the anterior and posterior faces of the luminous molecules, since on simply turning the ray round itself, and without changing the direction of these latter faces, we withdraw it from the repulsive power of the axis. But the lateral faces of the luminous molecules are not less exposed to the repulsive force emanating from the axis and acting per-

pendicularly to its direction, when the ray is parallel to the axis, than when it is perpendicular to it, and one does not see why this action should be nothing in the first case, whilst it attains its *maximum* in the second

If, leaving aside all inquiry into the mechanical cause of this singular law, it be considered as a necessary consequence of facts in the emission system, we are then embarrassed by other difficulties According to this system, a beam of ordinary light is composed of molecules whose planes of polarization are turned in all azimuths experiment, moreover, shows that the direction of the plane of polarization of an incident ray does not change abruptly at the moment when it penetrates into the crystal, but gradually and after having traversed a sensible thickness, much greater in general than that to which must be limited the sphere of activity of the ordinary and extraordinary refraction, or the limits of the curved portion of the trajectory This being established, in a beam of ordinary light, there can only be a very small portion of rays having their planes of polarization exactly parallel or perpendicular to the principal section those of nearly the whole of the luminous molecules will be found distributed through all the intermediate azimuths Now, if the repulsive influence of the axis is nothing on a ray polarized parallel to the principal section, and if it makes itself felt with its full energy when the ray is polarized in a perpendicular direction, this repulsive force must vary gradually for the intermediate directions, from the first, where it is nothing, up to the last, where it attains its maximum Thus, since the molecules which compose the direct light are polarized in an infinite number of different azimuths, they would be found subject to repulsive forces of different intensity, therefore their trajectories on entering the crystal ought to undergo different inflexions In order for them not to be sensibly affected by the differences of intensity which the diversity of the planes of polarization of the incident rays must cause in the repulsive energy of the axis, it would be necessary that this action, as well as the refracting power of the medium, should be sensible at much greater depths than that to which the luminous molecules preserve nearly the same plane of polarization Now, it is exactly the contrary which is most probable, for the thickness of crystal necessary to change the plane of polarization is too sensible, especially in certain cases, to allow of our admitting that the curved portion of the trajectory of the luminous molecule extends

so far this curve and the definitive direction of the refracted ray, must therefore vary according to the azimuth of the plane of polarization of the incident ray. Thus on following this hypothesis into its consequences, it would be found that the light instead of dividing itself simply into two rays, ought to separate itself into a multitude of rays, distributed according to all the inclinations comprised between the extreme directions of the ordinary and extraordinary beam.

The theory here combated, and against which many other objections might be brought, has not led to a single discovery. The skilful calculations of M. de Laplace, however remarkable for an elegant application of mechanical principles, have taught nothing new on the laws of double refraction.

Now, we do not think that the assistance to be derived from a good theory is to be confined to the calculation of the forces when the laws of the phænomena are known: it would contribute too little to the progress of science. There are certain laws so complicated or so singular, that observation alone, aided by analogy, could never lead to their discovery. To divine these enigmas we must be guided by theoretical ideas founded on a *true* hypothesis. The theory of luminous vibrations presents this character and these precious advantages: for to it we owe the discovery of optical laws the most complicated and most difficult to divine; whilst all the other discoveries, numerous and important no doubt, which have been made in this science by experimenters adopting the emission system, are much rather the fruit of their observation and sagacity, commencing with those of Newton, than mathematical consequences deduced from his system.*

The theory of vibrations, which had suggested to Huygens the idea of ellipsoidal waves, by means of which he has so

* For the labours of Newton and M. de Laplace I entertain the most lively and sincere admiration; but I do not admire equally all which they have done; and I do not consider, for instance, as many persons do, that Newton's Optics is one of his chief titles to fame. It contains many grave errors, and the truths comprised were much less difficult to discover than the mechanical explanation of the celestial motions. What a difference in fact between the so easy analysis of light and that profound glance by which Newton saw that the precession of the equinoxes was occasioned by the oblateness of the earth! It is his immortal Principia and the discovery of the method of fluxions which have placed him in the first rank of geometers and natural philosophers. But however great the intellectual superiority of so prodigious a man, he is not the less subject to error: it cannot be too often repeated, *Errare humanum est*. Nothing can be more fatal to the progress of science than the doctrine of infallibility.

happily represented the movement of extraordinary rays in uni axal crystals, has led us to the discovery of the true laws of double refraction in the general case of bi axal crystals Undoubtedly an important part of these laws was already known,* Sir David Brewster and M Biot, by numerous observations and a skilful use of analogy, had already succeeded in discovering the law of the direction of the planes of polarization of the two beams and of their difference of velocity, but they were mistaken with regard to their absolute velocities, in supposing that of the ordinary ray to remain constant, as in uni axal crystals The experiments made by M Biot on topaz to verify this hypothesis, had not presented to him any sensible difference in the refraction of the ray termed "*ordinary*," but we are no longer surprised that these variations escaped the attention of so accurate an observer, when it is known how small they are in almost all directions except those in which they attain their *maximum*, and which could not be indicated but by theory or a lucky chance

The mechanical considerations on the nature of luminous vibrations and the constitution of doubly refracting media, which I have set forth in the *Annales de Chimie et de Physique*, tom xvii p 179 *et seq*, have enabled me at the same time to explain the changes of the extraordinary refraction and the constant velocity of the ordinary ray in uni axal crystals

I soon perceived that the reason which I had assigned to myself for the uniformity of the velocity of the ordinary ray in uni-axal crystals was not applicable to crystals with two axes, and, constantly following the same theoretical views, I perceived that in these latter neither of the two rays ought to be subject to the laws of ordinary refraction This is exactly what I verified by experiment, a month after having announced it to M Arago I did not indeed present to him this result of my reflections as a thing certain, but as a consequence of my theoretical views so necessary, that I should be obliged to abandon them if experiment did not confirm this singular character of double refraction in bi axal crystals The theory did not announce to me in a vague manner the variations of velocity of the ordinary ray, it gave me the means of deducing their extent from the elements of double refraction of the crystal, that is to say, from its degree of energy and the angle between the two axes I had made beforehand this calculation for limpid topaz, according to data derived from the observations of M Biot The experimen

agreed in a satisfactory manner with the calculation or, at least, the difference which I have observed is sufficiently small to be attributed to some inaccuracy in the cleavage of the crystal or the direction of the rays, and perhaps also to some slight difference of optical properties between my topaz and those of M Biot

But before entering into the detail of these experiments, I shall endeavour to exhibit clearly the reasonings which have led me to it In this memoir I shall follow the synthetical method I shall first explain the mechanical theory of double refraction and afterwards make known the observations and calculations which have enabled me to verify it, and which form in some sort its experimental demonstration

Mechanical Theory of Double Refraction

This theory rests on two hypotheses one relative to the nature of the luminous vibrations, and the other to the constitution of the media possessing the property of double refraction According to the first, the luminous vibrations, instead of being performed in the direction of the rays themselves as has been generally supposed by those who have applied the wave system to optics are perpendicular to the rays or, more strictly speaking, parallel to the surface of the waves According to the second hypothesis the vibrating molecules of doubly refracting media do not exhibit the same mutual dependence in all directions, so that their relative displacements will give rise to different elasticities according to their directions

This second supposition has nothing in it but what is very probable, it is more general than the contrary supposition, namely that which makes the mutual dependence of the molecules, or the elasticity the same in every direction If there are many bodies which do not present the phænomena which ought to follow on this supposition, it is no doubt generally owing to the compensation of opposite effects produced by the molecular groups being turned in all directions With regard to the hypothesis as to the nature of the luminous vibrations, it appears at first much more difficult to admit because one does not easily see how transversal vibrations are capable of indefinite propagation in a fluid

Nevertheless, if the facts which already furnish so many probabilities in favour of the wave system, and so many objections

against that of emission, compel us to recognise this character in the luminous vibrations. It is safer to trust ourselves here to experiment than to the notions, unfortunately too incomplete, hitherto presented to us by the calculations of geometers on the vibrations of elastic fluids. Before showing how we may conceive the propagation of these transversal vibrations in an elastic fluid such as that by which light is transmitted, I must prove that their existence becomes a necessary consequence of facts as soon as the system of waves is admitted.

When M. Arago and myself had remarked that rays polarized at right angles always produce the same quantity of light by their reunion, whatever be their difference of route, I thought that this particular law of the interference of polarized rays might be easily explained by supposing that the luminous vibrations, instead of pushing the ethereal molecules parallel to the rays, caused them to oscillate in perpendicular directions, and that these directions were at right angles to each other for two beams polarized at a right angle. But this supposition was so contrary to the received ideas on the nature of the vibrations of elastic fluids, that I was a long time before adopting it entirely; and even when the assemblage of facts and new reflections had convinced me that it was necessary to the explanation of the phenomena of optics, I waited till I had assured myself that it was not contrary to the principles of mechanics before submitting it to the examination of philosophers. Mr. Young, more bold in his conjectures and less confiding in the views of geometers, has published it before me (although perhaps he thought of it after me); and therefore the priority belongs to him with regard to this theoretical view, as on many others. It was the experiments of Sir David Brewster on biaxal crystals which led him to think that the vibrations of light, instead of being executed longitudinally, in the direction of the rays, might in truth be transversal, and similar to the undulations of an indefinite cord agitated by one of its extremities. It was, at any rate, on the occasion of Sir David Brewster's observations that he published this hypothesis, that is to say three years after the discovery of the particular characteristics of the interference of polarized rays. Resting on the first law of interference of these rays, I shall endeavour to prove that the luminous vibrations are performed solely in a direction parallel to the surface of the waves.

Demonstration of the exclusive existence of Transversal Vibrations in the Luminous Rays

It was in 1816 that M. Arago and myself discovered that two beams of light, polarized in planes at right angles to each other, no longer exert any influence on each other, in the same circumstances in which rays of ordinary light present the phenomena of interference; whilst as soon as their planes of polarization approach each other a little, the dark and bright bands resulting from the concourse of the two beams reappear, and become by so much the more distinct as these planes are brought nearer to coincidence.

This experiment teaches us that two rays polarized in perpendicular planes always give by their reunion the same intensity of light, whatever be the difference of the paths which they have run over starting from their common source. Now, from this fact, it necessarily results that in the two beams, the vibrations of the ethereal molecules are performed perpendicularly to the rays and in rectangular directions. To demonstrate this I shall first call to mind that in the rectilinear oscillations produced by a small derangement of equilibrium, the absolute velocity of the vibrating particle is proportional to the sine of the time reckoned from the origin of the motion, the duration of a complete oscillation answering to a whole circumference. If the oscillation is curvilinear, it may always be decomposed into two rectilinear oscillations perpendicular to each other, to which the same theorem will apply.

In the luminous wave produced by the oscillation of the illuminating particle, the absolute velocities animating the molecules of ether are proportional to the corresponding velocities of the illuminating particle, and therefore also to the sine of the time. Moreover, the space described by each of the elementary disturbances of which the wave is composed is proportional to the time; and as many times as this space contains the length of an undulation, so many entire oscillations have been performed since the disturbance set out. If therefore (π) represent the ratio of the circumference to the diameter, (t) the time elapsed since the origin of the motion, if also (λ) denote the length of an undulation, and (x) the space described by the disturbance in order to reach the point of æther which we are considering, the absolute velocity with which this point is animated at the

end of the time (t) will be represented by $a \sin 2\pi\left(t-\frac{x}{\lambda}\right)$ (a) being here a constant coefficient proportional to the amplitude of the oscillations of the ethereal molecules or to the intensity of their absolute velocities.* This being established, let us consider one of the two interfering rays. Whatever be the direction of the absolute velocity of the ethereal molecule, we may always decompose this velocity at each instant in three constant directions at right angles to each other, the first, for example, being the direction of the normal to the wave, and the other two perpendicular to this being, the one parallel, the other perpendicular, to the plane of polarization. By the general principle of small motions, we may consider the oscillations performed by the ethereal molecules, of whatever nature they may be, as resulting from the combination of three series of rectilinear oscillations whose directions coincide with these three rectangular axes, oscillations which, for the greater generality, we shall suppose to have commenced at different epochs.

Call (t) the time elapsed since a common epoch, and represent by (u), (v) and (w) that which must be added to (t) to obtain the whole time reckoned from the origin of the motion in each of the three modes of rectilinear vibration, then the absolute velocities belonging to the instant we are considering will be

$$a \sin 2\pi\left(u+t-\frac{x}{\lambda}\right),$$

$$b \sin 2\pi\left(v+t-\frac{x}{\lambda}\right),$$

$$c \sin 2\pi\left(w+t-\frac{x}{\lambda}\right),$$

a, b and c being constant coefficients, which denote the intensity of the absolute velocities in each system of rectilinear oscillation.

Now let us consider the second polarized ray, and decompose its absolute velocities in the direction of the same rectangular axes. If we represent by (x') the path which it has passed over

* A demonstration of these formulæ, and a more detailed explanation of their usage will be found in the *Mémoires de l'Académie des Sciences*, tom. v. Those readers who are not familiar with the theory of luminous waves, may first study its elementary principles in an article on light in the supplement to the French translation of the fifth edition of Thomson's 'Chemistry.' [This article has been translated by Dr. Young in Brande's 'Quarterly Journal of Science' for Jan. 1827 and following numbers.—Tr. N.]

to arrive at the same point, we shall have similarly for the three components referred to the instant (t)

$$a' \sin 2\pi\left(u' + t - \frac{x'}{\lambda}\right)$$

$$b' \sin 2\pi\left(v' + t - \frac{y'}{\lambda}\right)$$

$$c' \sin 2\pi\left(w' + t - \frac{z'}{\lambda}\right)$$

These three velocities having respectively the same directions as the preceding, it is sufficient to add them in order to have their resultants, which gives—

$$a \sin 2\pi\left(u + t - \frac{x}{\lambda}\right) + a' \sin 2\pi\left(u' + t - \frac{x'}{\lambda}\right)$$

$$b \sin 2\pi\left(v + t - \frac{y}{\lambda}\right) + b' \sin 2\pi\left(v' + t - \frac{y'}{\lambda}\right)$$

$$c \sin 2\pi\left(w + t - \frac{z}{\lambda}\right) + c' \sin 2\pi\left(w' + t - \frac{z'}{\lambda}\right)$$

If we transform each of these expressions so that it may contain only one sine, according to the method indicated in my Memoir on Diffraction (*Mémoires de l'Académie des Sciences* tom. v. p. 379) we find that the square of the constant coefficient multiplying this sine is, for each of these respectively equal to

$$a^2 + a'^2 + 2aa' \cos 2\pi\left(u - u' + \frac{x' - x}{\lambda}\right)$$

$$b^2 + b'^2 + 2bb' \cos 2\pi\left(v - v' + \frac{y' - y}{\lambda}\right)$$

$$c^2 + c'^2 + 2cc' \cos 2\pi\left(w - w' + \frac{z' - z}{\lambda}\right)$$

Now it is the square of the constant coefficient of the absolute velocities which represents in each system of vibrations, the intensity of the light which is always proportional to the sum of the *vires vivæ*, and as these velocities are at right angles to each other, it is sufficient to add the three preceding squares to have the total sum of the *vires vivæ* resulting from the three systems of vibration, that is to say, the intensity of the whole light.

Experiment shows that this intensity remains constant, whatever variations are undergone by the difference $(x' - x)$ of the

paths described, when the two interfering beams have their planes of polarization perpendicular to each other

Thus, in this case, the sum of the three foregoing expressions remains the same for all values of $(x' - x)$ We must therefore have

$$a^2 + b^2 + c^2 + a'^2 + b'^2 + c'^2 + 2aa' \cos 2\pi \left(u - u' + \frac{x' - x}{\lambda}\right)$$

$$+ 2bb' \cos 2\pi\left(v - v' + \frac{x' - x}{\lambda}\right) + 2cc' \cos 2\pi\left(w - w' + \frac{x' - x}{\lambda}\right) = C,$$

an equation in which the only variable is $(x' - x)$

Now, since this equation must be satisfied whatever be the value of $(x' - x)$, it is clear that all the terms containing $(x' - x)$ must disappear, since otherwise we should obtain from the equation particular values of $(x' - x)$ Therefore we have

$$aa' = 0, \qquad bb' = 0, \qquad cc' = 0$$

The two polarized beams which interfere differ only in the azimuths of their planes of polarization, that is to say, if we turn one of them about its axis so that its plane of polarization may be parallel to that of the other these two luminous beams will present in every direction exactly the same properties, they will be reflected and refracted in the same manner and in the same proportions at the same incidences We must therefore admit that if one has no vibratory movements perpendicular to the waves, no more has the other Now (a) and (a') are the constant coefficients of the absolute velocities normal to the waves in these two beams, and since $aa' = 0$, which requires that we have at least $a = 0$ or $a' = 0$, we must conclude from this that both (a) and (a') are equal to zero

There cannot therefore be in polarized light any other than vibratory movements parallel to the surface of the waves

Let us now consider the other two equations, $bb' = 0$ and $cc' = 0$, which contain the constant coefficients of the velocities perpendicular to the rays, or, more generally, parallel to the waves (b) is for the first luminous beam the component parallel to its plane of polarization, and (c) that which is perpendicular to it, whilst for the second, (b') being parallel to (b), is perpendicular to the plane of polarization, and (c') is parallel to it thus (b') and (c') are respectively for the second beam that which (c) and (b) are for the first Therefore, according to the remark just made on the perfect similitude between the properties of the two interfering beams, if in the former $b = 0$, in the second

c' will be nothing; or if it is the component (c) which is nothing in the former, (b') in the second will equal zero. Thus we must conclude, from the two preceding equations,

$$b = 0 \text{ and } c' = 0, \text{ or } c = 0 \text{ and } b' = 0,$$

that is to say, that in each of the two beams there are only vibrations parallel or perpendicular to its plane of polarization.

When we have explained the mechanical causes of double refraction, we shall show that these vibrations are perpendicular to the principal section in the ordinary ray, that is to say, to the plane which it has been agreed to call the *plane of polarization*.

Having demonstrated that in polarized light the ethereal molecules cannot have any vibration normal to the waves, we must suppose that neither does this mode of vibration exist in ordinary light. In fact, when a beam of ordinary light, falling perpendicularly on a doubly refracting crystal, is divided into two polarized beams, they no longer contain vibrations normal to the waves. If then there were any such in the incident light they must have been destroyed, whence there must have been a diminution of *vis viva*, and therefore a weakening of the light, which would be contrary to observation, for, when the crystal is perfectly transparent the two emergent beams when reunited reproduce a light equal to that of the incident beam, if there be added to them the small quantity of light reflected at the faces of the crystal. Now we cannot suppose that it is into this small quantity of light that the vibrations normal to the waves have betaken themselves, since on causing it to traverse the crystal it could also be transformed almost entirely into two polarized beams, where we are certain that this kind of vibration does not exist. It is therefore natural to suppose that ordinary light also contains only vibrations parallel to the waves, and to consider it as the assemblage and rapid succession of a multitude of systems of waves polarized in all azimuths. According to this theory, the act of polarization does not consist in the creation of transversal vibrations, but in the decomposition of these vibrations into two fixed rectangular directions, and in the separation of the rays resulting from this decomposition.

Theoretical Explanation of the Laws of Interference of Polarized Rays

According to what we have just said concerning the nature of the vibrations of polarized rays, it is clear that they cannot pro-

sent the phænomena of interference, except so far as their planes of polarization are parallel or approach to parallelism. When these planes are perpendicular, the absolute velocities of the æthereal molecules are also perpendicular to each other; if, therefore, at each point of the common direction of the two rays we wish to obtain the resultant of the two velocities impressed by them on the molecule of æther, we must take the sum of the squares of the two velocities; this will be the square of the resultant. The same calculation applies to all the points of the two systems of waves, whatever may be in other respects their difference of route; thus the sum of the squares of the absolute velocities impressed on the æthereal molecules by the union of the two systems of waves will always be equal to the sum of the squares of the absolute velocities caused by each of the luminous rays; or, in other words, the intensity of the whole light will always be equal to the sum of the intensities of the two interfering rays, whatever may be their difference of route. Variations therefore in this difference cannot produce those alternations of brightness and obscurity which are observed in ordinary light, or in rays polarized in parallel directions. The ease with which our hypothesis explains the first law of interference of polarized rays is then seen, and this is what might be expected, since it was from this law itself that we have derived it.

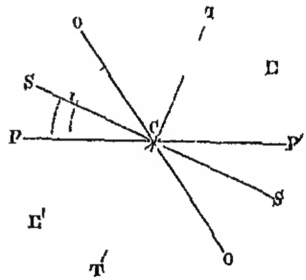
We may regard it as sufficiently established by the demonstration just given; but it will not be without use to show that the same hypothesis agrees quite as well with the other laws of interference of polarized rays which become the immediate consequences of it. These theoretical developments on the properties of polarized light will not appear out of place in an essay on double refraction, and will moreover find their application in the memoirs which we intend to publish afterwards on the colours of crystalline plates.

When the interfering luminous beams have their planes of polarization parallel, their vibratory movements have the same direction, and therefore are added to each other along the whole course of the rays if the difference of route is nothing, or equal to an even number of semi-undulations; and are subtracted one from the other when the number of semi-undulations is uneven. In general, to obtain in this case the intensity of the light resulting from the concourse of the different systems of waves, we may use the formulæ already cited from my Memoir on Dif-

fraction which have been calculated on the supposition that the vibrations of the interfering rays were performed in a common direction

I come now to the third principle of interference of polarized rays When two portions of a luminous beam, which had at first the same plane of polarization, PP′ receive a new polarization in two different planes OO′ and EE′, and are afterwards brought back to a common plane of polarization, SS′ or TT′, then agreement or disagreement answers precisely to the difference of the routes described, when the two planes of polarization OC and E′C, starting from the primitive direction CP, after having been separated one from the other, approach each other afterwards by a contrary movement so as to reunite in CS but when the two planes CO and CE′ continue to widen their distance from each other until they become situated one on the prolongation of the other, in CT and CT′ for example, it is no longer sufficient to take into account the difference of paths described it is necessary also to change the signs of the absolute velocities of one of the interfering beams by giving a contrary sign to their constant coefficient, or, which comes to the same thing, adding a semi undulation to the difference of paths described

Fig 1



It is easy to see the reason of this rule In order not to complicate the figure, we shall suppose that the lines there drawn, instead of representing the planes of polarization, indicate the direction of the luminous vibrations which are perpendicular to those planes This is as if we had turned the figure through a quarter of a circumference round its centre C, which alters nothing in the relative positions of the planes of polarization Let us consider, at any point whatever of the luminous ray projected in C, the absolute velocity which animates the ethereal molecules at a given instant in the primitive beam, whose vibrations are performed in the direction PP′, and suppose that at this instant the molecule C is pushed from C towards P, that is to say, that its absolute velocity acts in the direction CP, its components along CO and CE′ will act, one in the direction CO,

the other in the direction CE′. Now, according to the general principle of small motions, these components are the absolute velocities in the two systems of waves which result from the decomposition of the first. If we suppose OO′ and EE′ at right angles, as is the case for the directions of the ordinary and extraordinary vibrations in a doubly refracting crystal, the component CO will be equal to the first absolute velocity multiplied by cos i, and the component CE′ to the same velocity multiplied by sin i. We are thus led to a very simple explanation of the law of Malus, on the relative intensities of the ordinary and extraordinary images by passing from the absolute velocities to the *vires vivæ*, which are proportional to their squares, $\cos^2 i$ and $\sin^2 i$. But let us return to the components CO and CE′. If we decompose them each into two others in the directions SS′ and TT′, there will result for the former CO two velocities in the directions CS and CT, and for the second CE′, two components acting in the directions CS and CT′. It is seen that in the plane SS′ the two resulting components act in the same direction and are added to each other, whilst they act in contrary directions in the plane TT′, and must therefore be affected with contrary signs, which justifies the rule we have announced, for what we have just said applies equally to all the points taken on the ray projected in C, and therefore to the constant coefficient which multiplies all the absolute velocities of each system of waves. This law, the enunciation of which may at first sight have appeared complicated, is in substance, as we see, only a very simple consequence of the decomposition of forces.*

The principles just established with regard to the interference of polarized rays suffice for the explanation and calculation of all the phenomena of the colours of crystalline plates. We

* I think it needless to give here the explanation of the fourth law of interference of polarized rays, which is a result of the present one, as I have shown it to be in a Note joined to the Report of M. Arago in the *Annales de Chimie et de Physique*, tom. xvii. p. 101. This law consists in this, that the rays which have been polarized at right angles and are afterwards brought back to the same plane of polarization, cannot present phænomena of interference except in so far as the primitive beam has received a previous polarization. Not that they do not necessarily exert a mutual influence on each other as soon as their vibratory motions are brought back to a common direction; but the light which has not received any previous polarization, and which may be considered as the union of an infinity of systems of waves polarized in all directions, when analysed by a rhomboid of calcareous spar after its passage across a crystallized plate, produces at the same time in each of the two images opposite effects which mutually disguise each other (*se masquent*), as may be easily deduced from the law just explained.

might therefore limit here the development of these considerations whose special object it is to give the theoretical demonstration of the rules for calculating the tints of crystalline plates We think however that it will not be useless to point out here some of the most simple consequences of these principles

I suppose that a beam of polarized rays falls perpendicularly on a crystalline plate situated in the plane of the figure Let, as before, PP′ denote the direction parallel to which the vibrations of the incident beam are performed OO′ and EE′ those of the vibrations of the ordinary and extraordinary beams into which it is divided after having penetrated into the crystal Suppose this crystalline plate to be sufficiently thin, that there may be no sensible difference of route between the two emergent beams, or that it has such a thickness that the difference of route may contain a whole number of undulations, which comes to the same thing all the points taken on the ray projected in C, for example, are simultaneously urged in the two systems of waves by velocities which correspond to the same epochs of the oscillatory movement they will have therefore at each point of the ray the same ratio of intensity, that namely of the constant coefficients of the absolute velocities of the two systems of waves therefore their resultants will be parallel and will all be projected in the direction PP′, since the components are all two and two in the ratio of cos i to sin i Thus the light arising from the union of the two emergent beams will still be polarized since all its vibrations will be performed in parallel directions and its plane of polarization will be the same as that of the incident beam

Suppose now the difference of route of the ordinary and extraordinary beam on emergence from the crystal, to be a semi undulation or an uneven number of semi undulations, this is as if, the difference of route being nothing, we were to change the sign of all the absolute velocities of one of the two systems of waves, thus the velocity which urges the molecule C at a given instant, in the first beam pushing it from C towards O for example, that which is caused by the second beam, instead of pushing this molecule from C towards E′, as in the preceding case, will push it from C towards E so that the resultant of these two impulses, instead of being directed along C P, will have the direction of a line situated on the other side of C O, and making with this latter an angle equal to the angle (i) con

tained between C O and C P. The same will be the case for all the other points taken along the ray projected in C. Thus the whole light composed of the two emergent beams will still be polarized on leaving the crystal, since all its vibrations will be parallel to a constant direction, but its plane of polarization, instead of coinciding with the primitive plane, as in the preceding case, will be found separated from it by an angle equal to $(2i)$. It is this new direction of the plane of polarization which M. Biot has called *the azimuth* $2i$.

It is seen with what simplicity the theory we have set forth explains how the union of two beams of light, polarized at right angles, the one in a direction parallel, the other perpendicular, to the principal section of a crystal, form by their reuniting a light polarized in the primitive plane or in the azimuth $(2i)$, according as the difference of route between the two beams is equal to an even or uneven number of semi-undulations. We cannot imagine how one could conceive, on the emission system, this remarkable phænomenon, which nevertheless cannot be called into doubt after it has been proved by an experiment so decisive as that of the two rhomboids, given in the *Annales de Chimie et de Physique*, tom. xvii p. 94 *et seq.*

Let us consider now the case in which the difference of route is no longer a whole number of semi-undulations; then the corresponding velocities in the two systems of waves are no longer applied simultaneously to the same points of the ray projected in C; the result is, that the two forces, which solicit each of these points at the same instant, have not the same ratio of magnitude along the whole length of the ray, and consequently that their resultants are no longer in the direction of the same plane, thus, the reunion of the two systems of waves presents no longer the characters of polarized light. Call their difference of route (a), the constant coefficients of their absolute velocities are respectively equal to $\cos i$ and $\sin i$, taking for unity that of the primitive beam, whose vibrations are performed in a direction parallel to P P′.

Then, the absolute velocities excited by the two component beams, in the same point of the ray projected in C, at the instant (t), will be $\cos i \,.\, \sin 2\pi\,(t)$ and $\sin i \,.\, \sin 2\pi\left(t - \frac{a}{\lambda}\right)$: and the square of the resultant of these two rectangular forces will be equal to

$$\cos^2 i \ \sin^2 2\pi \ t \ + \sin^2 i \ \sin \ 2\pi\left(t - \frac{a}{\lambda}\right) \qquad \text{(A)}$$

From this formula may also be obtained the displacements of the vibrating molecule relative to its position of rest, by changing the time (t) by a quarter of a circumference, or the common point of departure by a quarter of an undulation: for these displacements follow the same law as the velocities, with this difference only, that the velocity is nothing at the moment of the molecule's being at its greatest distance from its position of rest, and that the instant of its passing through this position is that of maximum velocity.

For the same reason the displacements of the vibrating molecule measured parallel to the rectangular directions OO′ and EE′ are proportional to the expressions

$$\cos i \ \cos 2\pi \ t \qquad \text{and } \sin i \ \cos 2\pi\left(t - \frac{a}{\lambda}\right)$$

If we wish to find the curve described by the molecule referred by parallel coordinates to OO′ and LL′, it is sufficient to write

$$\cos i \ \cos 2\pi t = x, \qquad \text{and } \sin i \ \cos 2\pi\left(t - \frac{a}{\lambda}\right) = y$$

and to eliminate (t) between these two equations, which gives

$$x^2 \sin^2 i + y^2 \cos^2 i - 2xy \sin i \cos i \cos \frac{2\pi a}{\lambda}$$
$$= \sin^2 i \ \cos^2 i \ \sin^2 \frac{2\pi a}{\lambda}$$

an equation of a curve of the second degree referred to its centre.

Without discussing this equation, we are certain beforehand that the curve can only be an ellipse, since the excursions of the molecule in the direction of x and y have for limits the constant quantities $\sin i$ and $\cos i$. This curve becomes a circle when $i = 45°$, and (a) contains the fourth part of an undulation an uneven number of times; or in other words when the two systems of waves polarized at right angles have the same intensity, and differ in their route by an uneven number of quarter undulations. We have then

$$\sin i = \cos i = \sqrt{\frac{1}{2}}, \ \cos 2\pi \frac{a}{\lambda} = 0 \text{ and } \sin 2\pi \frac{a}{\lambda} = 1,$$

which reduces the above equation to

$$x^2 + y^2 = \frac{1}{2}$$

It would have been easy to arrive at the same consequence without the aid of the general equation, by remarking that, since in this particular case

$$\sin i = \cos i \text{ and } \cos 2\pi\left(t - \frac{a}{\lambda}\right) = \sin 2\pi\ t,$$

the two coordinates

$$\cos i\ \cos 2\pi t \text{ and } \sin i\ \cos 2\pi\left(t - \frac{a}{\lambda}\right)$$

are always proportional to the sine and cosine of the same variable angle $2\pi\ t$

Another remarkable peculiarity of the oscillatory motion in the same case is, that the velocity of the molecule is uniform In fact, the formula (A), which expresses the square of this velocity, becomes

$$\frac{1}{2}\sin^2 2\pi t + \frac{1}{2}\cos^2 2\pi t, \text{ or } \frac{1}{2}$$

This uniform circular motion takes place in the same direction for all the molecules situated along the ray projected in C, but they do not occupy at the same instant the corresponding points of the circumferences which they describe, that is to say, the molecules, which in their state of rest were situated on the straight line projected in C, instead of remaining on a straight line parallel to this, and which would describe round it a cylinder on a circular base, form a helix whose radius is that of the small circles described by the vibrating molecules, and the distance from thread to thread ("*le pas*") is equal to the length of an undulation If we turn this helix round its axis with a uniform motion, so that it describes a circumference in the interval of time during which a luminous undulation is performed, and if we conceive, besides, that in each infinitely thin slice perpendicular to the rays all the molecules perform the same movements as the corresponding point of the helix and preserve the same relative situations, we shall have a correct idea of the kind of luminous vibration which I have proposed to call *Circular Polarization*, giving the name of *Rectilinear Polarization* to that which was observed first by Huygens in the double refraction of Iceland spar, and which Malus has reproduced by simple reflexion at the surface of transparent bodies

These circular vibrations are performed sometimes from right to left and sometimes from left to right, according as the plane of polarization of the system of waves preceding (*en avant*) is to

the right or left of that of the system of waves succeeding (*en arrière*), the difference of route being equal to a quarter of an undulation, or to a whole number of undulations plus a quarter; it is the inverse when this difference is three quarters of an undulation, or a whole number of undulations plus three quarters.

There are certain refracting media, such as rock crystal in the direction of its axis, the essential oil of turpentine, of lemons, &c., which have the property of not transmitting with the same velocity the circular vibrations from right to left and those from left to right. Such a result may be conceived to arise from a particular constitution of the refracting medium or of its integrant molecules, which produces a difference between the direction from right to left and that from left to right; such would be, for example, a helicoidal arrangement of the molecules of the medium, which would offer contrary properties according as the helices were *dextrorsum* or *sinistrorsum*.

The mechanical definition which we have just given of circular polarization enables us to conceive how the singular double refraction presented by rock crystal in the direction of its axis may take place, namely, that the arrangement of the molecules of this crystal is not the same apparently from right to left and from left to right, so that the luminous beam whose circular vibrations are performed from right to left puts into play an elasticity or force of propagation slightly differing from that excited by another beam whose vibrations are performed from left to right.

Such is the principal theoretical advantage which may be derived from the geometrical considerations we have just given on the circular vibrations of light resulting from the combination of rectilinear vibrations. But in the calculation of the phænomena presented by light polarized rectilinearly or circularly after having traversed the media by which it is modified, it is useless to investigate, for example, what are the curvilinear vibrations resulting from the reunion of two systems of waves on leaving a crystalline plate; we are, on the contrary, obliged to decompose into rectilinear motions the circular vibrations of the two systems of waves emerging from a plate of rock crystal perpendicular to its axis, when we wish to determine the intensities of the ordinary and extraordinary images produced by this emergent light across a rhomboid of calcareous spar. The cal

culations of the intensities of the ordinary and extraordinary images, for a homogeneous light, or that of the tints developed by polarized white light, always lead us back to the consideration of rectilinear vibrations and to the employment of the formulæ of interference which refer to them

In indicating the mechanical cause of the altogether peculiar double refraction exerted by rock crystal on light in the direction of its axis, we have wandered from the object of this memoir, in which we shall treat solely of the case in which the particles of the vibrating medium have their homologous faces parallel, and thus exhibit the same molecular arrangement from right to left and from left to right We hope the reader will pardon us this digression on circular polarization, to which we were naturally led by what we had just said on rectilinear polarization It is, besides, useful to familiarize ourselves with these different modes of luminous vibrations, the whole of which we find in the most simple kind of double refraction, such as that of uniaxal crystals, as soon as we, instead of separating in thought the ordinary from the extraordinary waves, consider the complex effect which results from their simultaneous existence

After having proved that the transversal direction of the luminous vibrations is a necessary consequence of the absence of the ordinary phænomena of interference in the reunion of rays polarized at right angles, it is necessary to show that this hypothesis established by facts, in the wave system, is not contrary to the principles of mechanics, and to explain how such vibrations may be propagated in an elastic fluid

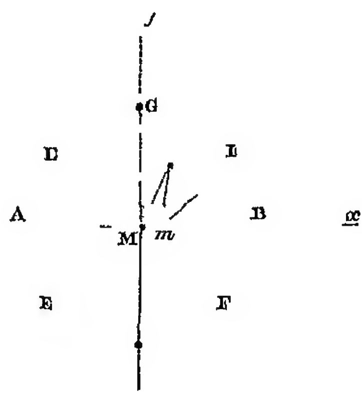
Possibility of the propagation of Transversal Vibrations in an Elastic Fluid

An elastic fluid is by all philosophers conceived as the assemblage of molecules or material points separated by intervals which are very great relatively to the dimensions of these molecules, and kept at a distance by repulsive forces which are in equilibrium with other contrary forces resulting from the mutual attraction of the molecules or from a pressure exerted on the fluid This being established, let us, for the sake of fixing our ideas, imagine the regular arrangement of molecules represented by fig 2, and consider the case of a plane and indefinite wave whose surface is parallel to the plane projected in A B If the portion of the medium above this plane has undergone a small

displacement parallel to the row of molecules A M B these molecules will become urged on to a similar motion

In fact, let us consider one of them in particular, the molecule M for example, and examine what change has been operated in the actions excited upon it by the superior portion of the medium And, in the first place I observe that these will be the same as if it were the molecule M which had been displaced to the same extent and in the same direction the superior portion of the medium remaining fixed I suppose then M to be displaced in the direction AB by a small quantity M m

Fig 2



The molecules L and F for example situated at equal distances from M and from the perpendicular M G dropped upon AB, acted equally on the molecule M in the direction MA and in the direction MB before its displacement that is to say the component of their actions along AB mutually destroyed each other, whilst the components perpendicular to AB were added to each other but were counterbalanced by the opposite actions of the molecules L′ and F′ situated below AB When the material point M is transported to m, the components parallel to AB of the two actions excited on it by the molecules L and F are no longer generally equal to each other, and the small changes which they have undergone, or their differentials, act in the same direction, and tend to bring back the point m to its original position M, if this was one of stable equilibrium

In fact, represent by $\phi(r)$ the action excited by a molecule situated at a distance (r), such as the molecules L and F Take M as the origin of coordinates, and the straight lines AB and MG for the axes of x and y denote by x and y the coordinates of the point L, those of F will be y and x The distances EM and FM or (r) are equal to $\sqrt{x^2+y^2}$ and therefore the

forces which act along FM and along EM are each represented by $\phi(\sqrt{x^2+y^2})$. Moreover, the sine of the angle FMB is equal to $\frac{y}{\sqrt{x^2+y^2}}$ and its cosine to $\frac{x}{\sqrt{x^2+y^2}}$, therefore the two components of the force acting along FM are, parallel to x,

$\frac{x}{\sqrt{x^2+y^2}}\ \phi(\sqrt{x^2+y^2})$, or $x\ \psi\ (x^2+y^2)$, and parallel to

$y\ \frac{y}{\sqrt{x^2+y^2}}\ \phi\ (\sqrt{x^2+y^2})$ or $y\ \psi\ (x^2+y^2)$, if we take for the positive direction of the forces parallel to the axes of coordinates, that in which each of these two components acts. Similarly, the two components of the action exerted by the molecule E are respectively $-x\ \psi(x^2+y^2)$ and $y\ \psi\ (x^2+y^2)$, that is to say, they only differ from the former in the sign of (x). Now, to calculate the small quantities by which these components are altered in consequence of the displacement of the point M, we must differentiate their expressions with respect to x; we find thus, for the differentials of the components of the force FM,

$$\begin{array}{ll} \text{parallel to } x & [\psi(x^2+y^2) + 2x^2\psi'(x^2+y^2)]\,dx, \\ \text{parallel to } y & 2xy\ \psi'(x^2+y^2)\,dx. \end{array}$$

The expression for the force EM differing only from that for the force FM by the sign of x, we may obtain at once the variations of its components by simply changing the sign of x in the two preceding expressions, without changing, be it understood, that of the small displacement dx, which takes place in the same direction for both forces. Now, by the mere inspection of these formulæ, it is seen that the differential of the component parallel to x will preserve the same sign, and will therefore be added to that of the force FM, whilst the differential of the component parallel to y will be subtracted from the corresponding variation of the other force, and will destroy it. There results, therefore, from the small displacement of the point M along AB, a force parallel to the same line, and which tends to bring back this point towards its position of equilibrium.

Therefore if, the point M remaining fixed, the superior portion of the medium be slightly displaced parallel to AB (which comes to the same thing), the point M will be pushed in the direction AB, as well as all the other molecules of this layer, which will therefore be urged throughout its whole extent to slide along its

own plane AB By the displacement of this layer the same effect will be produced successively on the parallel layers $A'B'$, $A''B''$, &c and in this manner the transversal vibrations of the incident wave may be transmitted throughout the whole extent of the medium

The force which urges the point M along AB, in consequence of the displacement of the layer E and of the superior layers sliding in their own planes is owing to this that their material elements are not contiguous if they were each point M of the layer AB would remain indifferent to a simple sliding of the superior layers, which in that case would produce no alteration in the action excited by them on this point But if the displacement of these layers took place in the perpendicular direction GM, it is clear that the contiguity of the elements of each of them would not prevent the force with which they tend to repulse each point of AB from increasing in proportion as the distance diminished, so that on this supposition, the resistance opposed by the layers to their approximation would be infinitely greater than the force necessary to give a sliding motion to an indefinite layer Without proceeding to this limit, which doubtless does not exist in nature, we may suppose that the resistance of the æther to compression is much greater than the force opposed by it to the small displacements of these layers along their own planes now, by help of this hypothesis it is possible to conceive how the molecules of æther may have no sensible oscillations except in a direction parallel to the surface of the luminous waves

How it may happen that the Molecules of Æther do not undergo any sensible agitation in the direction of the Normal to the Wave

The resistance to compression being in fact much greater than the other elastic force put in play by the simple sliding of the layers, the wave produced by the former will extend itself much further than that which results from the second, during the same oscillation of the illuminating particle by the vibrations of which the æther is agitated, thus, even if the small movements of the molecules of this fluid were performed in such a manner that their *vires vivæ* were equally distributed between the two modes of vibration, the *vires vivæ* comprised in the wave of condensation or dilatation being distributed over a much greater extent

of fluid than those of the other wave, the oscillations parallel to the rays would have much less amplitude than those perpendicular to them, and consequently could only impress on the optic nerve much smaller vibrations; for the amplitude of its vibrations cannot exceed that of the vibrations of the æther in which it is plunged (*qui le baigne*). Now it is natural to suppose that the intensity of the sensation depends on the amplitude of the vibrations of the optic nerve, and that thus the sensation of light resulting from vibrations normal to the waves will be sensibly nothing compared to that produced by the vibrations parallel to their surface.

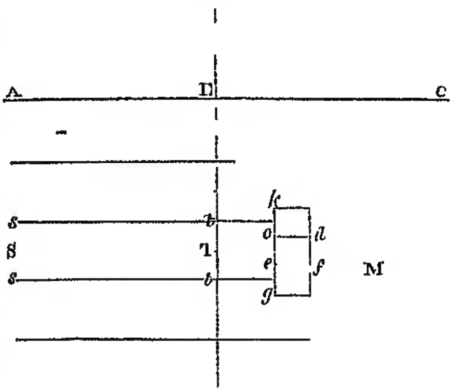
Moreover, it will be conceived that during the oscillation of the illuminating molecule, the equilibrium of tension is restored so rapidly between that portion of the æther which it approaches and that which it recedes from, that there is no sensible condensation or dilatation, and that the displacement of the æthereal molecules which surround it is reduced to an oscillatory circular movement, which bears them on the spherical surface of the wave from the point which the illuminating molecule approaches towards that from which it recedes.

I think I have sufficiently proved that there is no mechanical absurdity in the definition of luminous vibrations which the properties of polarized rays have compelled me to adopt, and which has led me to the discovery of the true laws of double refraction. If the equations of motion of fluids imagined by geometers are not reconcilable with this hypothesis, it is because they are founded on a mathematical abstraction, the contiguity of the elements, which, without being true, may nevertheless represent a part of the mechanical properties of elastic fluids, when it is admitted besides that these contiguous elements are compressible. But from this very circumstance, that such is not the reality and merely a pure abstraction, we ought not to expect to find in it all the kinds of vibration of which elastic fluids are susceptible, and all their mechanical properties; thus, for example, according to the equations of which we speak, there would be no friction between two indefinite layers of fluid which slide one on the other. It would then be but little philosophical to reject an hypothesis to which the phænomena of optics so naturally lead, for no other reason than because it does not agree with these equations.

How Transversal Vibrations are extinguished at the extremity of the Waves.

Hitherto we have only considered indefinite waves; let us suppose them limited, and examine what happens at their extremities, admitting the æther to be sensibly incompressible.

Fig. 3.



I suppose that a part of the wave AE (fig. 3.) has been arrested by a screen EC; let M be a point situated behind the screen at a distance very great relative to the length of an undulation. However small may be the sensible magnitude of the angle TEM which the straight line TM makes with the direct ray ET, the light sent to M will be very little, as we know by experience and as may be easily concluded from the theory of diffraction. If therefore the angle TEM is rather large, the point M will be nearly at rest, whilst the point T and all the rest of the wave ST will undergo sensible oscillations along the plane STM. It would seem that there ought to result from this alternate condensations and dilatations of the æther between T and M; but remark, in the first place, that at the same instant when the face (ce) of the small parallelopiped $cdef$ is pushed towards M by the semi-undulation whose middle corresponds to ST, the homologous faces ck, eg of the two contiguous parallelopipeds move off from M by the contrary movements of the two semi-undulations whose middle points correspond to the lines st, $s't'$, so that whilst the volume of $cdef$ diminishes, those of the two similar parallelopipeds between which it is situated increase by the same quantity, and so on in succession in the direction kg. If then the æther strongly resists compression, it is possible that the equilibrium of tension may continually re-establish itself, and almost instantaneously, between the neighbouring elements parallel to gk. Moreover, the points which remain at rest during the oscillations of the extremities of the waves, are sufficiently

distant from ET to cause the molecular displacements occasio
by these oscillations to diminish very slowly up to the po
which may be regarded as immovable, so that the condensati
and dilatations of the consecutive strata will be almost insensi
even if the equilibrium of pressure were not rapidly resto
from one stratum to another

Demonstration of two Statical Theorems, on which depends mechanical explanation of Double Refraction

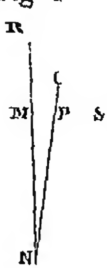
After having deduced from facts the hypothesis which I l
adopted on the nature of luminous vibrations, and having pro
that it is not contrary to the principles of mechanics, I s
now demonstrate two theorems belonging to general statics
which depends the theoretical explanation of the mathema
laws of double refraction

First Theorem

In any system of molecules in equilibrium, and whatever
be the law of their reciprocal actions, the minute displace
of a molecule, in any direction whatever, produces a repu
force equal in magnitude and direction to the resultant of t
repulsive forces which would be separately produced by t
rectangular displacements of this material point equal to
statical components of the first displacement

Let M (fig 4) be one of the material points of the mole
system, when the equilibrium comes to be
turbed by the small displacement M C o
molecule M, the resultant of all the forces ex
upon it, which before was equal to zero, acc
a certain value to calculate it, it is suffici
determine the variations which these forces
undergone in magnitude and direction, a
find the resultant of all these differentials
the next place, then, I consider the part
action of any other molecule N on the

Fig 4



M which has been displaced through M C, which I su
very small relative to the distance M N which separates tl
molecules On M N I draw the perpendicular M S in the
C M N, if C N be joined, C P will be the small quanti
which the distance M N has increased, or the differential
distance, and $\frac{MP}{MN}$ will be the sine of the small angle by

the direction of the force has varied. If therefore we refer the original force and the new one to two rectangular directions, M R and M S, the differential in the direction M R will only arise from the small increase C P of the distance and will be proportional to C P whilst the differential in the direction M S will result solely from the small change of direction of the force and will be proportional to $\frac{MP}{MN}$ or simply to M P, the distance M N remaining the same thus the first differential may be represented by A × C P, and the second by B × M P, A and B being two factors which remain constant so far as the action exerted by the same molecule N is concerned.

Let us consider as yet only the particular action of this molecule, and suppose M to be displaced successively in three rectangular directions and by quantities equal to the projections of M C on these three directions through the point M draw a plane perpendicular to M N, which will cut that of the figure, that is the plane N M C, in the straight line M S. The displacement M C has produced the two differential forces A × C P and B × M P the former in the direction M R and the second in the direction M S. The displacements on the three rectangular directions situated any how in space will likewise produce each a differential force parallel to M R, with another force perpendicular to this line, and comprised also in the normal plane M S drawn through the point M the former will be obtained by multiplying by the same coefficient A the distance of the new position of the molecule from the normal plane, and the second by multiplying by the same coefficient B the distance of M from the foot of the perpendicular dropped from this new position on the normal plane. Next let us find the resultant of three differential forces parallel to M R which have the same coefficient A, and the resultant of three differential forces contained in the normal plane which have B for their common coefficient. The displacements in question being the projections of the displacement M C on the three rectangular directions which have been chosen, the sum of their projections on the direction M R must be equal to C P, and consequently the resultant of the three differential forces parallel to M R will be equal to A × C P that is to the force produced by the displacement M C in this direction. It is easy to see in the same way that the resultant of the three differential forces comprised in the

normal plane is equal to B × M P In fact, they are expressed by the same coefficient B multiplied by the projections of the three rectangular displacements on this plane, hence, to find their resultant consists in finding the statical resultant of these three projections considered as representing forces Now, in this point of view, the three rectangular displacements are the statical components of the displacement M C, and consequently their projections on the normal plane M S the statical components of M P, which is therefore their resultant, so that the resultant of the three differential forces contained in the normal plane is directed along M P, and represented by B × M P, that is it is equal in magnitude and in direction to the differential force arising from the displacement M C comprised in the same normal plane

Therefore, finally, we find the molecule M urged by the same differential forces, whether we make it undergo the small displacement M C, or, supposing it successively displaced in three rectangular directions and by quantities equal to the statical components of M C in these directions, we find the resultant of the forces produced by these three rectangular displacements

This principle, being true for the action excited by the molecule N, is equally so for the actions excited by all the other molecules of the medium on M, so that we may rightly pronounce that the resultant of all the small forces arising from the displacement M C, or the total action of the medium on the molecule M after its displacement, is equal to the resultant of the forces which would be separately produced by three rectangular displacements equal to the statical components of the displacement M C

Second Theorem

In any system whatever of molecules or material points in equilibrium, there exist always for each of them three rectangular directions, along which every small displacement of this point, by slightly changing the forces to which it is subject, produces a total resultant whose direction coincides with the line of displacement itself

To demonstrate this theorem, in the first place I refer the various directions of the small displacements of the molecule to three rectangular axes, arbitrarily chosen, as axes of x, y, z I suppose that the molecule is displaced successively along these three directions by the same small quantity, which I take as the

unity of these differential displacements. I call a, b, c the three components along these axes of the force excited by the displacement parallel to the axis of x, a' b' c' the three components of the force excited by the displacement parallel to y, and lastly, a'' b'' c'' the components of the force excited by the displacement parallel to z.

To obtain the force which results from a small displacement equal to unity, along any other direction whatever making angles X, Y, Z with the axes of x, y, z, we must first, in accordance with the preceding theorem, take on these axes the statical components of the displacement, which will be respectively cos X, cos Y, cos Z, and determine the forces separately produced by each of these displacements, then calculate the resultant of all these forces.

Now to obtain the components of the force produced by the displacement along the axis of x equal to cos X, we must multiply successively cos X by the coefficients a, b, c, since they represent the components of the force excited by a displacement equal to unity, and because, as we are here considering only very small variations, the forces developed are proportional to the lengths of these differential displacements; so that the components of the force resulting from the displacement cos X are

	x	y	z
parallel to	$a\cos X$,	$b\cos X$,	$c\cos X$.

Similarly, the components of the force produced by the displacement cos Y along the axis of y are

	x	y	z
parallel to	$a'\cos Y$,	$b'\cos Y$,	$c'\cos Y$.

And the components of the force excited by the displacement cos Z, which takes place along the axis of z, are

	x	y	z
parallel to	$a''\cos Z$,	$b''\cos Z$,	$c''\cos Z$.

Adding together those components whose directions are along the same axis, we have for the total components

parallel to x	$a\cos X + a'\cos Y + a''\cos Z$
parallel to y	$b\cos X + b'\cos Y + b''\cos Z$
parallel to z	$c\cos X + c'\cos Y + c''\cos Z$

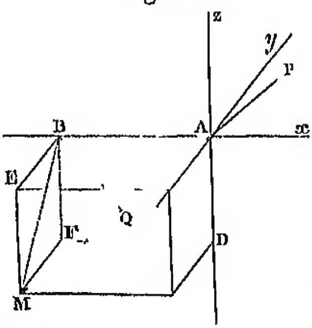
These components determine the magnitude and direction of the total resultant.

It might at first sight be thought that the nine constants $a, b, c, a' b' c', a'' b'' c''$ are independent, but it is easy to perceive that there exists amongst them a relation which reduces their number to six.

In fact, let A x, A y, A z (fig. 5.) be the three rectangular axes along which the molecule A is successively displaced by a very small quantity equal to unity; let A P be the direction on the prolongation of which is situated another material point M, which acts on A, and which I always suppose separated from this point by a quantity very great relative to the extent of the displacements.

Let us first suppose that it is displaced along the axis of x by a quantity A B equal to unity; this small displacement will cause to vary at the same time the direction and the intensity of the force exerted by the point M by bringing the other molecule nearer; if from the point B the perpendicular B Q be dropped on the direction A P M, A Q will be the variation of the distance, and B Q may be considered as proportional to the variation of the direction. The former variation will produce a differential force A × A Q along the direction A P M, and the second a differential force B × B Q in the direction B Q, the coefficients A and B remaining constant so long as we consider the action exerted by the same molecule M.

Fig. 5.



To fix the direction in which these differential forces push the point A, suppose the molecule M to exert a repulsive action on this point. The distance A M being diminished by A Q, this action is increased, and the differential A × A Q acts in the direction M A; in the same manner the differential B × B Q, resulting from the small change of direction of the force, acts in the direction Q B. If then we regard as positive the directions of action A x, A y, A z for forces parallel to the axes of coordinates, the component parallel to x of this second differential will be negative, whilst the components parallel to y and z will be positive, as well as the three rectangular components of the first differential.

Let us now seek for the components of the two differential

forces, and first those of the former $A \times AQ$. If we represent by X, Y, Z the angles made by the straight line APM with the axes of x, y and z, AB being equal to unity by hypothesis, $AQ = \cos X$, and the differential force in the direction AM is represented by $A \cos X$; its components therefore are

$$\begin{array}{llll} \text{parallel to} & x & y & z \\ & A\cos^2 X, & A\cos X\cos Y, & A\cos X\cos Z \end{array}$$

Let us now find what are the components of the second differential force $B \times BQ$ acting along BQ. Since $AB =$ unity $BQ = \sin X$, and this force is represented by $B \sin X$. I decompose it in the first place into two others in the directions, one of BA and the other of BP* perpendicular to BA: the first component, which is parallel to the axis of x, is equal to $B \sin X \times \cos ABQ$, or $-B \sin^2 X$, and the second has for its value $B \sin X \times \sin ABQ$, or $B \sin X \cos X$. I resolve this second component into two other forces in the directions EB and FB, that is parallel to the axes of y and z: the first will be equal to $B \sin X \cos X \times \dfrac{BE}{BP}$ and the second to $B \sin X \cos X \times \dfrac{BF}{BP}$; but $\dfrac{BE}{BP} = \dfrac{\cos Y}{\sin X}$ and $\dfrac{BF}{BP} = \dfrac{\cos Z}{\sin X}$; hence the values of the components parallel to y and z become respectively $B \cos X \cos Y$ and $B \cos X \cos Z$. We have then for the three components of the second differential force,

$$\begin{array}{llll} \text{parallel to} & x & y & z \\ & -B\sin^2 X, & B\cos X\cos Y, & B\cos X\cos Z \end{array}$$

Adding together the parallel components of the two differential forces, we find for the total components

$$\begin{array}{llll} \text{parallel to} & x & y & z \\ & A\cos^2 X - B\sin^2 X, & (A+B)\cos X\cos Y, & (A+B)\cos X\cos Z \end{array}$$

If we now suppose the material point A to be displaced along the axis of y by a quantity equal to unity, we shall find in the same manner the following components,

$$\begin{array}{llll} \text{parallel to} & y & x & z \\ & A\cos^2 Y - B\sin^2 Y, & (A+B)\cos X\cos Y, & (A+B)\cos Y\cos Z \end{array}$$

And for a similar displacement along the axis of z we should have

* [There is evidently a misprint of P for M in the original in this page.—TRANS.]

parallel to . . z x y

$$A\cos^2 Z - B.\sin^2 Z,\quad (A+B)\cos X.\cos Z,\quad (A+B)\cos Y.\cos Z.$$

The simple inspection of the components of the differential forces excited by these three small displacements, shows that the displacement parallel to x gives in the direction of the axis of y the same component as the displacement parallel to y produces in the direction of the axis of x, and gives in the direction of the axis of z the same component as the displacement parallel to z produces in the direction of x, and lastly, that the component parallel to z of the force excited by the displacement along the axis of y is equal to the component parallel to y of the force excited by the displacement along the axis of z; that is to say, generally, *the component parallel to one axis produced by the displacement along one of the two others is equal to that which results parallel to this latter from a similar displacement parallel to the former axis.*

This theorem being demonstrated for the individual action of each molecule M on the point A, is consequently proved also for the resultant of the actions exerted by all the molecules of a medium on the same material point; hence there exists always between the nine constants $a, b, c, a', b', c', a'', b'', c''$ the three following relations:—

$$b = a',\qquad c = a'',\qquad c' = b'';$$

which reduces the number of arbitrary constants to six.

We may then in general represent as follows the components of the three forces resulting from three small displacements equal to unity, and operated successively along the axes of x, y and z:—

For the displacement along the axis of x,—

Components . . . . . . a, h, g.
Parallel to . . . . x, y, z.

For the displacement along the axis of y,—

Components . . . . . . b, h, f.
Parallel to . . . . y, x, z.

For the displacement along the axis of z,—

Components . . . . . . c, g, f.
Parallel to . . . . z, x, y.

Thus the three components of a similar displacement in any direction whatever, making with the axes of x, y and z angles respectively equal to X, Y, Z, will be—

Parallel to x	$a\cos X + h\cos Y + g\cos Z = p$
Parallel to y	$b\cos Y + h\cos X + f\cos Z = q$
Parallel to z	$c\cos Z + g\cos X + f\cos Y = r$

I now proceed to show that there exists always a direction for which the resultant of these three components coincides with this very direction of the displacement itself, that is to say, that real values may be given to the angles X, Y, Z such that the resultant of the three components shall make with the axes of x, y and z angles respectively equal to X, Y and Z, or, in other terms, such that these three components shall be to one another in the same ratio as the quantities cos X, cos Y, cos Z.

To find the direction which satisfies this condition, I shall substitute for the three unknown quantities cos X, cos Y, cos Z (which are reduced to two by the equation

$$1 = \cos^2 X + \cos^2 Y + \cos^2 Z)$$

the tangents of the angles which the projections of the straight line on the planes xz and yz make with the axis of z, in order to be able to decide as to the reality of the angles from that of the values of the trigonometrical lines given by the calculation. Let then $x = mz$ and $y = nz$ be the equations of the straight line: we have $m = \dfrac{\cos X}{\cos Z}$, $n = \dfrac{\cos Y}{\cos Z}$; now the three above components which I shall represent by p, q and r, must be to one another in the same ratio as the quantities cos X, cos Y, cos Z, in order to satisfy the condition just mentioned.

We have therefore $\dfrac{p}{r} = \dfrac{\cos X}{\cos Z} = m$, $\dfrac{q}{r} = \dfrac{\cos Y}{\cos Z} = n$, or, putting for p, q, r their values,

$$m = \frac{a\cos X + h\cos Y + g\cos Z}{c\cos Z + g\cos X + f\cos Y} = \frac{a\dfrac{\cos X}{\cos Z} + h\dfrac{\cos Y}{\cos Z} + g}{c + g\dfrac{\cos X}{\cos Z} + f\dfrac{\cos Y}{\cos Z}}$$

And

$$n = \frac{b\cos Y + h\cos X + f\cos Z}{c\cos Z + g\cos X + f\cos Y} = \frac{b\dfrac{\cos Y}{\cos Z} + h\dfrac{\cos X}{\cos Z} + f}{c + g\dfrac{\cos X}{\cos Z} + f\dfrac{\cos Y}{\cos Z}}$$

Or lastly,

$$m = \frac{am + hn + g}{c + gm + fn}, \qquad (1)$$

and

$$n = \frac{b\,n + h\,m + f}{c + g\,m + f\,n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

From equation (2.) we get

$$m = \frac{-f n^2 + (b - c)\,n + f}{g\,n - h};$$

substituting this value of m in equation (1.), and getting rid of the denominators, we have

$$g\;[-f.n^2 + (b-c)\,n + f]^2 + fn\,(gn-h)\,[-f.n^2 + (b-c)\,n + f] + c(-a)\,(gn-h)\,[-fn^2 + (b-c)n + f] - hn(gn-h)^2 - g(gn-h)^2 = 0.$$

This equation in (n), which under this form appears of the fourth degree, falls to the third on effecting the multiplications, because then the two terms containing (n^4) mutually destroy each other, hence we are sure that it has at least one real root. There is therefore always one real value of (n), and consequently one real value of (m). Consequently there is always at least one straight line which satisfies the condition that a small displacement of a material point along this straight line gives rise to a repulsive force—the general resultant of the molecular actions—the direction of which coincides with that of the displacement.

To those directions which possess this property we give the name of *Axes of Elasticity.*

Proceeding from this result, it is easy to prove that there are still two other axes of elasticity perpendicular to one another and to the former. In fact, take this last-mentioned one for axis of x, the components parallel to y and z, produced by a displacement in the direction of the axis x, will be nothing; so that we shall have $g = 0$, $h = 0$; and the equations (1.) and (2.) become

$$m\,(c - a + f n) = 0,$$

and

$$n^2 - \left(\frac{b - c}{f}\right) n - 1 = 0.$$

The former equation gives $m = 0$; and the second gives for (n) two values which are always real, the last term ($-$ 1) being a negative quantity. Hence we see that besides the axis of x there are two other axes of elasticity; they are perpendicular to the axis of x, since for both one and the other $m = 0$, that is to say their projections on the plane $x\,z$ coincide with the axis of z,

they are moreover perpendicular to each other, for the product of the two values of (n) when multiplied by each other is equal to the last term (-1) of the second equation. *Therefore there exist always three rectangular axes of elasticity for every material point in any molecular system whatever, and whatever may be the laws and the nature of the actions which these material points exert on each other.*

If we suppose that in a homogeneous medium the corresponding faces of the particles or the homologous lines of the molecular groups are all parallel to each other, the three axes of elasticity for each material point will have the same direction throughout the whole extent of the medium. This is the most simple case of a regular arrangement of molecules, and that which seemingly should be always exhibited by crystallized substances according to the idea one forms of regular crystallization; nevertheless the needles of rock crystal present optical phænomena which show that this condition of parallelism of homologous lines is not always rigorously fulfilled by it. It is in fact conceivable that there may be without this condition many different sorts of regular arrangements; but as yet I have sought only the mathematical laws of double refraction on the supposition that the axes of elasticity have the same direction throughout the whole extent of the vibrating medium, and consequently shall confine myself to the consideration of this particular case, the most simple of all, and which appears to be that of the greater number of crystallized substances; for as yet rock crystal is, I believe, the only known exception to this rule.

Application of the preceding Theorems to the complex displacement of the Vibrating Molecules which constitutes Luminous Waves.

Hitherto we have only considered the displacement of a material point, supposing all the other molecules immoveable; we have been allowed to suppose, without altering the problem in any way, that it is the medium which displaces itself and the material point alone which remains fixed. But the relative displacements of the molecules in which consist the vibrations of luminous waves are more complex. Let us first consider the most simple case, that of an indefinite plane wave. All the molecules comprised in the same plane parallel to the surface of the wave, have remained in the same positions relative to each other,

but they have been displaced relative to the rest of the vibrating medium, or if you like, it is this medium which has been displaced relative to them, but not by the same quantity for the different strata or layers of molecules; the neighbouring stratum is the least displaced, and the molecules of the succeeding strata are found so much the more displaced from their positions corresponding to those of the molecules comprised in the first plane, as they are further off from it. If we consider all the molecules which were originally situated on the same straight line perpendicular to this plane or to the surface of the wave, they will be found transported, in consequence of the vibratory movement, along a "*sinusoidal*" curve on one side and the other of this perpendicular, which will be the axis of the curve; its ordinates parallel to the wave, that is to say the small displacements of the molecules, will be proportional to the sines of the corresponding abscissæ; such at least will be the nature of this curve in all cases where the illuminating particle which has produced the waves, having been slightly displaced from its position of equilibrium, is brought back to it by a force proportional to the displacement Confining ourselves then to the hypothesis of small movements, we may represent the absolute velocity which animates an æthereal molecule after a time (t) by the formula $u = a \cdot \sin 2\pi\left(t - \frac{x}{\lambda}\right)$, in which (u) represents this velocity, (a) a constant coefficient which depends on the energy of the vibrations, (2π) the circumference to radius unity, (x) the distance of the molecule from the luminous point, (λ) the length of an undulation, and (t) the time elapsed since the origin of the motion. If we suppose that these plane and indefinite waves are totally reflected at a plane parallel to their surface, that is to say that on this plane the æthereal molecules are restrained to remain completely immovable, then the reflected waves will have the same intensity as the incident waves, to which they will moreover be parallel; so that the same coefficient (a) must be employed in expressing the absolute velocities caused by these waves in the æthereal molecules. Calling (z) the distance of the direct wave from the reflecting plane, and (c) the constant distance of this plane from the source of movement, the space described by the direct wave is $(c - z)$, and the space described by the reflected wave which comes to meet it is $(c + z)$. Hence the velocities, brought in the same time and to the same point

of the æther by the direct and the reflected waves, are respectively equal to $a \sin 2\pi\left(t - \frac{c}{\lambda} + \frac{z}{\lambda}\right)$ and to $-a \sin 2\pi\left(t - \frac{c}{\lambda} - \frac{z}{\lambda}\right)$. This second expression has necessarily the negative sign, since the ætherial molecules remaining immovable against the reflecting plane, the luminous vibrations also change their sign by reflexion. Consequently the absolute velocity resulting from the superposition of the direct and reflected wave is at the instant (t)

$$a\left[\sin 2\pi\left(t - \frac{c}{\lambda} + \frac{z}{\lambda}\right) - \sin 2\pi\left(t - \frac{c}{\lambda} - \frac{z}{\lambda}\right)\right],$$

which expression may be put under the form

$$2a \sin 2\pi\left(\frac{z}{\lambda}\right) \cos 2\pi\left(t - \frac{c}{\lambda}\right).$$

Such is the general expression of the absolute velocity which animates at the instant (t) an ætherial molecule situated at a distance (z) from the reflecting plane. It teaches us, in the first place, that at certain distances from this plane, for which $\sin 2\pi\left(\frac{z}{\lambda}\right) = 0$, the ætherial molecules remain constantly at rest; now $\sin 2\pi\left(\frac{z}{\lambda}\right)$ becomes nothing when $z = 0$, or a whole number of times $\frac{1}{2}\lambda$, hence the nodal planes, that is the planes of rest, are separated from each other and from the reflecting surface by intervals equal to $\frac{1}{2}\lambda$. On the contrary, the bellyings, that is the points where the vibrations have the greatest amplitude, have intermediate positions and at equal distances from the nodal planes: in fact, $\sin 2\pi\left(\frac{z}{\lambda}\right)$ attains its maximum when (z) is equal to an uneven number of times $\frac{1}{4}\lambda$.

The above formula may be used also to represent the molecular displacements by merely changing (t) into $(t - 90^{\circ})$, or $\cos 2\pi\left(t - \frac{c}{\lambda}\right)$ into $\sin 2\pi\left(t - \frac{c}{\lambda}\right)$, it becomes then

$$y = 2b \sin 2\pi\left(\frac{z}{\lambda}\right) \sin 2\pi\left(t - \frac{c}{\lambda}\right).$$

If (y) be taken as the ordinate corresponding to the abscissa (z),

we see that the curve represented by this equation always cuts the axis of (z) in the same points for every instant (t), that these are the points for which

$$z = 0,\ z = \frac{1}{2}\lambda,\ z = \lambda,\ z = \frac{3}{2}\lambda,\ \&c.$$

The greatest displacements of the molecules, or the greatest values of y, correspond, on the contrary, to the values of z, which contain $\frac{1}{4}\lambda$ an uneven number of times. Considering now the changes undergone by the curve from one moment to another, consequent on the different values of t, we see that the ordinates always preserve the same proportion to each other as in the oscillations of a vibrating cord, and the preceding formula shows that the velocities which animate the molecules at each instant follow also the same law as those of the elements of a vibrating cord We may therefore assimilate each portion of the medium comprised between two consecutive nodal planes to an assemblage of vibrating cords perpendicular to these planes, and attached to them by their extremities; the tension of these cords would produce the same effect as the elasticity of the medium, since like this latter it would incessantly tend to restore the straight lines which had become curved by the small displacements of the molecules perpendicular to these lines, and that with a force proportional to the angle of contingence. Hence, since the direction of the oscillatory movements, their law and that of the accelerating forces, are the same in the two cases, the rules which apply to the one necessarily apply to the other. Now, we know that in order for a cord to execute always its vibrations in the same time, when its tension varies it is necessary that its length increase proportionally to the square root of its tension; therefore the length of the same luminous waves (which must remain isochronous in all mediums which they traverse) is proportional to the square root of the elasticity which urges the molecules of the vibrating medium parallel to their surface; hence the velocity of propagation of these waves, *measured perpendicularly to their surface*, is proportional to the square root of this same elasticity.

Without recurring to the known laws of the oscillations of vibrating cords, it is easy to demonstrate directly, by geometrical considerations, the principle just announced.

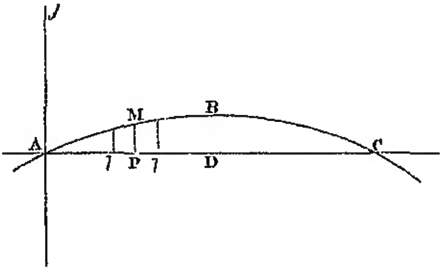
Let A B C (fig. 6) be the curve formed by a row of molecules

of the vibrating medium, which were originally situated on the straight line A D C, this curve may be represented, as we have seen, by the equation

$$y=2\,b\ \sin 2\pi\left(\frac{z}{\lambda}\right)\ \sin 2\pi\left(t-\frac{c}{\lambda}\right)$$

which becomes $y=2\,b\ \sin 2\pi\left(\frac{z}{\lambda}\right)$ when the molecules arrive at the limit of their oscillation, at this moment their velocity is nothing and we may consider it as the origin of motion for the ensuing oscillation which must result from the accelerating forces tending to bring back the molecules into their relative positions of equilibrium Let m and m' be two material points very near to and equally distant from the molecule M, denote by $d\,a$ the constant length of the interval p P or Pp', comprised between two consecutive ordinates The difference between the ordinates M P and $m'\,p'$ is the quantity by which the point M is displaced from its primitive position *relatively* to the molecules comprised in the plane drawn through m' perpendicularly to the axis A C of the curve, hence the accelerating force exerted on M by this stratum of the medium in consequence of this displacement, is proportional to $m'\,p'$—M$'$ P If we consider the molecules comprised in the plane passing through the point m and perpendicular to A C, their action on M resulting from their relative displacement will also be proportional to the extent of this displacement M P — $m\,p$ but will act in the contrary direction to that of the other accelerating force, so that the resulting action of these two equidistant strata on the molecule M will be proportional to the difference of the two relative displacements, or to $d^2 y$, if the distance Mp or Mp' is very small with regard to the length of an undulation*

Fig 6



* In the note on the dispersion of light placed at the end of the first part of this memoir I have examined the mechanical consequences which result from the supposition that the mutual action of the molecules one on the other extends to sensible distances relative to the length of an undulation for the present I confine myself here to the more simple case treated by geometers who have

On differentiating twice the value of y, we find

$$d^2 y = -8 b \frac{\pi^2}{\lambda^2} \cdot \sin\left(2\pi \frac{z}{\lambda}\right) d z^2.$$

Hence the accelerating forces, and consequently the velocities impressed at each point of the curve A B C, at the instant when the oscillation recommences, are proportional to the corresponding ordinates, therefore the small spaces described during the first instant will also be in the same ratio and will not alter the nature of the curve; hence, after the first instant dt the new accelerating forces will still be proportional to the corresponding ordinates, and since the acquired velocities are so likewise, the spaces described during the second instant will still preserve amongst each other the same ratio. The same will hold true after the third, fourth instant, &c. Consequently all the points of the curve A M C will arrive at the straight line A D C together, from which they will afterwards deviate by quantities equal to those of their primitive deviation, to re-commence afterwards an oscillation in the contrary direction. We see that the law of these vibrations will be similar to that of the small oscillations of a pendulum, since the accelerating force which urges each material point is always proportional to the space which remains for it to describe in order to arrive at its position of equilibrium. Hence the duration of the vibrations will be in the inverse ratio of the square root of the elasticity of the medium, an elasticity which is measured, in the case we are considering, by the energy of the force resulting from the relative displacements of the parallel strata of the medium, supposing them equal to a small constant quantity taken for unity.

It is easy to see also that the duration of the oscillations of the point M will be proportional to the length (λ) of an undulation. In fact, to compare the durations of an oscillation corresponding to different values of (λ), we must always suppose $d z$ constant, in order that, the distances being the same, the molecular actions and the masses to be moved may be similar on one part and on the other. On substituting for $\sin\left(2\pi \cdot \frac{z}{\lambda}\right)$ its value, in the expression for $d^2 y$, we have

always supposed the sphere of activity of the elastic force to be infinitely small with regard to the extent of the disturbance [No such note to the memoir.—TRANSLATOR.]

$$d^2y = -4\,\frac{\pi^2}{\lambda^2}\,y\,d$$

For one and the same degree of elasticity of the vibrating medium, d^2y measures the energy of the force which tends to bring back the point M to P, and (y) is the space which this point must describe. Hence for equal displacements of the point M, the accelerating force is proportional to $\frac{1}{\lambda^2}$; therefore the duration of its oscillation will be proportional to λ. Consequently the duration of the vibrations of the assemblage of particles represented by the curve A B C (*concamerations* (is proportional to $\frac{\lambda}{\sqrt{\varepsilon}}$, denoting by ($\varepsilon$) the elasticity of the medium. Now as this duration must remain constant for the same luminous waves, whatever medium they traverse, it is necessary that the length of an undulation (λ) or the velocity of propagation be proportional to the square root of the elasticity put in play. It is sufficient therefore to determine the law according to which this elasticity varies in one and the same medium, to know all the velocities of propagation with which light may be affected in it.

The law which I have found for the case where the axes of elasticity have parallel directions throughout the whole extent of the medium, is founded on the theorems of general statics which have been demonstrated, and on the following principle:—*The elasticity put into play by the relative displacements of molecules remains always the same in the same medium, so long as the direction of these displacements does not change, and whatever moreover may be that of the plane of the wave.* I shall now endeavour to give the theoretical reason of this principle, the accuracy of which I have moreover verified by very precise experiments.

The elasticity put into play by Luminous Vibrations depends solely on their direction, and not on that of the Waves.

Let us consider the molecules comprised in one and the same plane parallel to the surface of the wave: they preserve always the same relative positions, and the resultant of all their actions upon one of their number does not tend to impress on it any movement. The same is not the case for the action of the next stratum of the medium on this molecule, which being no longer in its primitive position of equilibrium with regard to it, exerts upon it a small action parallel to the plane of the wave. Con-

tinuing to subdivide in this way the vibrating medium by parallel planes infinitely near and equidistant, in proportion as they are further off from the first, the molecules which they contain are found further removed from their original position relatively to the material point which we are considering; but this effect is more than counterbalanced by the enfeebling of the forces resulting from the increase of distance, and it ceases to be sensible at a certain distance, which, without being probably altogether to be neglected with regard to the length of an undulation, can only be but a very small fraction of it. Whatever be the law according to which the molecular forces vary with the distance, it is natural to suppose that this law remains the same for the same medium in all directions. I do not mean by this to say that the molecules situated at the same distance from the material point exert upon it in all directions equal repulsions; but only that these repulsions, though unequal, vary in the same manner with the distance.

Admitting this hypothesis, which is very probable from its simplicity, we may conclude from it, I think, that the elasticity put into play by the small displacements of the molecules does not change so long as the direction and the extent of these displacements remain the same at the same distance from the plane of the wave, whatever besides may be the direction of this plane.

Suppose, in fact, that the molecular displacements are always parallel to the same direction, and consider two different planes drawn through this direction, which shall represent successively the surface of the wave in two different situations. Subdivide the vibrating medium into infinitely thin and equidistant strata, first parallel to the former plane, and afterwards parallel to the second; call δ the small quantity by which the second stratum or the second row of molecules becomes displaced relative to that which is contained in the plane of departure; the molecules originally situated on straight lines perpendicular to this plane, now form curved lines in consequence of the undulatory movement; and the displacements are sensibly proportional to the squares of the distances from the plane of departure in those strata sufficiently near to exert an appreciable action. Hence 4δ will be the quantity by which the molecules of the third row will become displaced relatively to those of the plane of departure; and in the same way 9δ, 16δ, &c. will be the relative displacements of the succeeding strata. Similar displace-

ments, be it understood, are supposed on the other side of the plane.

If all these displacements instead of increasing with the distance, were equal to δ, the elasticity put in play would be the same as in the case where the medium remaining immovable the molecules only comprised in this plane had slided by the small quantity δ. It will be moreover remarked that if there were only one of these molecules displaced from its position of equilibrium the direction of the plane in question would have no influence on the force to which it would be subject.

Call this force F, it is the sum of the actions exerted on the molecule remaining fixed by all the strata of the medium. Now, to pass from this case to that with which we occupied ourselves in the first place, it would be necessary to multiply the action of the first stratum by zero that of the second by 1, that of the third by 4, that of the fourth by 9, &c. Since in this case the first stratum has not changed its position, the second is displaced by the quantity δ the third by 4δ instead of δ, the fourth by 9δ and so on we should have besides, the same progression whatever were the direction of the plane of the wave. Hence we must always multiply the individual actions of the strata situated in the same rank by the same numbers in order to take into account the extent of their displacements; moreover, the coefficients, which depend on the distance of each stratum from the fixed molecule, will also be the same at equal distances, supposing, as we have done the molecular actions to diminish in all directions according to the same function of the distances; consequently the total numerical series by which F must be multiplied to obtain the elastic force which results from the undulatory movement will remain constant for the different directions of the parallel strata, or of the plane of the wave and this force will depend only on the mere direction of the molecular displacements.

Application of the preceding principles to media where the Axes of Elasticity preserve the same direction throughout their whole extent.

If this principle be admitted, the theoretical probability of which I have just shown, and whose accuracy I have besides verified by very precise experiments on the velocities of light in

topaz, it becomes easy to compare the elasticities put into play by two vibratory movements which have different directions, and belong to two systems of luminous waves making any angle with each other. For this it is sufficient to compare in the first place the elasticity put into play by the former system with the elasticity put into play by vibrations whose directions are always in its plane, but parallel to the intersection of the planes of the two systems of waves, then, changing the plane of the waves without changing the direction of these new displacements, we shall compare in the plane of the second system of waves the elasticity which they develope with that excited by the vibrations of this second system In one word, the variations of inclination of the surface of the waves relatively to the axes of the vibrating medium, causing no change in the elastic force so long as the direction of the molecular displacements remains the same, the problem always reduces itself to the comparison of the elasticities put in play by two systems of waves whose surfaces are parallel, and whose vibrations make with each other any angle whatever. Now, the elasticities excited by two systems of similar waves which coincide as to their surfaces, but whose vibrations are performed in different directions, are evidently to each other as the forces produced by the successive displacements of a single molecule along the former and the latter direction. In fact, consider the stratum situated in the primitive position of equilibrium, and with regard to which the parallel strata have been displaced, in both cases it is the same strata of the medium which have become displaced and by equal quantities, but according to two different directions. Now, on considering these two modes of displacement, we may apply to the influence excited on each molecule of the immovable stratum by one of the other strata, the theorems we have demonstrated for the action of any molecular system whatever on a material point which has been slightly disturbed from its original position, since this is equivalent to leaving this point fixed and displacing all the other molecules of the system by the same quantity. Thus we may calculate and compare, according to these theorems, the actions excited by any stratum on the fixed stratum; and the actions of the other strata will be in the same ratio, since their displacements are supposed equal in the two cases. Consequently the elasticities put into play by the two undulatory movements are to each other as the elasticities which would be excited by the

two successive displacements of a single molecule along similar directions and we may apply to the complex displacements resulting from luminous waves the principles before demonstrated for the case where one molecule is disturbed from its position of equilibrium, whilst all the others remain fixed

This being established, let us take the three axes of elasticity of the vibrating medium as coordinate axes, and denote by a^2, b^2, c^2 the elasticities put into play by vibrations parallel to the axes of x, y, z, so that the corresponding velocities of propagation, which are proportional to the square roots of the elasticities, are represented by a b c we propose to determine the elastic force resulting from vibrations of the same nature but parallel to any other direction whatever making with these axes the angles X, Y, Z I take as unity the amplitude of these vibrations, or the constant coefficient of the relative displacements of the parallel strata of the medium, for in order to compare the elasticities, it is necessary to compare the forces resulting from equal displacements This coefficient being equal to 1, those of the components parallel to x y and z will be cos X, cos Y cos Z We know besides that these forces will have the same directions, according to the characteristic property of axes of elasticity

Hence, denoting by (f) the resultant of these three forces, we shall have

$$f = \sqrt{a^4 \cos^2 X + b^4 \cos^2 Y + c^4 \cos^2 Z}$$

and the cosines of the angles which this resultant makes with the axes of x, y, z, will be respectively equal to

$$\frac{a^2 \cos X}{f}, \quad \frac{b^2 \cos Y}{f}, \quad \frac{c^2 \cos Z}{f}$$

We see that in general this resultant has not the same direction as the displacements which have produced it But we can always decompose it into two other forces, one parallel and the other perpendicular to the direction of the displacements If the second force be found at the same time normal to the plane of the wave, it will no longer have any influence on the propagation of luminous vibrations since according to our fundamental hypothesis, the luminous vibrations are performed *solely* in the direction of the surface of the waves Now we shall take care to reduce to this case all calculations relative to the velocities of

propagation; for this reason we shall now confine ourselves to the determination of the component parallel to the displacements.

The angles which this direction makes with the axes are X, Y, Z; the cosines of the angles which the same axes make with the resultant are

$$\frac{a^2 \cos X}{f}, \quad \frac{b^2 \cos Y}{f}, \quad \frac{c^2 . \cos Z}{f},$$

consequently the cosine of the angle which this resultant makes with the direction of displacement is equal to

$$\frac{a^2 \cos^2 X + b^2 . \cos^2 Y + c^2 . \cos^2 Z}{f}.$$

Now this cosine must be multiplied by the force f to obtain its component parallel to this direction; the component sought is therefore equal to

$$a^2 . \cos^2 X + b^2 \cos^2 Y + c^2 . \cos^2 Z.$$

If we denote by v^2 this component of the elastic force, in order that the corresponding velocity of propagation may be represented by v, we shall have

$$v^2 = a^2 . \cos^2 X + b^2 . \cos^2 Y + c^2 . \cos^2 Z.$$

Surface of Elasticity, which represents the Law of the Elasticities and of the Velocities of Propagation.

I shall suppose a surface to be constructed according to this equation, each radius vector of which, making angles equal to X, Y, Z with the axes of x, y, z, has for its length the value of v: we may call it the *surface of elasticity*, since the squares of its *radii vectores* will give the components of the elastic force in the direction of each displacement.

If we conceive a system of luminous waves (always supposed plane and indefinite) which are propagated in the medium whose law of elasticity is represented by this surface, and draw through its centre a plane parallel to the waves, every component perpendicular to this plane must be considered as having no influence on the velocity of propagation of the luminous waves. The elastic force excited by displacements parallel to one of the radii vectores of this diametral section, may always be decomposed into two other forces, one parallel and the other perpen-

dicular to the radius vector, the former is represented in magnitude by the square of the length of this radius vector itself; the second, not being perpendicular to the plane of the diametral section except for two particular positions, may be generally decomposed into two other forces, one comprised in this plane and the other normal to the plane: this latter, as we have said, exerts no influence on the propagation of the luminous waves; but it is not so for the other component, which must be combined with the first component parallel to the radius vector to obtain the whole elastic force excited in the plane of the waves.

It will be remarked that in this general case, the elastic force which propagates the waves will not be parallel to the displacements which have produced it: whence would result, in the vibrations which pass from one stratum to another, a gradual change of their direction, and consequently of the intensity of the elastic force which they put in play, which would render very difficult the calculation of their propagation, and would prevent the application to it of the ordinary law, according to which the velocity of propagation is proportional to the square root of the elasticity put in play: a law whose applicability we have shown only for the particular case where the direction of the vibrations and the elasticity remain constant from one stratum to another.

But there exist always in each plane two rectangular directions, such that the elastic forces excited by displacements parallel to each of them being decomposed into two other forces, one parallel, the other perpendicular to this direction, the second component is found perpendicular to the plane; so that the vibrations are propagated solely by an elastic force parallel to the primitive displacements, which therefore preserves the same direction and the same intensity during their transit. Now whatever be the direction of the incident vibrations, they may always be decomposed along these two rectangular directions in the diametral plane parallel to the waves, and thus reduce the problem of their path to the calculation of the velocities of propagation of vibrations parallel to these two directions: a calculation easy to make according to the principle that the velocities of propagation are proportional to the square roots of the elasticities put into play, which principle then becomes rigorously applicable.

The small displacements parallel to the axes of any diametral section whatever of the surface of elasticity, do not tend to separate the molecules of the succeeding strata from the normal plane drawn through their direction.

I shall now demonstrate that the greatest and least radius vector, or the two axes of the diametral section, possess the property just announced; that is to say, the displacements along each of these two axes excite elastic forces, the component of which perpendicular to their direction is found at the same time perpendicular to the plane of the diametral section.

In fact, let $x = \mathrm{B}\,y + \mathrm{C}\,z$ be the equation of the cutting plane passing through the centre of the surface of elasticity; the equation of condition which expresses that this plane contains the radius vector, whose inclinations to the axes of x, y, z are respectively X, Y, Z, is

$$\cos \mathrm{X} = \mathrm{B}\,.\cos \mathrm{Y} + \mathrm{C}\ \ \cos \mathrm{Z}.$$

We have, besides, between the angles X, Y, Z, the relation

$$\cos^2 \mathrm{X} + \cos^2 \mathrm{Y} + \cos^2 \mathrm{Z} = 1;$$

and for the equation of the surface of elasticity,

$$v^2 = a^2 \cos^2 \mathrm{X} + b^2\,.\cos^2 \mathrm{Y} + c^2\,.\cos^2 \mathrm{Z}.$$

The radius vector (v) attains its maximum or its minimum when its differential becomes nothing; we have therefore in this case, differentiating the equation of the surface with respect to the angle X,

$$0 = a^2\,.\cos \mathrm{X} \sin \mathrm{X} + b^2\,.\cos \mathrm{Y} \sin \mathrm{Y}\,.\frac{d\mathrm{Y}}{d\mathrm{X}} + c^2.\cos \mathrm{Z}.\sin \mathrm{Z}\,.\frac{d\mathrm{Z}}{d\mathrm{X}}.$$

If we differentiate similarly the two preceding equations, we have

$$\cos \mathrm{X} \sin \mathrm{X} + \cos \mathrm{Y} \sin \mathrm{Y}\,.\frac{d\mathrm{Y}}{d\mathrm{X}} + \cos \mathrm{Z}.\sin \mathrm{Z}.\frac{d\mathrm{Z}}{d\mathrm{X}} = 0,$$

$$-\sin \mathrm{X} + \mathrm{B} \sin \mathrm{Y}\,.\frac{d\mathrm{Y}}{d\mathrm{X}} + \mathrm{C}.\sin \mathrm{Z}.\frac{d\mathrm{Z}}{d\mathrm{X}} = 0;$$

whence we obtain for $\frac{d\mathrm{Y}}{d\mathrm{X}}$ and $\frac{d\mathrm{Z}}{d\mathrm{X}}$ the following values:

$$\frac{d\mathrm{Y}}{d\mathrm{X}} = \frac{\sin \mathrm{X}\ (\mathrm{C} \cos \mathrm{X} + \cos \mathrm{Z})}{\sin \mathrm{Y}\ (\mathrm{B} \cos \mathrm{Z} - \mathrm{C} \cos \mathrm{Y})},$$

and

$$\frac{d\mathrm{Z}}{d\mathrm{X}} = \frac{-\sin \mathrm{X}\ (\mathrm{B} \cos \mathrm{X} + \cos \mathrm{Y})}{\sin \mathrm{Z}\ (\mathrm{B} \cos \mathrm{Z} - \mathrm{C}.\cos \mathrm{Y})}.$$

Substituting these two values in the first differential equation, which expresses the common condition for a maximum or for a minimum, we find for the equation determining the direction of the axes of the diametral section,

$$a^2 \cos X (B \cos Z - C \cos Y) + b^2 \cos Y (C \cos X + \cos Z)$$
$$- c^2 \cos Z (B \cos X + \cos Y) = 0 \qquad (A)$$

Let us now conceive a plane drawn through the radius vector, and the accelerating force developed by the displacements parallel to the radius vector; it is in this plane that we shall decompose this force into two others, the former in the direction of the radius vector, the second perpendicular to it and if this plane is perpendicular to the cutting plane, it is clear that the second component will be normal to this latter We proceed now to find the equation which expresses that these two planes are at right angles to each other and if it agrees with equation (A), we may conclude from it that the axes of the diametral section are precisely the two directions which satisfy the condition that the component perpendicular to the radius vector be at the same time perpendicular to the cutting plane

Let $x = B'y + C'z$ be the equation of the plane drawn through the radius vector, and the direction of the elastic force developed by the vibrations parallel to the radius vector The cosines of the angles made by this force with the three axes of coordinates are

$$\frac{a^2 \cos X}{f}, \qquad \frac{b^2 \cos Y}{f}, \qquad \frac{c^2 \cos Z}{f},$$

and since it is contained in the plane $x = B'Y + C'Z$, we have

$$\frac{a^2 \cos X}{f} = B' \frac{b^2 \cos Y}{f} + C' \frac{c^2 \cos Z}{f} \qquad \text{or}$$

$$a^2 \cos X = B' b^2 \cos Y + C' c^2 \cos Z$$

This plane containing the radius vector, we have, similarly,

$$\cos X = B' \cos Y + C' \cos Z$$

From these two equations we obtain

$$B' = \frac{(a^2 - c^2) \cos X}{(b^2 - c^2) \cos Y} \text{ and } C' = -\frac{(a^2 - b^2) \cos X}{(b^2 - c^2) \cos Z}$$

Substituting these values of B' and C' in the equation

$$BB' + CC' + 1 = 0,$$

which expresses that the second plane is perpendicular to the first, we find

$$B\,(a^2 - c^2)\cos X \cos Z - C\,(a^2 - b^2)\cos X \cos Y + (b^2 - c^2)\cos Y \cos Z = 0,$$

a relation similar to that of equation (A), which determines the direction of the axes of the diametral section, as may be easily seen by effecting the multiplications. Therefore the directions of these two axes do in reality possess the property announced; whence it results that the parallel vibrations preserving always the same direction, have a velocity of propagation proportional to the square root of the elasticity put into play, a velocity which may then be represented by the radius vector (v).

Determination of the Velocity of Propagation of plane and indefinite Waves.

By the aid of this principle and of the equation of the surface of elasticity, whenever the three semi-axes a, b, c are known, it will be easy to determine the velocity of propagation of plane and indefinite waves whose direction is given. To this end, in the first place, a plane parallel to the waves is to be drawn through the centre of the surface of elasticity, and their vibratory motion decomposed into two others in the directions of the greatest and least axis of this diametral section. If we denote by (α) the angle made by the incident vibrations with the former of these axes, $\cos \alpha$ and $\sin \alpha$ will represent the relative intensities of the two components; and their velocities of propagation, measured perpendicularly to the waves, will be respectively equal to half of the semi-axis of the diametral section to which the vibrations are parallel. These two semi-axes being in general unequal, the two systems of waves will traverse the medium with different velocities, and will cease to be parallel on emerging from the refracting medium if the surface of emergence is oblique to that of the waves, so that the difference of velocities causes a difference of refraction. With regard to the planes of polarization of the two divergent beams, they will be perpendicular to each other, since their vibrations are at right angles to each other.

There are two diametral planes which cut the Surface of Elasticity in circles.

It is to be remarked that the surface

$$v^2 = a^2 . \cos^2 X + b^2 . \cos^2 Y + c^2 . \cos^2 Z,$$

which represents the laws of elasticity of every medium whose molecular groups have their axes of elasticity parallel, may be cut in two circles by two planes drawn through its mean axis, and equally inclined to each of the other two axes. In fact, replace the polar coordinates by rectangular ones in this equation, which then becomes

$$(x^2 + y^2 + z^2)^2 = a^2 x^2 + b^2 y^2 + c^2 z^2,$$

the circular section made in this surface may always be considered as belonging at the same time to the surface of a sphere $x^2 + y^2 + z^2 = r^2$; its circumference therefore will be found at the same time in the cutting plane $z = Ax + By$, on the surface of the sphere and on the surface of elasticity. Combining the equations of these two surfaces gives

$$r^4 = a^2 x^2 + b^2 y^2 + c^2 z^2,$$

substituting in this equation the value of z obtained from the equation of the cutting plane, we have

$$x^2 (a^2 + A^2 c^2) + y^2 (b^2 + B^2 c^2) + 2AB\,c^2\,xy = r^4 \qquad (1)$$

On substituting this value of z in the equation of the sphere, we find for the projection of the same curve on the same plane of xy,

$$x^2 (1 + A^2) + y^2 (1 + B^2) + 2AB\,xy = r^2 \qquad (2)$$

Since the two equations (1) and (2) must be identical, we have

$$\frac{1+B^2}{1+A^2} = \frac{b^2 + B^2 c^2}{a^2 + A^2 c^2}, \quad \frac{2AB}{1+A^2} = \frac{2AB\,c^2}{a^2 + A^2 c^2}, \quad \frac{r^2}{1+A^2} = \frac{r^4}{a^2 + A^2 c^2}.$$

The second condition can be satisfied only by $A = 0$ or $B = 0$, since otherwise it would be necessary to make $c^2 + A^2 c^2 = a^2 + A^2 c^2$, or $a = c$, constant quantities of which we cannot dispose. If we suppose $A = 0$, we obtain from the first equation of condition $B = \pm\sqrt{\dfrac{a^2 - b^2}{c^2 - b^2}}$, an imaginary quantity if (b) be the middle axis, since in that case the two terms of the fraction placed under the radical are of different signs. Hence if we suppose $a > b$ and $b > c$, we must make $B = 0$, whence we obtain for A the real value $A = \pm\sqrt{\dfrac{a^2 - b^2}{b^2 - c^2}}$.

$B = 0$ indicates that the cutting plane must pass through the axis of y, or the mean axis of the surface of elasticity, the two equal values with contrary signs which we find for A, that is to

say, for the tangent of the angle which this plane makes with the axis of x, show that there are two planes equally inclined to the plane of $x y$, which satisfy the condition of cutting the surface of elasticity in a circle, and that there are only these two planes Every other diametral section has therefore two unequal axes, so that the waves which are parallel to it may traverse the same medium with two different velocities, according as their vibrations have the direction of one or the other of these axes.

The Double Refraction becomes nothing for Waves parallel to the two circular sections of the Surface of Elasticity.

On the contrary, waves parallel to the circular sections must always have the same velocity of propagation in whatever direction their vibrations be performed, since the radii vectores of each section are all equal to each other; and, moreover, their vibrations cannot undergo any deviation in passing from one stratum to another, because the component perpendicular to each of these radii vectores is at the same time perpendicular to the plane of the circular section; for we have demonstrated by the preceding calculations that this condition was fulfilled when the differential of the radius vector became equal to zero. Now this is what takes place for all the radii vectores of the circular sections, since their length is a constant quantity. Consequently, if a crystal be cut parallel to each of the circular sections of the surface of elasticity, and if we introduce into it perpendicularly to these faces rays polarized in any azimuth whatever, they will not undergo in the crystal either double refraction or deviation of their plane of polarization. Hence these two directions will possess the properties of what have been improperly called the *axes of the crystal*, and which I shall name *optic axes*, to distinguish them from the three rectangular axes of elasticity, which ought, in my opinion, to be considered as the true axes of the doubly-refracting medium

There are never more than two optic axes in refracting media whose axes of elasticity have everywhere the same direction.

A remarkable consequence of the calculation which we have made is, that a body constituted as we suppose it to be, that is whose particles are arranged in such a manner that the axes of elasticity for each point of the vibrating medium are parallel throughout its whole extent, cannot have more than two optic

axes. They are reduced to one only when two of the semi-axes a, b, c of the surface of elasticity are equal to each other: when $a = b$, for example, $\Lambda = 0$, the two circular sections coincide with the plane of $x y$, and the two optic axes which are perpendicular to them coincide with the axis of z, or the axis (c) of the surface of elasticity, which becomes then a surface of revolution.

This is the case of those crystals which are designated by the name of *uni-axal* crystals, such as calcareous spar. When the three axes of elasticity are equal to each other, the equation of the surface of elasticity becomes that of a sphere; the forces no longer vary with the direction of the molecular displacements, and the vibrating medium no longer possesses the property of double refraction. This is what appears to be the case in all bodies crystallizing in cubes.

As yet we have calculated only the velocity of propagation of luminous waves measured perpendicularly to their tangent plane, without seeking to determine the form of the waves in the interior of the crystal and the inclination of the rays to their surface. Whilst it is sought only to calculate the effects of the double refraction for incident waves which are sensibly plane, that is to say, which emanate from a luminous point sufficiently far off, it is sufficient to determine the relative directions of the plane of the wave within and without the crystal; since we thus find the angle which the emergent wave makes with the incident wave, and consequently the mutual inclination of the two lines along which the visual ray or the axis of a telescope must be successively directed, in order to obtain the line of sight (*voir le point de mire*), first directly, and then across the prism of crystal. I say the *prism*, for if the plate of crystal had its faces parallel, the emergent wave would be parallel to the incident wave in the case we are considering, where the luminous point is supposed at an infinite distance, whatever in other respects might be the energy of the double refraction and the law of the velocities of propagation in the interior of the crystal.

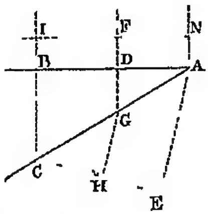
There cannot therefore be any sensible angular separation of the ordinary and extraordinary images in this case, except in as far as the crystallized plate is prismatic; and to calculate the angles of deviation of the ordinary and extraordinary beams, which by their difference give the angle of divergence of the two images, it is sufficient to determine the velocity of propagation

of each system of waves in the crystal from the direction of its plane relatively to the axes.

Demonstration of the Law of Refraction for plane and indefinite Waves.

Let, for example, I N (fig. 7) be the plane of the incident wave, which I suppose for greater simplicity parallel to the face by which it enters the prism of crystal B A C, whose axes moreover have any directions whatever; all the portions of this wave will arrive simultaneously at the plane A B, and it will not undergo any deviation of its plane in penetrating and traversing the crystal. This will no longer be the case when it emerges from the prism. to determine the direction of the plane of the emergent wave,

Fig. 7.



from the point A as centre and with radius A E equal to the path described by the light in the air in the time during which the wave advances from B to C, describe the arc of a circle, to which through C draw the tangent C E; this tangent will indicate precisely the plane of the emergent wave, as is easily proved*. If we consider each disturbed point of the surface A C as becoming itself a centre of disturbance, we see that all the small spherical waves thus produced will arrive simultaneously at C E, which will be their common tangent plane; now, I say that this plane will be the direction of the total wave resulting from the union of all these small elementary waves, at least at a distance from the surface of considerable magnitude relative to the length of an undulation; in fact, let H be any point of this plane for which I seek to determine in position and in intensity the resultant of all these systems of elementary waves. The first ray arrived at this point is that which has followed the direction G H perpendicular to C E; and the rays g H and g' H, starting from other points g and g' situated on the right and left of G, will be found behindhand in their route by a whole or fractional number of undulations, so much the greater as these points are further off from the point G If now C A be divided in such a manner that there may be always a difference of a semi-undulation between the rays emanating from two consecutive points of

* I suppose the plane of the figure perpendicular to the two faces of the prism

division, it is easy to see that by reason of the distance of H, which is very great relative to the length of an undulation, the small parts into which we have divided C A will become sensibly equal to each other for rays which make slightly sensible angles with G H. We may therefore admit that the rays sent by two consecutive parts will mutually destroy each other as soon as they have a sensible obliquity to G H; or, more rigorously that the light sent by one of these parts will be destroyed by the half of the light of that preceding it, and the half of the light of that succeeding it, for its magnitude differs only from the arithmetical mean of those between which it is situated by a very small quantity of the second order. Moreover, the rays sent by these three parts must have sensibly the same intensity whatever be the law of their variation of intensity round the centres of disturbance, since, being sensibly parallel to each other (by reason of the distance of H), they are in the same circumstances.*

Moreover, it results, from the nature of the primitive vibratory motion which gives rise to all these centres of disturbance, and the oscillations of which are necessarily repeated by them, that the elementary waves which they send to H will carry to that point absolute velocities alternately positive and negative, which will be the same in magnitude, and will differ only in sign. The same will be the case for the accelerating forces resulting from the relative displacements of the molecules, which will be equal and of contrary signs for the two opposite movements of the primitive wave. Now this equality between the positive and negative quantities contained in each complete undulation, is sufficient in order that two systems which differ in their route by a semi undulation may mutually destroy each other when they have besides the same intensity. Hence all the rays sensibly inclined to G H will mutually destroy each other and only those which are almost parallel to it will concur effectually in the formation of the resultant system of waves.

They may then be considered in the calculation as having equal intensities, and the integration be made between the limits of positive and negative infinity in the two dimensions, employing

* We may make the same observation with regard to the intensities of these rays as with regard to the extent of the portions of A C which send them, by remarking that the rays of the two consecutive portions differing only in intensity by an infinitely small quantity of the first order, the intensity of the rays of an intermediate part differ only by an infinitely small quantity of the second order from the mean between the intensities of the rays of the two adjacent parts.

the formulæ which I have given in my memoir on Diffraction. But, without recurring to these formulæ, it is evident beforehand that if the intensity of the incident wave A B is the same in all its parts, the elements of the integration will be the same for the different points h', H, h, &c. of the emergent wave situated at a sufficient distance from the surface C A, whatever in other respects may be the form of the integral, and that consequently the intensity and the position of the resultant wave will be the same in each of these points; it will therefore be parallel to C E, the geometrical locus of the primitive disturbances; the formulæ of integration place it at a quarter of an undulation behind this plane, but this does not alter its direction, which alone determines that of the visual ray, or of the axis of the telescope by which is observed the line of sight*.

Thus the sines of the angles B A C and C A E, made by the refracting surface with the incident and refracted waves, are to each other as the lengths C B and A E, that is to say, as the velocities of propagation of light in the two contiguous media.

We see, then, that in order to calculate the prismatic effects of doubly-refracting media, when the point of sight is at an infinite distance, and the incident wave consequently plane, it is sufficient to know the velocity of propagation of the ordinary and extraordinary waves in the interior of the crystal for each direction of the plane of the wave, this velocity being measured perpendicularly to this plane. Now these things are given by the greatest and smallest radius vector of the diametral section made in the surface of elasticity by the plane of the wave. But when the point of sight is very near the refracting medium, and we employ a crystal whose double refraction is very strong, such as calcareous spar, in which the curvature of the waves differs greatly from that of a sphere, it becomes necessary to know the form of these waves.

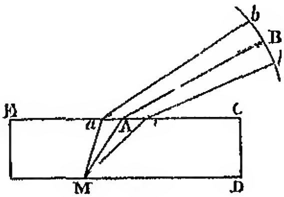
Principle which determines the direction of the refracted rays, when the point of sight is not sufficiently distant to allow of the curvature of the luminous waves being neglected.

In order that I may be more easily comprehended, I shall take a very simple case, that in which the point of sight (*point*

* I have thought it advisable to repeat here, in an abridged form, the explanation which I have given of the law of Descartes for ordinary refraction, in the last note of my memoir on Diffraction, in order to save the reader the trouble of referring to it

de mire) is situated in the interior of the crystal or else against its lower surface Let M (fig 8) be the luminous point, E C the upper surface of the plate by which the rays emerge, let M *a*, M A, M *a'* be rays starting from the luminous point, and following such a course as to strike against the opening *b b'* of the eye or of the object glass of the telescope I suppose that the curve *b* B *b'* represents the geometrical locus of the disturbances which arrive first, starting from the refracting surface E C it will be parallel, as we have seen, to the resultant wave of all the elementary disturbances Now it is on the direction of the element of the emergent wave which falls on the opening of the pupil that the position of the image of the luminous point on the retina depends, and consequently that on which depends the direction of the visual ray which is perpendicular to the element of the wave It is therefore the direction of this element or of its normal, that we have to determine This normal is the ray A B of swiftest arrival at the middle B of the element, since this element is the tangent to the sphere described from A as centre We have then only to seek amongst all the broken rays M *a* B M A B, M *a'* B, for that which will bring the first disturbance to B, and its direction outside the crystal will be that along which will be seen the object

Fig 8



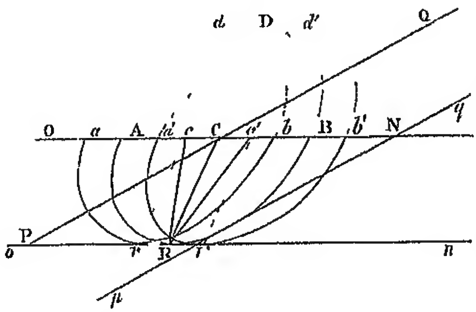
But the section made in the surface of elasticity does not furnish immediately the quantities necessary for determining the intervals of time comprised between the arrivals of the disturbance from M at the points *a*, A, *a'* for it does not give the velocity of propagation except the direction of the cutting plane, or of the element of the wave to which it is parallel, be known, and it is to be remarked, moreover, that the velocity of propagation has always in this construction been supposed to be reckoned on the perpendicular to the plane of the wave, whilst here it would be necessary to have it on the direction of the ray, for, as we have just said, the problem consists in finding the ray of first arrival * It is therefore necessary to calculate, in the first place the velocities of propagation of the wave, whose centre is in M, along the different rays M *a*, M A, M *a'*, that is to say, the lengths of these rays comprised between the centre M and the

surface of the wave at the end of a given time, or in other terms, the equation of the surface of the wave.

Theorem on which depends the calculation of the Surface of the Waves

Let C (fig. 9) be a centre of disturbance, A R B D the position of the wave emanating from C at the end of the unit of time, which I take sufficiently great for the distance of the wave from the point C to contain several undulations, or in other words, so that the length of an undulation may be neglected with regard to this distance.

Fig 9.



Now conceive a plane and indefinite wave O N passing through the same point C; at the end of the unit of time, I say, this wave will have been transferred parallel to itself into the position (on) tangent to the curve A R B D. In fact, let R be the point of contact, and let us seek for the resultant of all the systems of elementary waves emanating from the different points of O N which arrive at R; it is seen that, for the reasons previously explained, it will only be such rays as c R, c' R, of small inclination to C R, that will concur in an efficacious manner in composing the oscillatory motion in R. Let c and c' be two centres of disturbance, whence come these rays whose inclinations to C R are small, at the end of the unit of time they will have sent forth the two waves $a\,r\,b\,d$ and $a'\,r'\,b'\,d'$, absolutely parallel to the wave A R B D, and tangents to the same plane $o\,n$ in the points r and r'. Hence they will arrive at R rather later than the wave emanating from C; C R is therefore the path of quickest arrival of the disturbance at R. It is to be remarked, in the first place, that everything is symmetrical on all sides of the *minimum* throughout a small interval such as that we are considering, and that hence the oscillatory movements which come by the corresponding rays c R and c' R, and are slightly inclined to the plane $o\,n$, will together form resultant motions exactly

parallel to this plane, like the oscillatory movement which comes from C The same may be said of any other two corresponding points situated out of the plane of the figure therefore already the oscillatory motion will have the same direction as it must have in the plane $o\,n$ With regard to the position of the resultant wave, it will be found in arrears of the point R by a quarter of an undulation, on integrating parallely and perpendicularly to the plane of the figure but in a calculation where we have considered the length of an undulation as a quantity to be neglected with regard to the distance C R, we may say that the wave O N has in fact arrived at R at the end of the unit of time By going through a similar reasoning for each of the other points of $o\,n$ it might in the same way be proved that the disturbances resulting from all those which start from O N arrive there also at the end of the unit of time, and that consequently the entire wave is found at this instant transported to $o\,n$ We might demonstrate in the same way that every other plane wave P Q passing through the point C would, at the end of the unit of time, be in the parallel position $p\,q$, tangent to the same curve surface A R B D, therefore this surface must be a tangent at the same time to all the planes occupied at the end of the unit of time by all the plane indefinite waves which have started from C Now we know their relative velocities of propagation measured in directions perpendicular to their planes, and we may consequently determine their positions at the end of the unit of time, and obtain therefrom the equation of the surface of the wave emanating from the point C In this manner the question is reduced to the calculation of an enveloping surface

Calculation of the surface of waves in doubly refracting media

Consequently the equation of a plane which passes through the centre of the surface of elasticity being $z = m\,x + n\,y$, that of the parallel plane to which the surface of the wave must be a tangent, will be $z = m\,x + n\,y + C$, C being so determined that the distance of this plane from the origin of coordinates may be equal to the greatest or least radius vector of the surface of elasticity comprised in the diametral plane $z = m\,x + n\,y$

The equation of the surface of elasticity, referred to the three rectangular axes of elasticity, is

$$\rho^2 = a^2 \cos^2 X + b^2 \cos^2 Y + c^2 \cos^2 Z$$

Let $x = \alpha\, z$ and $y = \beta\, z$ be the equations of a straight line

passing through its centre, that is to say, of a radius vector; between α, β and X, Y, Z we have the following relations:—

$$\cos^2 X = \frac{\alpha^2}{1+\alpha^2+\beta^2}, \ \cos^2 Y = \frac{\beta^2}{1+\alpha^2+\beta^2}, \ \cos^2 Z = \frac{1}{1+\alpha^2+\beta^2}.$$

Substituting these values of $\cos^2 X$, $\cos^2 Y$, $\cos^2 Z$ in the above equation, it becomes

$$v^2 (1 + \alpha^2 + \beta^2) = a^2 \alpha^2 + b^2 . \beta^2 + c^2.$$

This is also the polar equation of the surface of elasticity, but in which the cosines of the angles X, Y, Z, which the radius vector makes with the axes, have been replaced by the tangents (α) and (β) of the two angles which its projections on the coordinate planes $x z$, $y z$ make with the axis of z.

When the radius vector (v) attains its maximum or its minimum, $d v = 0$, hence, on differentiating this last polar equation of the surface of elasticity, we have for the equation of condition,

$$v^2 \left(\alpha + \beta . \frac{d \beta}{d \alpha}\right) = a^2 . \alpha + b^2 . \beta . \frac{d \beta}{d \alpha}.$$

The radius vector whose equations are $x = \alpha z$, $y = \beta z$, being necessarily contained in the cutting plane $z = m x + n y$, we have

$$1 = m \alpha + n \beta;$$

an equation which gives by differentiation

$$0 = m . d \alpha + n . d \beta;$$

whence $\frac{d \beta}{d \alpha} = - \frac{m}{n}$, substituting in the above differential equation, we find

$$v^2 (\alpha . n - \beta . m) = a^2 . \alpha n - b^2 . \beta m.$$

If we combine this relation with the equation $1 = m \alpha + n \beta$, we find the following values for α and β:—

$$\alpha = \frac{(b^2 - v^2)\, m}{(a^2 - v^2)\, n^2 + (b^2 - v^2)\, m^2}, \ \beta = \frac{(a^2 - v^2)\, n}{(a^2 - v^2)\, n^2 + (b^2 - v^2)\, m^2}.$$

We shall observe in passing, that these expressions being of the first degree, (α) and (β) cannot have more values than (v^2). Now on substituting them in the place of (α) and (β) in the equation of the surface of elasticity, we find

$$(a^2 - v^2)\,(c^2 - v^2)\, n^2 + (b^2 - v^2)\,(c^2 - v^2)\, m^2 + (a^2 - v^2)\,(b^2 - v^2) = 0. \quad . \ . \ . \ . \ . \ . \ . \quad (A.)$$

This equation being only of the second degree with regard to (v^2), can give only two values for it; hence there are only two

different elasticities and two directions of the radius vector which satisfy the condition of a maximum or a minimum. It is easy to perceive, without calculating the double values of (α) and of (β), that these two directions must always be at right angles to each other; for it results from the general theorem concerning the three rectangular axes of elasticity that if we consider only the displacements which are performed in one plane and the components comprised in the same plane, not considering the forces which are perpendicular to it, it contains always two rectangular directions, for which the resultant of the components comprised in this plane acts along the line of the displacement itself. Now these directions are precisely those which we have just sought; since, as we have shown, every small displacement parallel to the greatest or least radius vector of any diametral section whatever, excites in the plane of this section a force parallel to the same radius vector, the other component being always perpendicular to this plane.

Media constituted as we have supposed cannot give more than two images of the same object.

Hence the two modes of vibration, which are propagated without deviation of their oscillations or change of velocity, are performed in directions at right angles to each other, that is to say, in the most independent manner; and since, besides, there are only two values of (v^2) or of the elasticity which they put in play, there can be only two systems of waves parallel to the plane of the incident wave, whatever be the original direction of the vibratory motion, since it can always be decomposed along these two directions. If therefore a crystal constituted as we suppose the vibrating medium to be, that is so that the axes of elasticity are parallel throughout its whole extent, be formed into a prism, there can never be seen but two images of a very distant point of sight. The same is also true when this point is so near to the crystal as to render it necessary to take into account the curvature of the wave.

In fact, it results from the principle of the path of quickest arrival, and from the construction deduced from this by Huygens for determining the direction of the refracted ray, that the number of images is equal to the number of points of contact of the tangent planes, which can be drawn on the same side through a straight line to the surfaces of the different waves into which

the light divides itself in traversing the crystal. Now it is evident that through the same straight line, and on the same side of their common centre, there can only be drawn to them two tangent planes; for if three of these could be drawn, it would be equally possible to draw three parallel tangent planes on the same side of the centre of the waves, whence would result three different distances of these tangent planes from the centre, and consequently three velocities of propagation for the indefinite plane waves parallel to one and the same plane, and we have just shown that there cannot be more than two of these. For the same reason there cannot be more than two points of contact, for the existence of three points of contact would render possible that of three parallel tangent planes.

Calculation of the surface of the waves, continued.

But in calculating the equation of the surface of the waves, the degree of this equation will show us still more clearly that it is impossible to draw to them, through one straight line, more than two tangent planes on the same side of the centre.

The equation of a plane passing through the centre of the surface of elasticity being

$$z = mx + ny,$$

that which determines the two values of the greatest and least radius vector comprised in this diametral section is, as we have seen,

$$(a^2-v^2)(c^2-v^2)n^2 + (b^2-v^2)(c^2-v^2)m^2 + (a^2-v^2)(b^2-v^2) = 0. \quad \text{(A.)}$$

We have already put for the equation of a plane parallel to the section,

$$z = mx + ny + C;$$

the square of the distance of this plane from the origin of coordinates is represented by $\frac{C^2}{1+m^2+n^2}$; hence to express that the plane parallel to the diametral section is distant from it by a quantity equal to the greatest or least radius vector, it is sufficient to write

$$\frac{C^2}{1+m^2+n^2} = v^2, \text{ or } C^2 = v^2(1+m^2+n^2).$$

Hence the equation of this plane, to which the luminous wave must be tangent, becomes

$$(z-mx-ny)^2 = v^2(1+m^2+n^2). \quad \text{(B.)}$$

The equation (A) gives (v^2) as a function of (m) and (n). If we make (m) and (n) vary successively by a very small quantity, we shall have two new tangent planes very near the former, and the common intersection of these three planes will belong to the surface of the wave. We must then, in the first place, differentiate equations (A) and (B) with regard to (m), supposing (n) constant, which gives

$$(z - mx - ny)\,x + v^2 m + (1 + m^2 + n^2)\frac{v\,dv}{dm} = 0 \qquad (B')$$

$$\frac{v\,dv}{dm}\left[(1+n^2)(a^2-v^2) + (1+m^2)(b^2-v^2) + (m^2+n^2)(c^2-v^2)\right] - (b^2-v^2)(c^2-v^2)\,m = 0 \qquad (A')$$

Differentiating afterwards with regard to (n), without making (m) vary, we find in the same way,

$$(z-mx-ny)\,y + v^2 n + (1+m^2+n^2)\frac{v\,dv}{dn} = 0 \qquad (B_1)$$

$$\frac{v\,dv}{dn}\left[(1+n^2)(a^2-v^2) + (1+m^2)(b^2-v^2) + (m^2+n^2)(c^2-v^2)\right] - (a^2-v^2)(c^2-v^2)\,n = 0 \qquad (A_1)$$

If we now eliminate $\frac{v\,dv}{dm}$ between equations (A′) and (B′), and $\frac{v\,dv}{dn}$ between equations (A_1) and (B_1), two new equations will be obtained, containing only the variable quantities (v), (m) and (n), besides the rectangular coordinates x, y, z; and joining them to equations (A) and (B), we shall have four equations between which we may eliminate v, m and n. The relation obtained by this elimination between the coordinates x, y and z will be the general equation of the waves, and will belong at the same time to the surface of the ordinary wave and to that of the extraordinary wave.

Another method of calculating the surface of the waves.

This direct method seems necessarily to lead into calculations of harassing length, in consequence of the number of quantities to be eliminated and the degree of the equations. We may, it is true, eliminate (v^2) between equations (A) and (B) before differentiating them, which gives an equation of the fourth degree in (m) and (n).

A more simple equation, and of the third degree only, is

arrived at by following another method An equation of the first degree in (v^2) is easily obtained by causing the cutting plane, and therefore the tangent plane which is parallel to it, to vary, so that $(d\,v)$ may be nothing; then the common intersection of the two successive positions of the tangent plane is the tangent which passes through the foot of the perpendicular dropped from the origin of coordinates on the tangent plane; and this tangent passing through the point of contact, may serve to determine its position as well as the tangent plane, and by the same method of differentiation and elimination

If we differentiate equation (A.), considering (v) as constant, we find

$$\frac{d\,n}{d\,m} = -\frac{m\,(b^2 - v^2)}{n\,(a^2 - v^2)}$$

Differentiating in the same way the equation (B.) of the tangent plane, we have

$$\frac{d\,n}{d\,m} = -\frac{v^2\,m + x\,(z - m\,x - n\,y)}{v^2\,n + y\,(z - m\,x - n\,y)}$$

Equating these two values, we get the relation

$$[v^2\,n + y\,(z - m\,x - n\,y)]\,(b^2 - v^2)\,m$$
$$= [v^2\,m + x\,(z - m\,x - n\,y)]\,(a^2 - v^2)\,n,$$

in which the two terms containing (v^4) destroy each other, and which becomes

$$m\,n\,(a^2 - b^2)\,v^2 + (z - m\,x - n\,y)\,(m\,y - n\,x)\,v^2$$
$$+ (z - m\,x - n\,y)\,(n\,a\,x^2 - m\,b\,y^2) = 0,$$

or, putting for v^2 its value $\frac{(z - m\,x - n\,y)^2}{1 + m^2 + n^2}$, and suppressing the common factor $(z - m\,x - n\,y)$,

$$(z - m\,x - n\,y)^2\,(m\,y - n\,x) + m\,n\,(a^2 - b^2)\,(z - m\,x - n\,y)$$
$$+ (n\,a^2\,x - m\,b^2\,y)\,(1 + m^2 + n^2) = 0 \qquad \text{(C.)}$$

Now, to obtain the surface of the wave, it is sufficient to differentiate this equation successively with respect to (m) and (n), and afterwards to eliminate (m) and (n) by aid of these two new equations

Having found the equation of the surface of the wave by a much shorter process, it was sufficient for me to verify it by its satisfying equation (C.), in which (m) and (n) represent the $\frac{d\,z}{d\,x}$ and $\frac{d\,z}{d\,y}$ of the surface sought I have followed this synthetical

method because it appeared to be simpler than elimination, yet nevertheless the calculations into which it led me are so long and tedious that I do not think it advisable to give them here. I shall content myself with saying that the condition expressed by the equation (C.) is satisfied by the following equation —

$$(x^2 + y^2 + z^2)(a^2 x^2 + b^2 y^2 + c^2 z^2) - a^2 (b^2 + c^2) x^2 - b^2 (a^2 + c^2) y^2 - c^2 (a^2 + b^2) z^2 + a^2 b^2 c^2 = 0 \quad \text{(D.)}$$

I had arrived at this equation by determining first the intersection of the surface of the wave with each of the coordinate planes, an intersection which presents the union of a circle with an ellipse. I remarked afterwards that a surface offering the same character was obtained by cutting the ellipsoid by a series of diametral planes, and drawing through its centre perpendicularly to each plane radii vectores equal to half of each of the axes of the diametral section, for the surface which passes through the extremities of all these radii vectores thus determined, gives also the union of a circle and an ellipse in its intersection with the three coordinate planes. It is moreover of the fourth degree only, and the identity of the sections made by the three rectangular conjugate diametral planes in these two surfaces would have been to me a sufficient proof of their identity if I had been able to demonstrate that the equation of the wave could not surpass the fourth degree, a result which seemed to follow from the conditions themselves of its generation, since there are only two values for the square (v^2) of the distance of the origin from the tangent plane, so that the surface cannot have more than two real sheets; but as it was not impossible that the equation sought might contain besides imaginary sheets (*nappes*), it was necessary to obtain direct proof, as I have done, that the equation of the fourth degree, to which the ellipsoid had conducted me, satisfied equation (C.), which expresses the generation of the surface of the wave.

Very simple process which leads from the equation of an ellipsoid to that of the wave surface.

The calculation by which I arrived at equation (D.) is so simple, that I think it right to give it here.

I take an ellipsoid which has the same axes as the surface of elasticity; its equation is

$$b^2 c^2 x^2 + a^2 c^2 y^2 + a^2 b^2 z^2 = a^2 b^2 c^2$$

Let $z = px + qy$ be the equation of the cutting plane, the squares of the two axes of the section are given by the following relation,

$$a^2 (b^2 - r^2) (c^2 - r^2) p^2 + b^2 (a^2 - r^2) (c^2 - r^2) q^2 \\ + c^2 (a^2 - r^2) (b^2 - r^2) = 0,$$

in which (r) represents the greatest and least radius vector of this elliptical section

The equations of a straight line drawn through the centre of the ellipsoid perpendicular to the cutting plane, are

$$x = -pz, \text{ and } y = -qz,$$

whence $p = -\frac{x}{z}$, $q = -\frac{y}{z}$, and substituting these values in the above equation, we have

$$a^2 x^2 (b^2 - r^2) (c^2 - r^2) + b^2 y^2 (a^2 - r^2) (c^2 - r^2) \\ + c^2 z^2 (a^2 - r^2) (b^2 - r^2) = 0,$$

or, effecting the multiplications,

$$(a^2 x^2 + b^2 y^2 + c^2 z^2) r^4 \\ - [a^2 (b^2 + c^2) x^2 + b^2 (a^2 + c^2) y^2 + c^2 (a^2 + b^2) z^2] r^2 \\ + a^2 b^2 c^2 (x^2 + y^2 + z^2) = 0$$

Finally, observing that $r^2 = x^2 + y^2 + z^2$, and suppressing the common factor $(x^2 + y^2 + z^2)$, we arrive at the equation (D),

$$(x^2 + y^2 + z^2) (a^2 x^2 + b^2 y^2 + c^2 z^2) - a^2 (b^2 + c^2) x^2 \\ - b^2 (a^2 + c^2) y^2 - c^2 (a^2 + b^2) z^2 + a^2 b^2 c^2 = 0$$

If we wish to refer the surface of the wave to polar coordinates, we must put (r^2) in the place of $(x^2 + y^2 + z^2)$, and substitute for x^2, y^2, z^2 their values $r^2 \cos^2 X$, $r^2 \cos^2 Y$, $r^2 \cos^2 Z$, which gives the following equation,

$$(a^2 \cos^2 X + b^2 \cos^2 Y + c^2 \cos^2 Z) r^4 \\ - [a^2 (b^2 + c^2) \cos^2 X + b^2 (a^2 + c^2) \cos^2 Y + c^2 (a^2 + b^2) \cos^2 Z] r^2 \\ + a^2 b^2 c^2 = 0,$$

by the aid of which we may calculate the length of the radius vector of the wave, that is to say, its velocity of propagation reckoned along the direction of the luminous ray itself, when we know the angles which this latter makes with the axes of elasticity of the crystal

It is easy to assure ourselves that the intersections of the surface represented by the equation (D) with the coordinate planes

are composed of a circle and an ellipse, in fact, if we, for example, suppose $z = 0$ in it, we find

$$(a^2 x^2 + b^2 y^2)(x^2 + y^2) - a^2 (b^2 + c^2) x^2 - b^2 (a^2 + c^2) y^2 + a^2 b^2 c^2 = 0,$$

or

$$(a^2 x^2 + b^2 y^2 - a^2 b^2)(x^2 + y^2 - c^2) = 0,$$

an equation compounded of the equation to a circle whose radius is (c), and of that to an ellipse whose semi axes are (a) and (b)

The equation of the Wave Surface cannot be decomposed into two rational factors of the second degree, except when two of the axes of elasticity are equal

But the general equation to the surface of the wave is not, like those of its intersections, always decomposable into two rational factors of the second degree, as I have assured myself by the method of indeterminate coefficients, this decomposition can only be effected when two of the axes are equal Suppose, for example, that $b = c$, the equation (D) then becomes

$$[a^2 x^2 + b^2 (y^2 + z^2)](x^2 + y^2 + z^2) - 2a^2 b^2 x^2$$
$$- b^2 (a^2 + b^2)(y^2 + z^2) + a^2 b^4 = 0$$

or

$$(x^2 + y^2 + z^2)[a^2 x^2 + b^2 (y^2 + z^2) - a^2 b^2]$$
$$- b^2 [a^2 x^2 + b^2 (y^2 + z^2) + a^2 b^2] = 0,$$

or, lastly,

$$(x^2 + y^2 + z^2 - b^2)[a^2 x^2 + b^2 (y^2 + z^2) - a^2 b^2] = 0,$$

an equation which is the product of that of a sphere by that of an ellipsoid of revolution

The construction of Huygens, which determines the path of swiftest arrival, or the direction of the refracted ray, is applicable to biaxal crystals as to calcareous spar, and in general to all waves of any form whatever

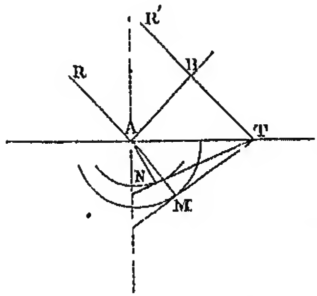
It is to these two surfaces that a tangent plane is successively drawn, in the construction given by Huygens for Iceland spar In the general case of biaxal crystals, that is to say, when the three axes of elasticity are unequal, we must draw a tangent plane to each of the two sheets of the surface represented by the equation (D), and by joining the points of contact with the centre of the surface, we shall have the directions of the two paths of swiftest arrival, and consequently of the ordinary and of the extraordinary ray I employ here the received express...

"*ordinary ray,*" although in reality in this general case neither of the two beams of light follows the laws of "*ordinary*" refraction, as we conclude from the equation.

The position of the straight line through which the tangent plane must be drawn is determined here, as in the construction of Huygens, that is to say, we must take on a direction R′ T (fig. 10.), parallel to the incident rays, a quantity B T equal to the space described by the light outside the crystal during the unit of time; then through the point B draw perpendicularly to these rays the plane A B, which will represent an element of the incident wave at the commencement of the unit of time, supposing A B very small relatively to the distance of the luminous point.

Now if through the point T a straight line be drawn parallel to the intersection of this plane with the face of the crystal, this line projected in T (the plane of the figure being supposed perpendicular to the intersection of the plane A B with the surface A T of the crystal) will be the intersection of the surface with the element A B of the wave at the end of the unit of time, it is therefore through this straight line that a tangent plane must be drawn to the waves formed in the crystal at the end of the same interval of time, and whose centres are situated on the first intersection A. The points of contact M and N with the two sheets of the surface of these waves, will determine the two directions A N and A M of the two refracted rays, which in general will not coincide with the plane of the figure.

Fig. 10.



The same construction will be applicable to waves of any form whatever; and the general principle of the path of swiftest arrival reduces all problems on the determination of refracted rays to the calculation of the surface which the wave assumes in the refracting medium.

Determination of the axes of elasticity, and of the three constants a, b *and* c *in the equation to the wave.*

For the case which forms the object of this memoir, the surface of the wave is represented by the equation (D.); the direc-

tions of its axes are given by observation, and will probably afford in each crystal a very simple relation with its lines of crystallization and its faces of cleavage.* two of these axes divide into two equal parts the acute and obtuse angles comprised between the two optical axes, the direction of which may be determined immediately by observation, and the third axis of elasticity is perpendicular to the plane of the two optical axes

The directions of the axes of elasticity may also be found by observing those of the planes of polarization of the emergent light by the aid of a very simple rule relative to these planes deduced by M Biot from his experiments, and which is found to be a consequence of our theory, as we shall soon demonstrate† As to the constants a b c, or the three semi axes of the surface of elasticity, they represent by hypothesis the velocities of propagation of vibrations parallel to the axes of x y and z, that is to say, the spaces which they describe during the unit of time These velocities may be determined in several ways The most direct is to measure successively the velocities of the rays refracted parallel to each of the axes of elasticity, and whose vibrations are parallel to one of the other two axes For this purpose may be employed the ordinary observations of refraction or the more delicate process furnished by the principle of interferences, and which allows of the most minute differences of velocity being estimated In traversing the crystal parallel to the axis of x, the light assumes two velocities, which being measured give (b) and (c) parallel to the axis of y these two velocities are (a) and (c), and parallel to the axis of z they are (a) and (b) Hence two of these measurements, made with care, are rigorously speaking, sufficient to determine the three quantities a, b and c

* It would seem that the axes of elasticity should always assume directions symmetrical with regard to the corresponding faces of the crystal that is to say that they should be axes of symmetry for the form as they are for the elasticity yet M Mitscherlich has observed several crystals in which the line which divides into two equal parts the angle of the two optic axes is not found directed symmetrically with regard to the corresponding faces of crystallization

† In saying that the simple and elegant construction given by M Biot for determining the planes of polarization is a consequence of our theory I do not mean it to be understood that I have any right to participate in the honour of this discovery since the labours of M Biot on double refraction are much earlier than mine I mean simply to state that the law which he had found flows necessarily from the theory which I have set forth and that we have here a striking confirmation and not merely a fact which by the help of an arbitrary constant or by the addition of a subsidiary hypothesis is made to coincide with the calculation

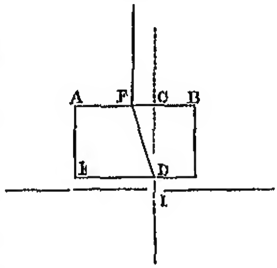
We may deduce from the construction of Huygens applied to equation (D.) general formulæ, which give the direction of the refracted rays for all directions of the incident rays, and of the surface of the crystal relatively to these axes, as Malus has done for Iceland spar, where the extraordinary wave is an ellipsoid of revolution. I have not calculated these formulæ, of which I had no need, in order to verify my theory on topaz. In general, so long as we are concerned with crystals whose double refraction is feeble, and when we confine ourselves to the investigation of the divergence of the two beams obtained by forming the crystal into a prism, it is sufficient to determine, in the first place, approximately the direction of the luminous ray in the interior of the crystal by the law of Descartes, with the index of refraction of the ordinary or extraordinary rays; and when we thus know the approximate direction of the refracted ray, we may calculate the two corresponding velocities by means of equation (D.), or the two velocities of the wave measured perpendicularly to its plane by means of equation (C.), which represents the section made in the surface of elasticity by a diametral plane parallel to the wave, and in which (m) and (n) are given as soon as we know the direction of the refracted wave. These two velocities once known, it becomes easy to deduce from them the direction and the divergence of the two beams, or of the two systems of emergent waves.

If greater accuracy however were desired, it would be necessary to determine with the velocity thus calculated a new and more approximate direction of the ray or of the plane of the wave in the crystal, and calculate afresh the corresponding velocity by the aid of equation (D.) or of equation (C.), according as we wish to obtain the velocity measured on the ray or the normal to the plane of the wave; then we can deduce from this the direction of each of the two emergent beams. This method is quite as accurate and much less laborious than employing the formulæ of which we have spoken, which would be doubtless very complicated. It may also be applied to crystals whose double refraction is more powerful, by repeating the operation a sufficient number of times.

When it is sought to verify the law of the velocities by an experiment of diffraction, it is sufficient to consider the velocity of propagation of the refracted wave measured perpendicularly to its plane. This is even the most simple method, since the ex-

periment gives immediately the difference between the numbers of the undulations performed within the thickness of the plates, whence it is easy to conclude immediately the difference of route of the two systems of waves since these numbers are equal to the thickness of the plate divided by the two lengths of undulation, or the two velocities measured perpendicularly to the plane of the waves, whatever besides may be the obliquity of the rays to the surface of the waves Suppose, for example, that a plate of crystal with parallel faces A B F D (fig 11) is traversed perpendicularly by a beam of light coming from a point so distant that we may consider as a plane the small extent of the incident wave A B, which undergoes refraction the refracted wave will be in all its successive positions plane and parallel to A B consequently it will be sufficient to know the velocity of propagation of this wave measured along C D perpendicularly to A B, to ascertain what relative time it has employed in traversing the thickness of the plate, or what number of undulations it has performed in it It is useless to calculate the oblique direction E D by which the *refracted rays* have arrived at D, opposite the slit F made in the screen but if this route were known, instead of employing the velocity deduced from the equation to which we have referred, and in which it is supposed to be reckoned on the normal to the wave, it would be necessary to make use of the velocity given by equation (D) where it is reckoned on the direction of the ray E D, and we should evidently arrive at the same result

Fig 11



Definition of the word "Ray"

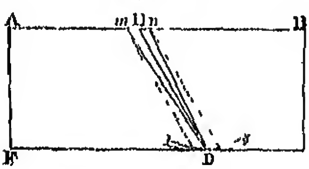
The word "*ray*" in the wave theory must always be applied to the line which goes from the centre of the wave to a point of its surface, whatever besides may be the inclination of this line to the element on which it abuts, as Huygens has remarked; for this line offers, in fact, all the optical properties of that which is called the *ray* in the emission system Hence, when it is wished to translate the results of the former theory into the language of the latter, it must always be supposed that the line

described by the luminous molecules on the emission hypothesis, has the same direction as the ray drawn from the centre of the wave to the point of its surface under consideration. That which we have previously said to establish this principle will have perhaps appeared sufficient, we think it useful nevertheless to support it yet further by a new consideration drawn from another mode of judging by experiment of the direction of the refracted ray.

New consideration, which shows further that the radius vector of the surface of the wave is really the direction of the luminous ray.

Suppose, as just now, that the incident wave is plane and parallel to the surface of entry of the crystal, but that the screen, pierced by a small hole, is placed on the first face, instead of on the second; and that we wish to judge of the direction of the ray refracted through the point D (fig. 12), where the light thus introduced strikes against the second face. The point which will be regarded as answering to the axis of the luminous beam will be the centre D of the small bright and dark rings projected on the face F D; and it is in this central point that the maximum of light will be found if the hole *m n* is sufficiently small relative to the distance E D. The position of the centre D is determined by the condition that the rays starting from the different points *m* and *n* of the circumference of the opening arrive at the same time at D. This point must be the most strongly illuminated spot so long as the diameter of the opening is sufficiently small with regard to the distance E D for the difference of route between the rays starting from the centre and circumference, not to exceed a semi-undulation. Now, in order to compare the route of the elementary disturbances which emanate from the various parts of the surface of the wave comprised within the extent of the small opening, we must consider the waves which they would produce separately in the same interval of time, and thence conclude the difference between their moments of arrival at D. Let *r* D *s* be the elementary wave, having for centre the middle E of the opening; if a tangent plane F D be drawn to it parallel to the incident wave A B, the point of contact D will satisfy the condition just announced; for the elementary wave

Fig. 12.



which has started from E will be that which will arrive there the first, and by reason of the general property of *maxima* and *minima* all the differences will be equal and symmetrical at a small distance round the shortest path L D that is to say, the elementary waves which have started from points (m) and (n) equally distant from E will be found behindhand by the same quantity at D relatively to the wave which started from E, and will therefore arrive at D in the same time It is also in the neighbourhood of a *minimum* or *maximum* of a function that its variations are the least sensible D will therefore be the point for which there will be the smallest possible differences between the paths described at the same instant by the elementary waves which have started from the opening $m\,n$ and consequently it is there that the most perfect accordance between their vibrations will exist, if as we have supposed, the greatest differences do not exceed a semi undulation It is at D therefore that the *maximum* of light will be found, and consequently E D will be, for this reason as well as for all the others, the direction of the *luminous ray in the crystal* Now if the screen be removed, it will still be true that the *refracted rays* which start from the various points of the incident wave, considered then as indefinite, are parallel to E D, that is to say, to the *radius vector* directed towards that point of the surface of an interior wave for which the tangent plane is parallel to the refracted wave

The meaning to be attached to the word "*luminous ray*" being thus settled we see that the ellipsoid constructed on the same rectangular axes as the surface of elasticity, gives *rigorously*, by the two semi axes of its diametral section, the velocities of the *refracted rays perpendicular to this section*, as the analogous construction made in the surface of elasticity gives the velocities of propagation of waves parallel to the diametral section, these velocities being reckoned perpendicularly to the plane of the waves Thus understood, the first construction is a mathematical consequence of the second, and represents the phænomena in as rigorous a manner, whatever may be the energy of the double refraction or the inequality of the three axes a, b, c

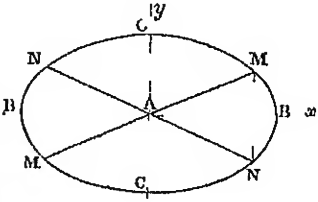
In translating into the language of the emission system the law of Huygens for the double refraction of Iceland spar M de Laplace has found, by an elegant application of the principle of least action, that the difference between the squares of the velo

cities of the two beams, ordinary and extraordinary, is proportional to the square of the sine of the angle which the extraordinary ray makes with the axis of the crystal. Guided by analogy, M. Biot has thought that in bi-axal crystals the same difference ought to be proportional to the product of the sines of the angles which the extraordinary ray makes with each of the optic axes, a product which becomes equal to the square of the sine when these two axes are united into one only. M. Biot has verified this law by numerous experiments, having for their object to determine the angle of divergence of the ordinary and of the extraordinary beam. He has compared these measures with the numbers deduced from the law of the product of the sines by the principle of least action, and has always found a satisfactory accordance between the results of calculation and those of experiment In transforming the formulæ given previously by Sir David Brewster, M. Biot has discovered that the law of the product of the sines to which he had been led by analogy, was implicitly contained in the more complicated formulæ deduced by Sir David Brewster from his observations, hence the experiments of the Scotch experimenter, as well as those of M. Biot, establish the accuracy of the law of the product of the sines. In order to translate it into the language of the wave theory, it must be recollected that in it the velocities of the incident and refracted rays are in the inverse ratio of that which they would have in the emission system; hence, the difference of the squares of the velocities of the ordinary and extraordinary beam, considered under the point of view of this system, answer in that of the wave system to the difference of the quotients of unity divided by the squares of the velocities of the same rays. Now I shall demonstrate that this latter difference must be in reality equal to a constant factor multiplied by the product of the two sines, according to the construction which I have given for determining the velocity of the luminous rays by a normal section made in the ellipsoid constructed on the three axes of elasticity.

Theoretical Demonstration of the Law of MM. Biot *and* Brewster *on the difference of the squares of the velocities.*

Let B B′ and C C′ (fig. 13) be the greatest and least diameters of the ellipsoid. the former I always take for the axis of (x), and

Fig 13



the second for the axis of (x), the mean diameter coinciding with the axis of (y), projected in A, the centre of the ellipsoid If we give the name of *optic axes* of the medium to the directions along which the luminous rays which traverse it can have only one velocity, those which possess this property are, according to the construction which determines the velocity of the luminous rays the two diameters of the ellipsoid perpendicular to the circular sections Next, let the equation to the ellipsoid be

$$f x^2 + g\ y^2 + h\ z^2 = 1$$

If in this we put $y = 0$, we shall have $f x^2 + h z^2 = 1$ for the equation to the ellipse C M B N C′ M′ B′ N′ situated in the plane of the figure which we shall suppose to coincide with that of $x\ z$ The two diametral planes M M′ and N N′, which cut the ellipsoid in a circle, pass through the mean axis projected in A, and must be inclined to the axis of x at an angle (i) such that the semi diameters A M and A N may be equal to the mean semi axis of the ellipsoid or that the squares of the former may be equal to the square of the latter, which is $\frac{1}{g}$ Denote A M or A N by (r), we shall have

$$z = r \sin i, \text{ and } x = r \cos i$$

Substituting these values in the equation to the ellipse $f x^2 + h z^2 = 1$, we have

$$f r^2 \cos^2 i + h r^2 \sin^2 i = 1,$$

or, since $r^2 = \frac{1}{g}$,

$$f \cos^2 i + h\ \sin^2 i = g,$$

whence we obtain

$$\sin^2 i = \frac{f-g}{f-h}, \quad \cos^2 i = \frac{g-h}{f-h}, \quad \tan^2 i = \frac{f-g}{g-h}$$

Hence the equation to the plane A M is $z = x\sqrt{\frac{f-g}{g-h}}$, and that to the plane A N of the other circular section $z = -x\sqrt{\frac{f-g}{g-h}}$

Let $y = px + qz$ be the equation to a diametral plane drawn perpendicularly to a luminous ray of any direction, we have to calculate the difference between the two quotients of unity divided successively by the squares of the semi-axes of its elliptical section, as a function of the angles which this plane makes with the two circular sections; for these angles are equal to those which the normal to this plane, or the luminous ray, makes with the normals to the two circular sections, that is to say, with the two optic axes of the crystal. Now if we denote by (m) the angle contained between the plane $y = px + qz$, and the circular section M M′, and by (n) the angle which it makes with the other circular section N N′, we have

$$\cos m = \frac{p\sqrt{f-g} - q\sqrt{g-h}}{\sqrt{f-h} \times \sqrt{1+p^2+q^2}},$$

and

$$\cos n = \frac{p\sqrt{f-g} + q\sqrt{g-h}}{\sqrt{f-h} \times \sqrt{1+p^2+q^2}},$$

whence we have

$$\frac{q^2}{p^2} = \frac{(f-g)(\cos n - \cos m)^2}{(g-h)(\cos n + \cos m)^2},$$

and

$$\frac{1}{p^2} = \frac{-(f-h)(g-h)(\cos n+\cos m)^2-(f-g)(f-h)(\cos n-\cos m)^2+4(f-g)(g-h)}{(f-h)(g-h)(\cos n+\cos m)^2}.$$

Let us now calculate the two diameters of the elliptical section, which give the velocities of the ordinary and extraordinary ray perpendicularly to the plane of this section. To this end it is sufficient to form the polar equation to the ellipsoid, and to seek the maximum and minimum values of the radius vector in this plane. Let $x = \alpha y$ and $z = \beta y$ be the general equations to the radius vector; the square of its length will be equal to $x^2+y^2+z^2$ or to $y^2(1+\alpha^2+\beta^2)$, (y) corresponding to the point of intersection of the straight line with the surface of the ellipsoid. The equations to the straight line and to the surface being true at the same time for this point, we have $y^2(f\alpha^2 + h\beta^2 + g) = 1$; whence we obtain $y^2 = \dfrac{1}{f\alpha^2 + h\beta^2 + g}$, and consequently the square of the radius vector is equal to $\dfrac{1+\alpha^2+\beta^2}{f\alpha^2+h\beta^2+g}$, an ex-

pression which we shall put equal to $\frac{1}{t}$ so that the variable (t) may represent unity divided by the square of the radius vector We obtain thus the polar equation of the ellipsoid

$$f\alpha^2+h\beta^2+g = t\,(1+\alpha^2+\beta^2),$$

of which Petit has made so elegant an application to the general discussion of surfaces of the second degree

To express that the particular radius vector we are considering is contained in the plane $y=px+qz$ we must write $1=p\alpha+q\beta$, an equation which being differentiated with respect to (α) and (β) gives

$$\frac{d\beta}{d\alpha} = -\frac{p}{q}$$

If we differentiate in the same way the polar equation of the ellipsoid, considering (β) and (t) as functions of (α), we have

$$2f\alpha+2h\beta\,\frac{d\beta}{d\alpha}=(1+\alpha^2+\beta^2)\,\frac{dt}{d\alpha}+2t\alpha+2t\,\frac{d\beta}{d\alpha},$$

or, putting for $\frac{d\beta}{d\alpha}$ the above value $-\frac{p}{q}$,

$$2qf\alpha-2ph\beta-2tq\alpha+2tp\beta=(1+\alpha^2+\beta^2)\,\frac{dt}{d\alpha},$$

whence we get

$$\frac{dt}{d\alpha}=\frac{2qf\alpha-2ph\beta-2tq\alpha+2tp\beta}{1+\alpha^2+\beta^2}$$

When the radius vector attains its maximum or its minimum (t) is at its maximum or minimum, and consequently $\frac{dt}{d\alpha}=0$, therefore

$$2qf\alpha-2ph\beta-2tq\alpha+2tp\beta=0,$$

or

$$\alpha q\,(t-f)-\beta p\,(t-h)=0$$

If we join to this relation the equation of condition,

$$p\alpha+q\beta=1,$$

which expresses that the radius vector is contained in the plane of the elliptical section, we obtain the following values of (α) and (β), corresponding to the maximum and minimum values of the radius vector,

$$\alpha=\frac{p\,(t-h)}{p^2\,(t-h)+q^2\,(t-f)}\qquad\beta=\frac{q\,(t-f)}{p^2\,(t-h)+q^2\,(t-f)}$$

We may put the polar equation of the ellipsoid under the form

$$\alpha^2 (t-f) + \beta^2 (t-h) + t - g = 0,$$

and substituting for (α) and (β) their values, we have

$$p^2 (t-h)^2 (t-f) + q^2 (t-f)^2 (t-h) \\ + (t-g) [p^2 (t-h) + q^2 (t-f)]^2 = 0;$$

or

$$(t-f)(t-h) [p^2 (t-h) + q^2 (t-f)] \\ + (t-g) [p^2 (t-h) + q^2 (t-f)]^2 = 0;$$

or lastly, suppressing the common factor $p^2 (t-h) + q^2 (t-f)$,

$$(t-f)(t-h) + p^2 (t-g)(t-h) + q^2 (t-f)(t-g) = 0,$$

an equation of the second degree, which ought to give at the same time the maximum and minimum values of (t), that is to say, the two values of (t) which correspond to those of the semi-axes of the elliptical section.

We may divide this equation by (p^2), and put it under the form

$$(t-f)(t-h) \cdot \frac{1}{p^2} + (t-g)(t-h) + \frac{q^2}{p^2} (t-f)(t-g) = 0.$$

And substituting for $\frac{1}{p^2}$ and $\frac{q^2}{p^2}$ the values which we have above found in functions of the angles (m) and (n), we arrive, after several reductions, at the equation

$$t^2 - t \cdot [f + h - (f-h) \cos n . \cos m] + fh + \frac{1}{4} (\cos^2 n + \cos^2 m)(f-h)^2 \\ - \frac{1}{2} \cos n . \cos m \, (f^2 - h^2) = 0;$$

whence we obtain

$$t = \frac{1}{2} \cdot (f+h) - \frac{1}{2} (f-h) \cos n . \cos m \\ \pm \frac{1}{2} (f-h) \sqrt{1 + \cos^2 n \cos^2 m - \cos^2 n - \cos^2 m},$$

or

$$t = \frac{1}{2} (f+h) - \frac{1}{2} (f-h) \cos n \cos m \pm \frac{1}{2} (f-h) \sin n . \sin m^*;$$

* The two values of (t), which give the quotients of unity divided successively by the squares of the velocities of the ordinary and of the extraordinary ray, may be put under the following form —

$$t = \frac{1}{2} (f+h) - \frac{1}{2} (f-h) \cos (m+n),$$

and

$$t = \frac{1}{2} (f+h) - \frac{1}{2} (f-h) \cos (m-n).$$

therefore the difference between the two values of (t), or the quantity sought, is equal to

$$(f-h)\sin n \cdot \sin m$$

consequently this difference is proportional to the product of the sines of the two angles (m) and (n), which was to be proved.

The angles concerned are those which the common direction of the ordinary and extraordinary rays makes with the two diameters of the ellipsoid perpendicular to the circular sections, which diameters we have called *optic axes*, admitting that this name ought to be given to the two directions along which the *luminous rays* traverse the crystal without undergoing in it any double refraction. But it is to be remarked that in general these rays meet the element of the surface of the luminous waves to which they correspond obliquely. Now we have previously pointed out, that if the surface of the crystal were parallel to this element or to its tangent plane the normal direction would be that which must be given to the incident beam in order that it might not undergo double refraction in penetrating into the crystal; whence it would appear that we ought also to give the name of *optic axes* to these two directions of the incident rays which do not coincide with the two normals to the circular sections of the ellipsoid. Hence the direction of the optic axes would be different according as we determined it by the direction of the *incident rays* perpendicular at the same time to the surface of the incident waves and the refracted waves, or by the direction of the *refracted rays corresponding to these waves*. In truth this difference is very slight in almost all crystals with two axes, but there are some of them in which it becomes more perceptible, and where the two directions can no longer be confounded. That to which it appears most fitting to give the name of *optic axis of the crystal* is the direction of the *refracted rays* which traverse it without undergoing double refraction. Adopting this definition, the law of the product of the sines of the angles which any ray makes with the two optic axes, becomes a rigorous consequence of our theory, as we have just proved.

Hitherto we have occupied ourselves solely with the velocity and the direction of the waves and rays; we now proceed to investigate their planes of polarization.

Planes of Polarization of Ordinary and Extraordinary Waves.

According to what we have said at the commencement of this memoir, in deducing our hypothesis as to the nature of luminous vibrations from the phænomena presented by the interference of polarized rays, the plane of polarization must be parallel or perpendicular to the direction of the luminous vibrations. It remains only to choose between these two directions that which agrees with the usual acceptation. Now the name *plane of polarization* of the ordinary beam in uni-axal crystals is given to the plane drawn through this beam parallel to the axis of the crystal; and it is clear that the ordinary vibrations, that is to say, those which always call into play the same elasticity, are the vibrations perpendicular to the axis of the crystal; in fact, in the case of crystals with one axis, the surface of elasticity becomes a surface of revolution, and each diametral section has always its greatest or least radius vector situated on the intersection of its plane with the equator; it is therefore this radius vector which remains constant, since the equator is a circle, and which consequently gives the direction of the ordinary vibrations; whence we see that these vibrations are always perpendicular to the axis of the crystal. Hence the plane drawn through this axis and the ordinary ray is perpendicular to these vibrations, since they are also perpendicular to the ordinary ray by reason of the sphericity of the wave to which they belong; but this plane is precisely, as we have just said, that which it has been agreed to call the *plane of polarization of the ordinary ray*; hence we shall give the name of *plane of polarization of a luminous wave* to the plane normal to the direction of its vibrations. This theoretical definition agrees with the meaning attached to the expression "*plane of polarization*" in the emission system, so long as the wave is spherical and its vibrations perpendicular to the luminous ray, because then the plane of polarization always passes through the ray; but when the vibrations are oblique to the ray, the plane of polarization, which ought to be perpendicular to them according to our definition, no longer contains the luminous ray, whilst in the emission system it is supposed to be always directed along this ray. Hence, the same direction precisely would not be assigned in the two theories to the planes of polarization of luminous rays in media where their waves no longer have the spherical form. But, in the first place,

this difference would be always very slight, because the surface of the luminous waves does not deviate much from the spherical form even in those crystals whose double refraction is most powerful in the second place it becomes useless to take any account of it for the experiments made by M Biot and the other experimenters on the direction of the planes of polarization of the ordinary and extraordinary rays, since it is always outside the crystal and by the direction of the planes of polarization of the incident or emergent rays, that they have judged of the direction of the planes of polarization of the refracted rays Thus for example, suppose that we wished to determine the planes of polarization of the ordinary and extraordinary refraction in a crystallized plate with parallel faces perpendicular to the incident rays For this purpose it is sufficient to employ light previously polarized and to turn the plate in its plane until the emergent beam, analysed by a prism or rhomboid of Iceland spar, no longer presents any trace of depolarization in consequence of its passage across the crystallized plate When this condition is fulfilled, we may conclude from it that the plane of polarization of the refracted wave coincides with that of the incident wave There are always two positions of the plate which satisfy this condition and thus afford the means of tracing on the crystal the direction of the planes of polarization of the ordinary and extraordinary refraction In this experiment the incident wave being parallel to the faces of the crystallized plate preserves this parallelism in traversing it and if the direction of the vibrations of the incident wave coincides with that of one of the axes of the parallel diametral section made in the surface of elasticity they will suffer no further deviation in traversing the crystal in that case, the incident refracted and emergent waves have all three the same plane of polarization and their surfaces are parallel, although moreover the refracted rays may be oblique to their wave, and thus not be found on the prolongation of the incident and emergent rays In this case the definition of the plane of polarization according to the emission system no longer gives rigorously for the plane of polarization of the refracted rays the same direction as the definition drawn from our theory although they agree in other respects as to the direction of the planes of polarization of the incident and emergent rays the only ones which can be determined immediately by observation

Considering always as the true plane of polarization that which

is perpendicular to the luminous vibrations, I shall now prove that the planes of polarization of the ordinary and extraordinary waves divide into two equal parts the dihedral angles formed by the two planes drawn along the normal to the wave, and the two normals to the planes of the circular sections of the surface of elasticity

The rule given by M Biot *for determining the direction of the planes of polarization of the ordinary and extraordinary rays agrees with the theory set forth in this memoir*

Suppose, in fact, that this surface be cut by a diametral plane parallel to the wave, the two axes of this section will give the directions of the ordinary and extraordinary vibrations, if then we draw through the centre two planes perpendicular to these two diameters, these will be the planes of polarization respectively of the ordinary and extraordinary vibrations Now it must be remarked,—1st, that they will each pass through one of the axes of the section, since these latter are perpendicular to each other, 2nd, that the axes of the diametral section cutting it each into two symmetrical portions, must divide into equal parts the acute and obtuse angles formed by the two lines along which the plane of this section meets those of the circular sections, since in these two directions the radii vectores of the diametral section are equal to each other, as belonging at the same time to two circular sections which have the same diameter

This being established, conceive a sphere concentric with the surface of elasticity, the plane of the diametral section, and the two planes of the circular sections, will trace on this sphere a spherical triangle, of which the side contained in the first plane will be divided into two equal parts by one of the planes of polarization, its supplementary triangle will be that formed by the normals of these three planes drawn through the common centre, that is to say, which will result from the intersection of the spherical surface with the three planes drawn along these three normals taken two and two Now the planes which divide into two equal parts the sides of the first triangle also divide into two equal parts the angles of the second, this is an easily proved property of supplementary triangles Therefore the plane of polarization, which divides into two equal parts the side of the first triangle comprised in the diametral section, divides also into two equal parts the corresponding angle of the

second triangle, that is, the dihedral angle formed by the two planes drawn along the normal to the wave and the diameters perpendicular to the two circular sections; and for the same reason, the other plane of polarization will divide into two equal parts the supplement of this dihedral angle.

M. Biot has deduced from his observations on the double refraction of topaz and several other bi-axal crystals, the following rule for determining the direction of the planes of polarization of the ordinary and extraordinary rays:

"Conceive a plane drawn through each of the axes of the crystal, and through the ray which undergoes the ordinary refraction. Conceive through this same ray a third plane, which bisects the dihedral angle formed by the two former. The luminous molecules which have undergone the ordinary refraction are polarized in this intermediate plane, and the molecules which have undergone the extraordinary refraction are polarized perpendicularly to the intermediate plane drawn through the extraordinary ray according to the same conditions." (*Précis Élémentaire de Physique Expérimentale*, vol. ii. page 502.)

The lines which M. Biot here calls the *axes of the crystal*, are those which we have called *optic axes*. We have remarked, that in order to assimilate in the best manner possible the language of the undulatory system with that of the emission theory, we ought to give the name of *optic axis* to the direction along which the luminous rays traverse the crystal without undergoing double refraction; and adopting this definition, we have proved that the law of the product of the two sines is a necessary consequence of our theory. The same is not true for the rule of M. Biot relative to the determination of the planes of polarization. His enunciation does not exactly agree with the construction which we have deduced from the properties of the surface of elasticity, because the dihedral angles bisected by the planes of polarization according to this construction are drawn along the normal to the wave and the two normals to the circular sections of the surface of elasticity, and in general the normal to the wave does not coincide exactly with the direction of the refracted ray, nor the normals to the circular sections of the same surface with the true optic axes, which are the perpendiculars to the circular sections of the ellipsoid. In truth, the geometrical theorem which we have demonstrated for the surface of elasticity applies equally to the ellipsoid; but the greatest and least radius vector of the

diametral section made in the ellipsoid perpendicularly to the direction of the luminous ray, do not now give the direction of its vibrations; so that the planes which are perpendicular to them are no longer the true planes of polarization of the refracted waves The rule of M. Biot therefore does not rigorously agree with our theory. But it must be recollected,—1st, that in the crystals employed by him, the normals to the circular sections of the surface of elasticity differ so little from the direction of the true optic axes, that they might be confounded without producing any sensible error in the direction of the planes of polarization; 2nd, that in the same crystals the rays directed along the optic axes are nearly normal to the corresponding waves; 3rd, that this skilful experimenter could only determine directly the plane of polarization of the incident or emergent beams, and not that of the refracted rays. The small differences which are here indicated to us by the theory, would doubtless be very difficult to observe, even in those bi-axal crystals whose double refraction is most powerful, for we cannot determine very accurately by the known methods the direction of the plane of polarization of a luminous ray, and there is here an additional difficulty, that of fixing the direction of the plane of polarization in the interior of the crystal from observations made on the emergent rays. Hence, far from seeing an objection against our theory in the rule given by M Biot, it ought rather to be considered as being a confirmation of it, since the small discordance which exists between them must necessarily have escaped his observations.

Most crystals present but little difference between the planes of the circular sections of the surface of elasticity and of the ellipsoid constructed on the same axes.

The two circular sections of the surface of elasticity are equally inclined to the plane of xy, which passes through the mean axis, and the tangent of this inclination is, as we have seen, $\sqrt{\frac{a^2-b^2}{b^2-c^2}}$, the tangent of the angle which the two circular sections of the ellipsoid make with the same plane is equal to $\frac{c}{a}\sqrt{\frac{a^2-b^2}{b^2-c^2}}$ We see by these formulæ, that when the double refraction is not very powerful, that is, when (c) differs little from (a), $\frac{c}{a}$ being nearly equal to unity, the planes of the circular

sections of the two surfaces are sensibly coincident For topaz the ratio $\frac{c}{a}$ is 0 9939, for anhydrous sulphate of lime one of the biaxal crystals whose double refraction is most powerful this same ratio according to the observations of M Biot, is equal to 0 9725*

Observations on the route of the Waves and luminous rays in the direction of the optic axes

It is to the circular sections of the surface of elasticity that a plane wave must be parallel in the interior of a crystal, in order that it may be there susceptible of only one velocity of propagation and this condition is satisfied when the plate of crystal, cut parallel to the circular sections of the surface of elasticity, is presented perpendicularly to the luminous beam But it is to be remarked that the ordinary and extraordinary rays resulting from it do not follow the same direction, and deviate a little, both the one and the other, from the normal to the circular section of the ellipsoid This is more easily seen by a reference to fig 14, which represents the intersection of the plane of x z with the two sheets of the wave surface, and in which the ellipticity of one of them is exaggerated to render the divergence of the rays more perceptible

This intersection is composed of a circle and an ellipse, whose equations are

$$x^2 + z^2 = b^2 \quad \text{and} \quad a^2 x^2 + c^2 z^2 = a^2 c^2$$

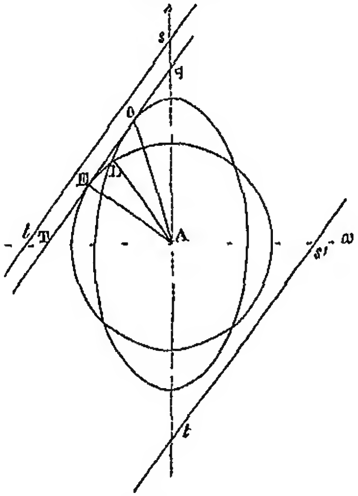
The plane T S, drawn parallel to the circular section of the surface of elasticity, and distant from the centre A by a quantity equal to (b), touches at the same time the circle and the ellipse

* According to the observations of M Biot the angle between the two optic axes in limpid topaz is 63 11 2 and in anhydrous sulphate of lime 44° 41′ 2″ which gives 31° 37′ 1 and 22 20′ 41 for the value of the angle whose tangent is represented by $\frac{c}{a}\sqrt{\frac{a^2 - b^2}{b^2 - c^2}}$ it results from the same measures that the angle which has for its tangent $\sqrt{\frac{a^2 - b^2}{b^2 - c^2}}$ is in the former crystal 31 16′ 25″ and in the second 22 54′ 13 Hence the difference of direction between the circular sections of the ellipsoid and of the surface of elasticity is only 0′ 21″ for topaz and 34′ 2″ for anhydrous sulphate of lime

Note —The seconds marked in the value of the angles given by M Biot and which we have here transcribed do not signify that the precision of measurement can be carried to this extent for it is difficult even to determine the angle of the optic axes within half a degree nearly

in E and O, the points of contact of this plane with the surface of the wave, hence the radii vectores A O and A E are the directions of the ordinary and extraordinary rays which correspond to the plane wave TS, parallel to the circular section of the surface of elasticity; and they traverse the plate $t s t' s'$ in the same interval of time, although by following different routes. The radius vector A L, drawn to the point of intersection of the ellipse and circle, and for which the two values obtained from the equation to the wave become equal, is the direction along which the *luminous rays* can have only one velocity, and consequently that of the normal to the circular section of the ellipsoid, which we have called the *optic axis*. We find for the tangents of the angles which these three radii vectores make with the axis of x,

Fig. 14.



$$\tan \mathrm{OAT} = \frac{a^2}{c^2}\sqrt{\frac{b^2 - c^2}{a^2 - b^2}},\ \tan \mathrm{LAT} = \frac{a}{c}\sqrt{\frac{b^2 - c^2}{a^2 - b^2}},$$

$$\tan \mathrm{EAT} = \sqrt{\frac{b^2 - c^2}{a^2 - b^2}}$$

We see that these expressions differ only by the factors $\frac{a^2}{c^2}, \frac{a}{c}$, which in most crystals are very nearly equal to unity.

All the ordinary and extraordinary rays parallel to L A traverse the crystal in the same interval of time and with the same velocity*, because they also follow the same path; but they necessarily diverge outside the crystal, because the two tangent planes drawn through the point L to the two sheets of the wave surface make with each other a sensible angle. On the contrary, the rays A E and A O, which take also the same time in

* Whatever be the directions of the faces of entrance and emergence, since these rays follow the same route L A, whilst the rays E A and O A do not take exactly the same time to traverse the crystalline plate, except when its faces $t s$ and $t' s'$ are parallel to one of the circular sections of the surface of elasticity

traversing the plate $t\,s\,t'\,s'$, although both following different directions again become parallel to each other outside the crystal When the face of emergence of the refracting medium is made to vary its inclination the ray F A, and that one of the two rays L A which belongs to the same sheet F I, are refracted conformably to the law of Descartes, whilst the ray O A, and the other ray directed along L A which answers to the second sheet L O, are refracted extraordinarily This establishes a yet further difference between the characters of the optic axes of uni axal and bi axal crystals for in the former all the rays parallel to the optic axis in the interior of the crystal are refracted according to the law of Descartes, whatever be the direction and inclination of the face of emergence, because these rays being then parallel to one of the axes of elasticity, are perpendicular at the same time to the two sheets of the wave surface

Having dwelt on distinctions which the theory shows clearly, but which escape in most observations, and were not capable of being made evident by those of M Biot, we proceed to consider for a moment the planes of polarization in a less rigorous manner, and adopt the rule which he has given for determining their direction, without any alteration of his enunciation, in order that we may be enabled to explain ourselves in a more simple and clearer manner

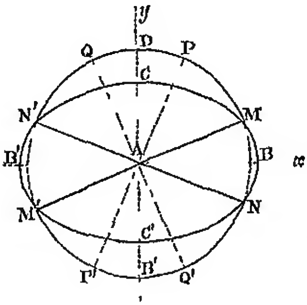
The rays named Ordinary *by* MM Biot *and* Brewster *are those whose variations of velocity have the least extent*

As we have already remarked, there is no longer any ordinary ray, properly so called, in crystals with two axes, since neither of the two beams of light traverses the crystal with the same velocity in all directions, but that which is called the ordinary beam, by analogy with the term adopted for uni axal crystals, is that whose variations of velocity are the least sensible Now it is easy to see that this is the one whose plane of polarization bisects the acute dihedral angle comprised between the planes drawn through the direction of the luminous rays and the two optic axes, whilst the plane of polarization of the beam which undergoes the greatest variations of velocity, bisects the obtuse dihedral angle which is the supplement of the former

In fact, whatever be the direction of the first beam, its plane of polarization passing within the acute angle Q A P (fig 15) of the two optic axes, its trace on the plane of the figure is con

tained in the interior of this angle; and consequently the projection of the diameter of the ellipsoid perpendicular to the plane of polarization, which is normal to the trace of this plane, is necessarily found to be contained in the acute angle M A N or M′ A N′ of the two circular sections, since they are normal to the optic axes P P′ and Q Q′; therefore this diameter cannot meet the surface of the ellipsoid outside of the two parts whose projections have for their limits M B′ N′ A and M B N A; but if from the point A as centre, and with radius equal to that of the circular sections, a sphere be described, its surface will pass beneath that of the ellipsoid in these two parts.

Fig 15.



Hence none of the diameters of the ellipsoid projected in the angular space M A N, M′ A N′ will be smaller than the diameter M M′ of the circular sections, which is equal to the mean axis of the ellipsoid, the length of the radii vectores corresponding to this part of the surface has therefore for limits, on one side the semi-major axis, and on the other the semi-mean axis.

In the same way it might be shown that the length of the radii vectores which give the measure of the velocities of the second luminous beam, is comprised between the semi-mean axis and the semi-minor axis. Now, in the case represented by fig. 15, where the minor axis of elasticity divides the acute angle of the two optic axes, and the major axis the obtuse angle, there is a greater difference between the minor axis and the mean axis than between this latter and the major axis, as we see by the expression $\frac{c}{a}\sqrt{\frac{a^2-b^2}{b^2-c^2}}$ for the tangent of the angle which the planes of the circular sections make with the major axis; for this angle being less than 45° by hypothesis, we have $c^2(a^2-b^2) \angle a^2(b^2-c^2)$, or nearly $(a-b) \angle (b-c)$, suppressing the common factors $c(a+b)$ and $a(b+c)$ as being sensibly equal.

The reasonings we have entered into for the ellipsoid may be applied just as well to the surface of elasticity, which gives by the axes of its diametral sections the true directions of the luminous vibrations, and consequently those of their planes of polari-

zation perpendicular to these vibrations Only, the velocities which we should then consider would no longer be those of the luminous rays, but those of the waves measured on the normal to their surface, and the two planes forming the acute and obtuse dihedral angles which the planes of polarization divide each into two equal parts, instead of passing through the luminous ray and the optic axes properly so called, would be drawn along the normal to the wave and the normals to the two circular sections of the surface of elasticity The tangent of the inclination of these sections to the semi major axis (a) is equal to $\sqrt{\dfrac{a^2-b^2}{b^2-c^2}}$, an expression less than unity when a^2-b^2 is $< b^2-c^2$, and greater when (a^2-b^2) is $> (b^2-c^2)$, or, which comes nearly to the same thing, when $(a-b) > (b-c)$ In this second case, the angle of the two circular sections, or of their normals which contains the minor axis (c), is therefore obtuse, whilst it is acute in the first case

Hence the waves whose planes of polarization are comprised in the acute angle between the two planes drawn along the normal to the wave, and the normals to the planes of the circular sections are those whose velocities of propagation vary between the narrowest limits, whilst the velocities of the waves whose planes of polarization pass within the obtuse dihedral angle undergo more extensive variations It is therefore natural to call the rays corresponding to the former *ordinary rays*, and those of the other waves *extraordinary rays*, as M Biot and Sir David Brewster have done

Particular case where there would no longer be any reasons for giving the name of ordinary ray to one of the two beams rather than to the other

A case is conceivable in which, the two beams undergoing variations of velocity equally extensive, there would no longer be any reason for giving the name of ordinary beam to one rather than the other, this would be the case if the two optic axes were perpendicular to each other, because then we should have $\dfrac{c}{a}\sqrt{\dfrac{a^2-b^2}{b^2-c^2}}=1$, or $c^2(a^2-b^2)=a^2(b^2-c^2)$, which supposes that $(a-b)$ is very nearly equal to $(b-c)$, since we may

suppress the factors $c^2(a+b)$ and $a^2(b-c)$ without sensibly altering the equation, so long as (a) does not differ much from (c), that is to say, so long as the double refraction has not a very great energy.

When we have given the angle between the two optic axes, it is sufficient to know two of three constants, a, b, c, *in order to determine the third.*

It is sufficient to know (a) and (c), that is, the greatest and least velocity of light in the crystal, together with the angle between the two optic axes, to determine the other semi-axis (b), since the tangent of half this angle is equal to $\frac{c}{a}\sqrt{\frac{a^2-b^2}{b^2-c^2}}$ a known function of three quantities, a, b and c. It was by pursuing this method that I calculated, with the elements of double refraction given by M. Biot for topaz, the variations of velocity which the ordinary beam must undergo in it, before seeking to verify them by experiment, and I found them very nearly such as the calculation had given me. The theory also pointed out to me in what direction the ordinary beam had the most different velocities.

For topaz it is the smallest axis of the surface of elasticity or of the ellipsoid which divides into equal parts the acute angle of the two optic axes, and the two limits of the velocities of the ordinary ray are (a) and (b); now the ordinary beam has the velocity (a) when it is parallel to the axis of (y), since (a) is the greatest radius vector of the perpendicular diametral section made in the ellipsoid, and since the corresponding plane of polarization, that is perpendicular to the radius vector (a), is also that of the ordinary beam, as passing within the acute angle of the two optic axes. The velocity of this same beam becomes equal to (b) when the light traverses the crystal parallel to the axis of (x), because then the diametral plane perpendicular to this direction cuts the ellipsoid in an ellipse whose greatest radius vector is (b). Moreover, the plane perpendicular to (b), or the corresponding plane of polarization, belongs to the ordinary refraction; for it is also contained in the acute angle formed by the two planes drawn along the luminous ray and each of the optic axes, a dihedral angle which then becomes equal to zero, these two planes becoming coincident with that of the two optic

axes Hence the theory announced that the ordinary beam must traverse the crystal, successively along the direction which bisects the obtuse angle between the two axes, and perpendicularly to their plane, in order to undergo the most perceptible variations of velocity, and in accordance with this indication it was that I made the first experiment, by which I have proved the existence of these variations

I have also in my experiments particularly endeavoured to assure myself that the velocity of propagation of luminous waves depends solely on the direction of their vibrations, or on the plane of polarization in the crystal, and that so long as this plane does not change, the velocity of the rays remains constant, whatever moreover may be their direction Diffraction afforded me very delicate methods for perceiving the slightest differences of velocity In truth, topaz is the only crystal on which I have operated as yet, but I have sufficiently varied and multiplied my observations to assure myself at least that this theorem is rigorously exact in topaz, and it must be supposed by analogy that it is equally true for all other bi axal crystals Besides, without giving a complete demonstration of it, the mechanical considerations which I have set forth on this subject establish in its favour very strong theoretical probabilities

Reflections on the probabilities presented by the Theory explained in this Memoir

The theorem which I have given, so admissible from its very simplicity, the mechanical definition of luminous vibrations deduced from the laws of interference of polarized rays, and the supposition that the homologous lines of crystallization are parallel throughout the whole extent of the refracting media which we have considered, are the three hypotheses, I might say the three principles, on which rests the theory of double refraction set forth in this memoir If we had only to calculate one phænomenon, such as that of interferences, which depends solely on the nature of luminous vibrations, their definition would have sufficed for the explanation of the facts But double refraction being the consequence of a particular constitution of the refracting medium, it was absolutely necessary to define this constitution, embodying however in the definition only that which was necessary for the explanation of the phænomenon

The theory which we have adopted, and the very simple constructions we have deduced from it, present this remarkable character, that all the unknown quantities are determined at the same time by the solution of the problem. We find at the same time the velocity of the ordinary ray, that of the extraordinary ray, and their planes of polarization. Philosophers who have studied with attention the laws of nature will feel that this simplicity, and these intimate relations between the various parts of the phænomenon, offer the greatest probabilities in favour of the theory by which they are established

A long time before having conceived it, and by the sole consideration of facts, I had perceived that the true explanation of double refraction could not be discovered without explaining at the same time the phænomenon of polarization which constantly accompanies it; thus it was after having found what mode of vibration constituted the polarization of light, that I first caught sight of the mechanical causes of double refraction. It appeared to me still more evident that the velocities of the ordinary and extraordinary beams ought to be in some sort the two roots of one and the same equation; I have never been able to admit for a single instant the hypothesis, according to which there would be two different media, the refracting body and the æther which it contains, by one of which the extraordinary rays are transmitted, by the other the ordinary ones; in fact, if these two media could transmit separately the luminous waves, one does not see why the two velocities of propagation should be rigorously equal in the greater number of refracting bodies, and why prisms of glass, water, alcohol, &c. should not thus divide the light into two distinct beams.

We have supposed it to be the same vibrating medium which, in bodies endowed with double refraction, propagates the ordinary and extraordinary waves, but without specifying whether the molecules of the body participate in the luminous vibrations, or whether these latter were alone propagated by the æther contained in the body; our theory is equally well conciliated with either hypothesis. It is indeed more easy to comprehend, in the first case, how the elasticity of one and the same refracting medium may vary with the direction along which the molecular displacements take place; but it is conceivable also, in the second, that the molecules of the body must influence the mutual

dependence of the strata of æther between which they are situated and that they may be arranged in such a manner that they weaken this mutual dependence or elasticity of the æther, more in one direction than another.

The phænomenon of dispersion proves that the rays of different colours or waves of different lengths do not traverse bodies with the same velocity, which arises doubtless from this, that the elasticity put in play by the luminous waves varies with their length. When the sphere of activity of the molecular actions is supposed infinitely small with regard to the extent of an undulation, analysis shows that the elasticity by which the waves are propagated does not vary with their breadth (*largeur*), but this is no longer true when the mutual dependence of the molecules extends to a sensible distance with regard to the length of an undulation. It is easy to prove that, in this case, the elasticity put in play is rather less for narrow waves (*étroites*) than for broader waves, and that consequently the former must be propagated rather more slowly than the second, conformably to experiment.* It hence results that the three semi axes *a*, *b*, *c*, which represent the square roots of the elasticities put in play by the parallel vibrations, or the corresponding velocities of propagation, must vary a little for waves of different breadths (*largeurs*) according to the theory as well as to experiment; now it is possible that this variation may not take place according to the same ratio between the three axes, in which case the angle formed by the two circular sections of the ellipsoid with each other, and therefore the angle between the two optic axes, may no longer be the same for rays of different colours, as Sir David Brewster and Sir J. Herschel have remarked in the greater number of bi axal crystals.

The phænomenon of dispersion has perhaps yet other causes than that which we have just indicated; but whatever they may be, we must still conclude from the observations of these two skilful experimenters, that the lengths of the semi axes (*a*), (*b*), (*c*) do not vary in the same ratio for waves of different breadths (*largeurs*) in crystals where the optic axes change their direction

* The demonstration of this consequence of the theory forms the object of Note II at the end of the memoir. [There are no notes at all to the memoir. Dr of Lloyd in his Report on Physical Optics to the British Association has remarked that this demonstration is more than once referred to by the author as contained in a note appended to his memoir on double refraction. The note, however, probably by some oversight, has never been printed.—TRANS.]

with the nature of the luminous rays; this is at least the only explanation which can be given of it according to the theory set forth in this memoir.

The following Note refers to page 239, sentence beginning "M. de Laplace, considering double refraction in the emission-point of view," &c. &c.

With reference to this, Prof. Lloyd has the following note in his 'Report on Physical Optics to the British Association' (Fourth Report, 1834, p 379).

"Fresnel states, in the commencement of his 'Memoir on Double Refraction,' that Laplace had derived the velocity of the extraordinary ray in uni-axal crystals from the hypothesis of a *resultant force* acting in a direction perpendicular to the optic axis, and varying as the square of the sine of the angle which the ray makes with that line. I have not been able to discover, in any of Laplace's writings, the discussion thus adverted to."

Laplace's investigation is contained in the second volume of the *Mémoires de Physique et de Chimie de la Société d'Arcueil*, page 111–143, from which the following extracts may probably interest the reader. After referring to the principle of least action, Laplace goes on to the particular hypothesis above alluded to:—

"Mais une condition à remplir dans le cas de la réfraction extraordinaire, est que la vitesse du rayon lumineux dans le cristal, soit indépendante de la manière dont il y est entré, et ne dépende que de sa position par rapport à l'axe du cristal, c'est-à-dire de l'angle que ce rayon forme avec une ligne parallèle à l'axe. En effet, si l'on imagine une face artificielle perpendiculaire à l'axe, tous les rayons intérieurs également inclinés à cet axe le seront également à la face, et seront évidemment soumis aux mêmes forces au sortir du cristal. Tous reprendront leur vitesse primitive dans le vide; la vitesse dans l'intérieur est donc pour tous la même. En partant de ces données, je parviens aux deux équations différentielles que donne le principe de la moindre action, et dans lesquelles la vitesse intérieure est une fonction indéterminée de l'angle que le rayon réfracté forme avec l'axe du cristal. J'examine ensuite les deux cas les plus simples auxquels je me borne, parce qu'ils renferment les lois de réfraction jusqu'à présent observées. Dans le premier cas, le carré de la vitesse de la lumière est augmenté dans l'intérieur du milieu, d'une quantité constante On sait que ce cas est celui des milieux diaphanes ordinaires et que cette constante exprime l'action du milieu sur la lumière. Les deux équations précédentes montrent qu'alors les rayons incident et

réfracté sont dans un même plan perpendiculaire à la surface du milieu, et que les sinus des angles qu'ils forment avec la verticale sont constamment dans le même rapport. Après ce premier cas, le plus simple est celui dans lequel l'action du milieu sur la lumière est égale à une constante plus un terme proportionnel au carré du cosinus de l'angle que le rayon réfracté forme avec l'axe; car cette action devant être la même de tous les côtés de l'axe, elle ne peut dépendre que des puissances paires du sinus et du cosinus de cet angle. L'expression du carré de la vitesse intérieure est alors de la même forme que celle de l'action du milieu. En la substituant dans les équations différentielles du principe de la moindre action, je détermine les formules de réfraction relatives à ce cas, et je trouve qu'elles sont identiquement celles que donne la loi d'Huygens; d'où il suit que cette loi satisfait à la fois au principe de la moindre action et à la condition que la vitesse intérieure ne dépende que de l'angle formé par l'axe et par le rayon réfracté; ce qui ne laisse aucun lieu de douter qu'elle est due à des forces attractives et répulsives dont l'action n'est sensible qu'à des distances insensibles. * * * * * *

Dans l'intérieur du cristal, la vitesse ne dépend que des angles formés par la direction du rayon et par des axes fixes dans l'intérieur du corps. Supposons qu'il n'y ait qu'un axe, et que V soit l'angle formé par cette axe et par la direction du rayon réfracté; *v* (the velocity of the refracted ray in the interior of the crystal) sera fonction de V. * * * * Ces deux équations donneront la loi de la réfraction extraordinaire lorsque *v* sera donné en fonction de cos V, et réciproquement. De plus, elles satisferont à la condition que la vitesse du rayon lumineux dans l'intérieur du cristal ne dépende que de sa position par rapport à l'axe du cristal. Nous observerons ici que non seulement *v* doit être fonction de cos V, mais qu'il ne doit dépendre que des puissances paires de cos V, car nous avons observé ci-dessus que la vitesse *v* est la même pour tous les rayons qui forment avec l'axe le même angle.

It will be seen from these extracts that Laplace supposes the action to be proportional to the square of the *cosine* of the angle made by the refracted ray with the axis, and not to that of the *sine* as Fresnel states.—TRANSLATOR'S NOTE.

Article VII

On Interpolation applied to the Calculation of the Coefficients of the Development of the disturbing Function. By U.-J. Le Verrier.

[From a separate Treatise Paris, 1841.]

1 The determination of the periodical and secular inequalities of the planets is carried back, by the theory of the variations of the arbitrary constants, to the investigation of the development of certain expressions which are functions of the time and elements of the orbits These functions are reduced into a series proceeding according to the sines and the cosines of the different multiples of the mean longitudes. And when the numerical values of the coefficients of the principal terms of these series have been calculated, we easily arrive at the knowledge of the perturbations themselves of the planets.

To obtain one of the coefficients in particular, the *Mécanique Céleste* supposes that we commence by forming its analytical expression in function of the disturbing mass, of the semi-major axes, of the excentricities and the inclinations of the orbits of the two planets under consideration, in function of the longitudes of their perihelia and their nodes. This algebraical development, which rests wholly on the employment of Taylor's theorem, offers no other difficulty than the length of the literal calculations. But this difficulty is immense. Thus, notwithstanding all the care of Burckhardt, the analytical expression which he determined for that part of the great inequality of Jupiter and Saturn, which depends on the fifth powers of the excentricities and inclinations, was found to contain some inaccuracies. Thus Mr. Airy, to obtain the expression of the inequality of a long period which Venus introduces into the mean movement of the Earth, had to go through a very long process; and other geometers, starting from the same data, have been unable to find again exactly the same results. This inequality is however only of the fifth order. To what difficult labour should we then be led by the method of the algebraic developments, if we recognized the necessity of having regard, in some theories, to inequalities of a higher order?

We might, it is true, attain to the seventh order, by means of

the investigation which M Binet presented in 1812 to the Academy of Sciences and in which he states the error which had crept into that part of the great inequality of Jupiter which depends on the fifth order But by the extent of that work, we may easily judge that all hope of further advancing the approximations by this path must be lost

2 Interpolation seems then alone capable of furnishing the coefficients corresponding to high multiples of the mean longitudes The calculations certainly are still very long but they are not impracticable, like those which result from algebraic developments The disturbing function depends on the mean longitudes of the disturbing planet and the disturbed planet and these two longitudes, in the development of the function, may be considered as independent variables By attributing to these variables particular values, we obtain numerical values of the disturbing function, a limited number of them serves for the determination of a similar number of the coefficients of the development effected according to the sines and cosines of the multiples of the mean longitudes

We may employ, according to the well known formulæ, all the numerical values of the function corresponding to mean longitudes equidistant from one another of an arc exactly dividing the circumference But this step is subject to an inconvenience which cannot always be easily avoided If, in fact, we know, for a given number of values of the disturbing function, what is the rank of the first term which is regarded as negligeable, frequently nothing indicates whether this term is really small enough to be neglected without altering the degree of accuracy which it is important to obtain And if we perceive after having effected the greater part of the calculations, that we should have preserved only two terms more, we are obliged either immediately to double the number of the numerical values employed or to recommence the whole work, *which will often present less inconvenience*

To avoid these difficulties I propose the employment of a method of interpolation in which I satisfy the following condition —

Having already executed the calculations necessary for the determination of n *of the coefficients if we find that* p *others must be preserved this may be done without having executed more calculations than if we had had regard, from the beginning, to the* (n + p) *coefficients*

3 Let us designate by R the disturbing function, by l and l' the mean longitudes of the disturbed planet and of the disturbing planet, and let us put

$$\left.\begin{aligned}R=C+\Sigma\,(i,i')\sin\,(i\,l+i'\,l')\\ +\Sigma\,[i,i']\cos(i\,l+i'\,l'),\end{aligned}\right\}\quad (1)$$

C being a constant The indices i and i' may have all entire positive and negative values from zero to infinity But it is sufficient also to give to one of them, i for example, positive values only This we shall suppose

Let us first leave the longitude l' constant, and give successively to the longitude l the equidistant values $0, \alpha, 2\,\alpha, 3\,\alpha, \ldots, p\,\alpha$, *$\alpha$ being an arc which does not exactly divide the circumference* If, for one of these arcs $l=p\,\alpha$, we attribute successively to i the values $0, 1, 2, 3, \ldots, i$, the corresponding numerical value R_p of the disturbing function may be written,—

$$\begin{aligned}R_p=C&+\Sigma\{(0,i')\sin i'\,l'+[0,i']\cos i'\,l'\}\\ &+\Sigma\{(1,i')\sin\,(p\,\alpha+i'\,l')+[1,i']\cos\,(p\,\alpha+i'\,l')\}\\ &\ldots\\ &+\Sigma\{(i,i')\sin\,(i\,p\,\alpha+i'\,l')+[i,i']\cos\,(i\,p\,\alpha+i'\,l')\},\end{aligned}$$

the sign Σ having now only relation to i' And in developing the sines and cosines, we shall have,

$$\left.\begin{aligned}R_p=C&+\Sigma\{(0,i')\sin i'\,l'+[0,i']\cos i'\,l'\}\\ &+\Sigma\{(1,i')\cos i'\,l'-[1,i']\sin i'\,l'\}\sin p\,\alpha\\ &+\Sigma\{(1,i')\sin i'\,l'+[1,i']\cos i'\,l'\}\cos p\,\alpha\\ &\ldots\\ &+\Sigma\{(i,i')\cos i'\,l'-[i,i']\sin i'\,l'\}\sin i\,p\,\alpha\\ &+\Sigma\{(i,i')\sin i'\,l'+[i,i']\cos i'\,l'\}\cos i\,p\,\alpha\end{aligned}\right\}\quad (2)$$

By only changing p in this expression, the quantities contained under the signs Σ remain constant We shall designate them more simply by putting

$$\left.\begin{aligned}C+\Sigma\{(0,i')\sin i'\,l'+[0,i']\cos i'\,l'\}&=B_0,\\ \Sigma\{(i,i')\cos i'\,l'-[i,i']\sin i'\,l'\}&=A_i,\\ \Sigma\{(i,i')\sin i'\,l'+[i,i']\cos i'\,l'\}&=B_i,\end{aligned}\right\}\quad (3)$$

and by giving successively to p the values $0, 1, 2, \ldots$ up to $2\,i$, the expression (2) will furnish the following relations —

$$\left.\begin{array}{llll} B_0 & +B_1+B & + & +B=R_0 \\ & A_1 \sin \alpha + A_2 \sin 2\alpha + & & \left.\begin{array}{l} +A_i \sin i\alpha \\ +B \cos i\alpha \end{array}\right\} = R_1 \\ +B_0+B_1 \cos \alpha + B \cos 2\alpha + & & & \\ & A_1 \sin 2\alpha + A_2 \sin 2\,2\alpha + & & \left.\begin{array}{l} +A_i \sin i.2\alpha \\ +B \cos i.2\alpha \end{array}\right\} = R_2, \\ +B_0+B_1 \cos 2\alpha + B \cos 2\,2\alpha + & & & \\ & A_1 \sin 2i\alpha + A_2 \sin 2\,2i\alpha + & & \left.\begin{array}{l} +A \sin i\,2i\alpha \\ +B \cos i.2i\alpha \end{array}\right\} = R_{2i} \\ +B_0+B_1 \cos 2i\alpha + B \cos 2\,2i\alpha + & & & \end{array}\right\} \quad (4)$$

in all $(2i+1)$ equations to calculate the $(2i+1)$ constants B_0, A_1 B_1 A_2, B_2 up to A_i and B_i. We have to resolve these equations

4 Let us substitute exponentials for the trigonometric lines and put in order to simplify

$$e^{\alpha\sqrt{-1}} = x \quad (5)$$

The second of the equations (4) for example by remarking that

$$\left.\begin{array}{l} \cos z = \dfrac{e^{z\sqrt{-1}} + e^{-z\sqrt{-1}}}{2} \\ \sin z = \dfrac{e^{z\sqrt{-1}} - e^{-z\sqrt{-1}}}{2\sqrt{-1}}, \end{array}\right\} \quad (6)$$

will become

$$\left.\begin{array}{r} 2\,B_0\sqrt{-1}+(B_1\sqrt{-1}+A_1)\,x^1+(B_2\sqrt{-1}+A_2)\,x^2\ldots \\ +(B_i\sqrt{-1}+A_i)\,x^i \\ +(B_1\sqrt{-1}-A_1)x^{-1}+(B_2\sqrt{-1}-A_2)\,x^{-2}\ldots \\ +(B_i\sqrt{-1}-A_i)\,x^{-i} \end{array}\right\}=2\,R_1\sqrt{-1}.$$

This equation, if for brevity sake we make

$$\left.\begin{array}{l} B_i\sqrt{-1}+A_i=a_i \\ B_i\sqrt{-1}-A_i=b_i \end{array}\right\}\quad\ldots\ldots\ldots\ldots\quad(7)$$

may be written

$$\left.\begin{array}{r} 2\,B_0\sqrt{-1}+(a_1\,x^1+b_1\,x^{-1})+(a_2\,x^2+b_2\,x^{-2})\ldots \\ +(a_i\,x^i+b_i\,x^{-i}) \end{array}\right\}=2\,R_1\sqrt{-1},$$

and by treating similarly all the equations of the system (4.), they will become—

$$\left.\begin{array}{l} 2\,B_0\sqrt{-1}+(a_1+b_1)+(a_2+b_2)\quad\ldots+(a_i+b_i)=2\,R_0\sqrt{-1}, \\ \left.\begin{array}{r} 2\,B_0\sqrt{-1}+(a_1\,x^1+b_1\,x^{-1})+(a_2\,x^2+b_2\,x^{-2})+(a_3\,x^3+b_3\,x^{-3})\ldots \\ +(a_i\,x^i+b_i\,x^{-i}) \end{array}\right\}=2\,R_1\sqrt{-1}, \\ \left.\begin{array}{r} 2\,B_0\sqrt{-1}+(a_1\,x^2+b_1\,x^{-2})+(a_2\,x^{2.2}+b_2\,x^{-2.2})+(a_3\,x^{3.2}+b_3\,x^{-3.2})\ldots \\ +(a_i\,x^{i.2}+b_i\,x^{-i.2}) \end{array}\right\}=2\,R_2\sqrt{-1}, \\ \left.\begin{array}{r} 2\,B_0\sqrt{-1}+(a_1\,x^3+b_1\,x^{-3})+(a_2\,x^{2.3}+b_2\,x^{-2.3})+(a_3\,x^{3.3}+b_3\,x^{-3.3})\ldots \\ +(a_i\,x^{i.3}+b_i\,x^{-i.3}) \end{array}\right\}=2\,R_3\sqrt{-1}, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \left.\begin{array}{r} 2\,B_0\sqrt{-1}+(a_1x^{2i}+b_1\,x^{-2i})+(a_2\,x^{2.2i}+b_2\,x^{-2.2i})+(a_3\,x^{3.2i}+b_3\,x^{-3.2i})\ldots \\ -(a_i\,x^{i.2i}+b_i\,x^{-i.2i}) \end{array}\right\}=2\,R_{2i}\sqrt{-1}. \end{array}\right\}\quad\ldots\quad(8)$$

When by means of these equations we shall have deduced the values of a_i and b_i, the equations (7) will give the values of A_i and B_i

5 It is indispensable to commence by elimination of the quantities a b which have the lowest index. Thus on arriving at a_i and b_i, if their coefficients cannot be neglected we may carry on the calculations without having to resume any one of the preceding operations and on executing only those which we should have had to do, if from the first we had wished to take account of the highest indices. Let us eliminate first B_0 by withdrawing from each equation that which follows it, and put

$$\left.\begin{array}{l} (R_0 \quad -R_1)=(1)_1 \\ (R_1 \quad -R_2)=(1)_2 \\ (R_2 \quad -R_3)=(1)_3, \\ \\ (R_{i-1}-R_i)=(1)_i \end{array}\right\} \qquad (9)$$

the index in the parenthesis of the second members being the lowest index of the quantities a and b in the equations which will follow and the index outside the parenthesis corresponding to the rank of the quantities (9) in these equations. The equations (8) will give

$$\left.\begin{array}{ll} \left.\begin{array}{l} (a_1 \quad -b_1 x^{-})(1-x)+(a \ -b \ x^{-})(1-x^{-}) \\ \quad -(a_3-b_3 x^{-})(1-x^{3})- \quad +(a \ -b \ x^{-})(1-x^{2}) \end{array}\right\} = 2(1)_1\sqrt{-1} \\ \left.\begin{array}{l} (a_1 x \ -b_1 x^{-})(1-x)+(a \ x^{-}-b \ x^{-}{}^{-})(1-x) \\ \quad +(a_3 x^{3}-b_3 x^{-}{}^{-})(1-x^{3})+ \quad +(a \ x^{2}-b \ x^{-}{}^{-})(1-x) \end{array}\right\} = 2(1) \ \sqrt{-1} \\ \left.\begin{array}{l} (a_1 x^{2}-b_1 x^{-3})(1-x)+(a \ x^{-} \ -b \ x^{-}{}^{-3})(1-x^{-}) \\ \quad -(a_3 x^{3} \ -b_3 x^{-3.3})(1-x^{3}) \quad +(a \ x^{2} \ -b \ x^{-3})(1-x^{2}) \end{array}\right\} = 2(1)_3\sqrt{-1}, \\ \left.\begin{array}{l} (a_1 x^{3}-b_1 x^{-})(1-x)+(a \ x^{-} \ -b \ x^{-} \ )(1-x^{-}) \\ \quad +(a_3 x^{3.3}-b_3 x^{-3})(1-x^{3}) \quad +(a \ x^{3}-b \ x^{-} \ )(1-x^{2}) \end{array}\right\} = 2(1)_4\sqrt{-1}, \\ \qquad \&c. \end{array}\right\} \qquad (10)$$

we have $2i$ equations of this kind

6. We shall be able to eliminate a_1, then b_1 between these equations. But the relations which would contain in general b_k without a_k would be wanting in symmetry, and we shall make no use of them. It is preferable to eliminate at once a_1 and b_1, and to form $(2i-2)$ new equations which will be deduced from the preceding combined three by three as follows

Let us designate by M_1, M_2 and M_3 the first members of three of the equations (10.) taken consecutively. On making the combination

$$M_1+M_3-M_2(x+x^{-1}), \quad . \quad . \quad . \quad . \quad . \quad (11.)$$

of these quantities, the terms in a_1 and b_1 will disappear. Any one may convince himself of this by substituting in this formula, in place of M_1, M_2 and M_3, the following expressions of the parts dependent on a_1 and on b_1 in the first members of three of the consecutive equations

$$\begin{aligned}&(a_1 x^{k} \quad -b_1 x^{-k-1})\ (1-x),\\&(a_1 x^{k+1}-b_1 x^{-k-2})\ (1-x),\\&(a_1 x^{k+2}-b_1 x^{-k-3})\ (1-x).\end{aligned}$$

Let us take three other corresponding terms in the first members of the same equations; for example the terms,

$$\begin{aligned}&[a_i x^{ik} \qquad -b_i x^{-i(k+1)}]\ (1-x^i),\\&[a_i x^{i(k+1)}-b_i x^{-i(k+2)}]\ (1-x^i),\\&[a_i x^{i(k+2)}-b_i x^{-i(k+3)}]\ (1-x^i).\end{aligned}$$

By submitting them to the combination (11.), it will easily be found that expressions of the following form will result,

$$[a_i x^{ik}-b_i x^{-i(k+3)}]\ (1-x^{i+1})\ (1-x^i)\ (1-x^{i-1}),$$

and this formula will serve to construct the first members of the new equations by giving to the index i values from 2 up to i, and to the quantity k values from 0 up to $(2i-3)$

With respect to the second members of these new equations, we shall designate their real parts by $(2)_1$, $(2)_2$, $(2)_3$, . . ., and by remarking that according to the first of the conditions (6.),

$$x^1+x^{-1}=2\cos\alpha,$$

we shall find

$$\left.\begin{aligned}&(2)_1=(1)_1+(1)_3-(1)_2\,2\cos\alpha,\\&(2)_2=(1)_2+(1)_4-(1)_3\,2\cos\alpha,\\&(2)_3=(1)_3+(1)_5-(1)_4\,2\cos\alpha,\\&\&c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (12.)$$